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FINITE ELEMENT ANALYSIS OF FLUID-FILLED ELASTIC PIPING SYSTEMS

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G.C. Everstine, M.S. Marcus, and A.J. Quezon
David W. Taylor Naval Ship R&D Center
Bethesda, Maryland 20084

Dedicated to the Memory of James M. McKee (1942-1983)

SUMMARY

Two finite element procedures are described for predicting the dynamic response of general 3-D fluid-filled elastic piping systems. The first approach, a low frequency procedure, models each straight pipe or elbow as a sequence of beams. The contained fluid is modeled as a separate coincident sequence of axial members (rods) which are tied to the pipe in the lateral direction. The model includes the pipe hoop strain correction to the fluid sound speed and the flexibility factor correction to the elbow flexibility. The second modeling approach, an intermediate frequency procedure, follows generally the original Zienkiewicz-Newton scheme for coupled fluid-structure problems except that the velocity potential is used as the fundamental fluid unknown to symmetrize the coefficient matrices. From comparisons of the beam model predictions to both experimental data and the 3-D model, the beam model is validated for frequencies up to about two-thirds of the lowest fluid-filled lobar pipe mode. Accurate elbow flexibility factors are seen to be crucial for effective beam modeling of piping systems.

INTRODUCTION

The vibrations that occur in fluid-filled piping systems are of interest in a variety of industrial, aircraft, and shipboard applications. The interesting dynamic behavior includes both water hammer (a transient phenomenon) and the steady-state (time-harmonic) vibrations caused by unbalanced rotating machinery such as pumps, for example.

Over 30 years ago, Callaway, Tyzzer, and Hardy (Ref. 1) recognized in their experimental work the importance of the coupling between the vibrations of the liquid and the pipe wall, even for nominally straight pipes. Since then, a number of investigators have proposed various techniques of mathematical modeling for design and analysis purposes.

Most of these techniques have been restricted to straight pipes. Some recent work, for example, was reported by El-Raheb (Ref. 2,3), who analyzed the acoustic propagation in a perfect, finite length, fluid-filled, thin elastic cylindrical shell. El-Raheb obtained eigenfunction expansions for Koiter's consistent shell equations and the Helmholtz equation governing the fluid field.

There have also been some finite element analyses of 2-D fluid cavities of general shape (Ref. 4,5). These approaches, however, avoid the fluid-structure coupling by requiring as input the impedance of the pipe wall.

There has been relatively little fluid-structure interaction work involving general 3-D piping systems containing joints such as elbows and tees. The first such analyses were probably performed by Davidson, Smith, and Samsury (Ref. 6,7). They recognized that a simplified beam model should suffice for the relatively low frequencies which are often of interest. Low frequency dynamic behavior is characterized by pipes which respond only in their beam (rather than lobar) modes and by fluid wavelengths which are large compared to the pipe diameter. Thus, for such situations, the fluid wave propagation through the pipes is essentially planar. This procedure, although fully general in concept, was implemented in a special purpose computer program limited as to the generality and size of problems which could be handled.

The same assumptions formed the basis of a finite element procedure developed by Howlett (Ref. 8) for aircraft hydraulic systems. This procedure modeled the fluid inside the pipe as a beam having zero bending stiffness. Elbows and tees were not modeled explicitly. Instead, compatibility at a joint (elbow or tee) was enforced as an additional constraint requiring the conservation of fluid mass passing through the joint. The Howlett analyses, however, apparently omitted two essential ingredients: (1) the correction to the fluid sound speed to account for the elasticity of the pipe walls, and (2) the flexibility factor correction for the elbows to account for the fact that curved pipes are considerably more flexible than straight pipes of the same cross section (Ref. 9).

More recently, the transfer matrix approach was used by El-Raheb (Ref. 10) to calculate the beam-type dynamic response of 3-D multiplane piping systems consisting of straight sections and elbows. One of El-Raheb's conclusions was that the one-dimensional acoustic assumption is valid for frequencies up to about one-half the frequency of the lowest acoustic mode having two waves around the circumference (n=2).

Schwirian and Karabin (Ref. 11) developed another finite element procedure which was similar to Howlett's (Ref. 8) except that the fluid inside an elbow was apparently modeled with a single straight axial member ("spar" element) with fictitious properties assigned to simulate properly the fluid mass and compressibility. This model also included the pipe hoop strain correction to the fluid sound speed and the flexibility factor correction to the elbow flexibility. No experimental validation of the model was included in the paper.

One additional modeling procedure was formulated by Hatfield and Wiggert (Ref. 12). They used a transfer function approach involving separate analyses of liquid and solid components, followed by synthesis of the component solutions. The scheme was validated for planar piping systems by comparison with experimental data.

In general, beam models of fluid-filled elastic piping systems are very attractive because of their simplicity. Finite element approaches have the added feature of allowing essentially arbitrary specification of geometry, boundary conditions, loadings, and output requests. In addition, finite element models of piping systems can easily be combined with models of the support structure.

In this paper, we will develop further the finite element approach for low frequency prediction by combining ideas from the papers just mentioned. Our model is mathematically equivalent to the Davidson-Smith-Samsury model (Ref. 6,7), a non-finite element approach. Our modeling scheme is similar to those of Howlett (Ref. 8) and Schwirian and Karabin (Ref. 11) except that we model elbows explicitly by a polygonal set of beam elements for the pipe and axial members for the fluid. We will also show the importance of assigning the correct flexibility factors to the elbows. (In general, most classical means of calculating flexibility factors are not adequate for elbows with straight pipe extensions at the ends (Ref. 13,14)). The modeling approach will be validated in two ways. First, for a simple planar system, comparisons will be made with both experimental data and a general 3-D finite element model which models the pipe as a shell and the contained fluid with 3-D fluid finite elements. Second, for a complex non-planar system, comparisons will be made between a beam solution and a general 3-D finite element solution. Since the latter includes the essential physics of the fully-coupled problem, it provides a good test for the approximate models.

The 3-D finite element solutions to be presented use the generally classical procedures which evolved from the work of Zienkiewicz and Newton (Ref. 15). In our analyses, we make use of the recent improvement (Ref. 16) which shows how to obtain symmetric matrix equations.

LOW FREQUENCY BEAM MODEL

For low frequency dynamic excitation of fluid-filled elastic piping systems, the pipes respond only in their beam (rather than lobar) modes, and the wave propagation in the fluid column is essentially planar. It is assumed either that the fluid is initially at rest, or that the average flow speed is so small compared to the sound speed that the acoustic response is unaffected. The fluid-structure coupling is assumed to occur only at pipe bends and other joints. Thus, the fluid is allowed to slide without friction in straight sections of pipe. The circular pipe cross section is assumed to remain circular. The equation satisfied by either the fluid pressure or the axial component of fluid displacement is the scalar wave equation; thus the fluid can be modeled by an axial structural member (rod).

Finite element models are prepared using the following procedure: Beam elements are used to model both the straight sections and the elbows. If straight beam elements are used, a minimum of three elements is recommended (on the basis of some numerical testing) for 90-degree bends, regardless of the spacing of grid points in adjacent straight sections. In straight sections, the grid point spacing is dictated by the need for accurate normal modes of vibration in the frequency range of interest.

Since a pipe bend is more flexible than an equivalent length of straight pipe, the moments of inertia for the beam elements in each elbow should be divided by the appropriate flexibility factor. For piping systems with straight sections not significantly longer than the arc lengths of the elbows, the elbow stiffness plays an important role in the dynamic response and must be accurately modeled. Thus the flexibility factors assigned to each elbow should apply to the elbow as it is configured in the piping system. In particular, the flexibility factors for 90-degree elbows with straight pipe extensions are sensitive to the length of those extensions (Ref. 13,14). As shown by Quezon and Everstine (Ref. 14), idealized approaches such as those used in the ELBOW computer program (Ref. 17) are generally not adequate for predicting the flexibility factors of 90-degree elbows with straight pipe extensions.

For the acoustic fluid inside the pipe, a duplicate set of grid points is defined to coincide with the pipe grid points. The fluid is modeled with elastic rod elements (sometimes called spars), which are equivalent to beam elements with zero flexural and torsional stiffness. These elements are assigned the actual mass density ρ for the fluid and a Young's modulus E given by

$$E = \rho c^2 \tag{1}$$

In Equation (1), c, the effective sound speed in a fluid column contained in an elastic circular thin-walled pipe, is obtained from

$$c = \sqrt{B/\rho} \;/\; \sqrt{1 + BD/E_s t} \tag{2}$$

where B is the fluid bulk modulus of elasticity, ρ is the fluid mass density, D is the mean diameter of the pipe, E_s is the Young's modulus of the pipe material, and t is the pipe wall thickness. Since the numerator in Equation (2) is the actual speed of sound in the fluid, the denominator (which is always greater than unity) is the corrective factor which accounts for the elasticity of the pipe. This correction is well known (Ref. 18); according to Krause, Goldsmith, and Sackman (Ref. 19), this relation was first derived by Joukowsky (Ref. 20) over 80 years ago. Equations (1) and (2) can be combined to yield

$$E = B/(1 + BD/E_s t) \tag{3}$$

The fluid, which is modeled with axial members, must have only one independent degree of freedom (DOF) at each grid point. The three rotational DOF are restrained at all fluid points. Both transverse translational DOF at each fluid point are constrained (using multipoint constraints or rigid links) to move with the corresponding structural point. The only remaining DOF, the axial DOF, is free to slide relative to the beam. These constraints are applicable in both the straight sections and the elbows. It is therefore convenient to define for each elbow a separate cylindrical coordinate system whose axis is perpendicular to the plane of the elbow and intersects the center of curvature. For elbows, the independent DOF is thus the azimuthal translation. For each straight section, it is convenient to define a separate Cartesian system with one axis coincident with the pipe axis. It is emphasized that the single independent fluid unknown is the axial displacement, not the pressure. The fluid pressure can be recovered from the finite element program in the usual way by requesting that stresses in the fluid elements be calculated and printed.

The modeling of fluid-filled tees is handled differently from that of elbows. Since fluid entering one leg of a tee can flow out both of the other two branches, the procedure must ensure that the total fluid mass flowing into one branch of the joint equals the total mass flowing out the other two branches (Ref. 8). (This condition is automatically satisfied for an elbow, a two-branch joint.) In the finite element model, we define at the intersection of the tee branches one structural grid point and three fluid grid points, as shown in Figure 1. As with other fluid grid points in the system, each of these three points is permitted to move only in the axial direction for the branch in which it lies. In addition, the three axial DOF

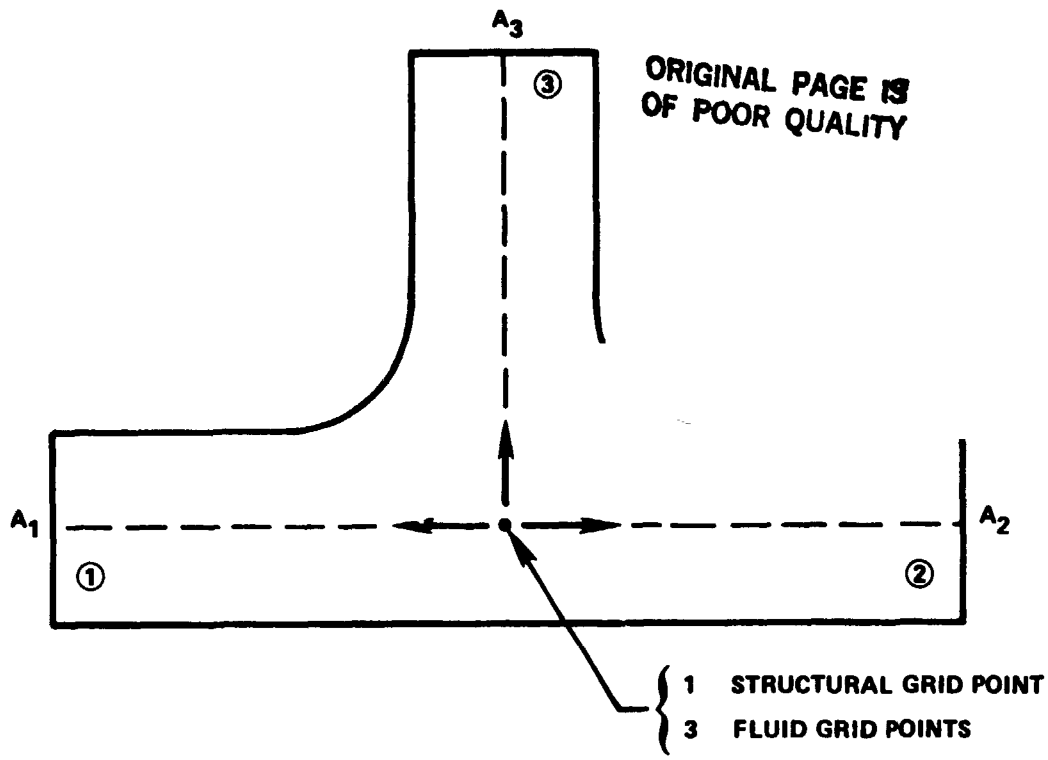

Figure 1 - Low Frequency Model of Piping Tee

ıre not independent (because of conservation of mass) and must satisfy the relation

$$\sum_{i=1}^{3} A_i \, (\xi_i - u_i) = 0 \qquad (4)$$

where u_i and ξ_i are the axial components of structural and fluid displacements, re-·pectively, and A_i is the fluid cross-sectional area for branch i of the tee. Thus, a tee introduces two independent fluid DOF into the model.

As with elbows, flexibility factors should be used with the beam elements which model the tee. Unfortunately, even less is known about tee flexibilities than about elbow flexibilities. The procedure that we find convenient for computing tee flexibilities is to perform a separate finite element analysis for a tee modeled as a shell. This analysis can be easily made since a tee data generation program (Ref. 21) has been interfaced with NASTRAN by Quezon (Ref. 22) so that, given a few basic parameters, an analysis can be performed within a few hours. No tees are included in the examples in this report.

Damping in the piping system can be directly incorporated in the finite element model by entering the damping loss factor as a material damping constant, which results in complex material moduli.

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Because of the versatility of finite element computer codes, various boundary conditions on the fluid column are possible. At a free surface (where the pressure vanishes), the fluid DOF (the axial displacement) is left free, a natural boundary condition. At a fixed boundary, the unknown is restrained. If the pipe modeled is part of a very long system, in which case a non-reflecting fluid boundary is needed, the plane fluid waves can be absorbed by attaching to the fluid DOF a dashpot whose constant (ratio of axial force to velocity) is ρcA, where ρ is the fluid mass density, c is the effective sound speed as given by Equation (2), and A is the cross-sectional area of the fluid column. For a pipe that opens into a large volume of fluid (e.g., the sea), the appropriate boundary condition is that of a piston in an infinite baffle (Ref. 23), in which case the scalar added mass

$$M_a = 8\rho a^3/3 \tag{5}$$

is applied, where ρ is the fluid mass density and 'a' is the radius of the opening. For a pipe with a closed end, the axial fluid displacement at the end is tied to the axial structural displacement using a multipoint constraint equation or rigid link.

The beam model just described can be applied using most general purpose finite element structural analysis codes without modification. The analysis is performed in a single pass with the fluid-structure coupling included in the model. The resulting model has seven independent degrees of freedom at each grid point location, six for the pipe and one for the axial component of fluid displacement. For matrix bandwidth reasons, each fluid grid point should be sequenced adjacent to its corresponding structural point. The major limitation of the model is frequency: this is a low frequency model. More will be said about this limitation later.

INTERMEDIATE FREQUENCY 3-D MODEL

For the dynamic response prediction of piping systems at frequencies for which beam models are not valid, general three-dimensional finite element models are required. In general, this approach models the pipe with shell elements and the contained fluid with 3-D acoustic finite elements. Thus the pipe need not respond only as a beam (for which the cross sections are rigid), and non-planar fluid response is allowed. Such a model generally requires thousands of degrees of freedom, even for simple piping systems. Thus, although 3-D models may find only limited use (given current computing power), it is worthwhile to describe the model's formulation and demonstrate its application. A 3-D model is particularly useful for validating approximate models such as beam models since the limitations of the approximate model can then be determined. The purpose of this section is to describe the general 3-D finite element modeling of non-planar fluid-filled elastic piping systems. We are not aware of any previous finite element analyses of the size to be considered here for the fully-coupled fluid-structure problem.

Most general finite element work involving an elastic structure coupled to an acoustic fluid (for which the fluid pressure satisfies the wave equation) can be traced to the work of Zienkiewicz and Newton (Ref. 15). In their work and in many subsequent papers by others, the fundamental fluid unknown was taken to be the pressure. A few investigators (e.g., Hamdi, Ousset, and Verchery (Ref. 24)) selected the fluid displacement as the unknown.

Everstine (Ref. 25-27) showed how general purpose structural analysis codes could be used, without modification, to solve the common field equations arising in mathematical physics (including the wave and Helmholtz equations of acoustics). He also showed how the codes could be used to solve mixed field problems such as coupled structural-acoustic problems.

More recently, he showed (Ref. 16) that, if the coupled fluid-structure problem were formulated with velocity potential rather than pressure as the fundamental fluid unknown, the nonsymmetric matrices of the pressure formulation would be symmetric. For some situations, including steady-state problems involving damped systems (which are of interest here), significant computational advantages result.

If both fluid and structure are modeled with finite elements, the following matrix equation arises (Ref. 16,28):

$$\begin{bmatrix} M & 0 \\ 0 & Q \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{q} \end{Bmatrix} + \begin{bmatrix} B & A \\ A^T & C \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{q} \end{Bmatrix} + \begin{bmatrix} K & 0 \\ 0 & H \end{bmatrix} \begin{Bmatrix} u \\ q \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} \qquad (6)$$

where q, the fundamental unknown in the fluid, is the time integral of pressure and hence proportional to the velocity potential. The unknown q is a vector with a single unknown at each fluid mesh point. In Equation (6), u is the vector of displacement components in the structure, M and Q are the mass matrices for the structure and fluid, K and H are the stiffness matrices for the structure and fluid, A is the area matrix which converts fluid pressure at interface points to structural loads, B and C are the damping matrices for the structure and fluid, and f_1 and f_2 are the structural and fluid applied loads. If the pressure gradient (or equivalently, fluid motion) is specified at a fluid boundary, f_2 takes the form

$$f_2 = -(A_s \partial q/\partial n)/\rho \qquad (7)$$

where A_s is the area matrix for the boundary surface. In Equation (6), the required "material constants" for the fluid elements are

$$G_e = -1/\rho, \; E_e = 10^{20} G_e, \; \rho_e = -1/\rho c^2 \qquad (8)$$

where G_e, E_e, and ρ_e are, respectively, the "shear modulus," "Young's modulus," and "mass density" assigned to the fluid finite elements (Ref. 16,28).

In the fluid-filled piping systems of interest here, damping is introduced by specifying an overall system loss factor. In that case, the matrices K and H in Equation (6) are complex, and B = C = 0. The loss factor η, if uniform, is given by

$$\eta = \mathrm{Im}\,(K) \,/\, \mathrm{Re}\,(K) = \mathrm{Im}\,(H) \,/\, \mathrm{Re}\,(H) \qquad (9)$$

All matrices in Equation (6) except A are automatically formed by the finite element program once the elements are defined. The area matrix A has nonzeros only at rows and columns corresponding to interface degrees of freedom. For piping systems which involve only cylinders and tori, the area contribution at each point is easily calculated analytically, so that A can be generated by an automatic data generation preprocessor.

This finite element scheme can be implemented on any general purpose structural code which allows the user to enter matrix elements directly from the input stream. We used NASTRAN for the analyses described in this paper.

EXAMPLE 1: A PLANAR PIPING SYSTEM

The formulations described in the preceding sections will be illustrated first on a simple planar piping system for which experimental data are available (Ref. 6). This system, shown in Figure 2, consists of two straight sections of standard 4-inch copper-nickel pipe connected by an elbow. The system is filled completely with lubricating oil. Table 1 summarizes the pertinent properties of the system.

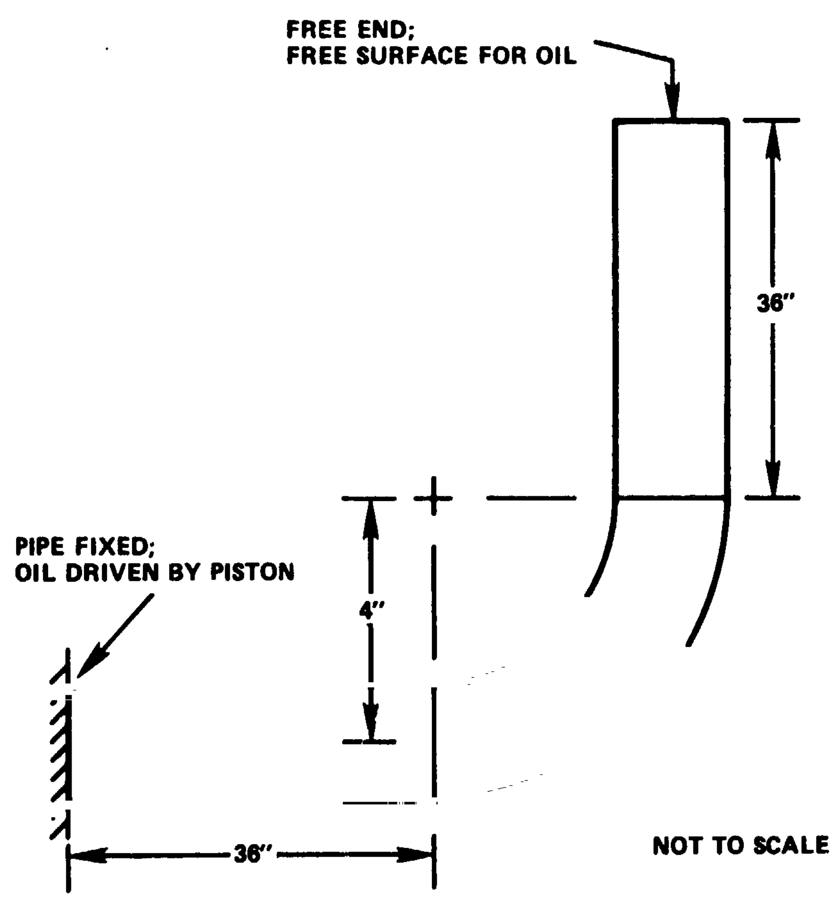


Figure 2 - Planar Piping System

TABLE 1 - CHARACTERISTICS OF PLANAR PIPING SYSTEM

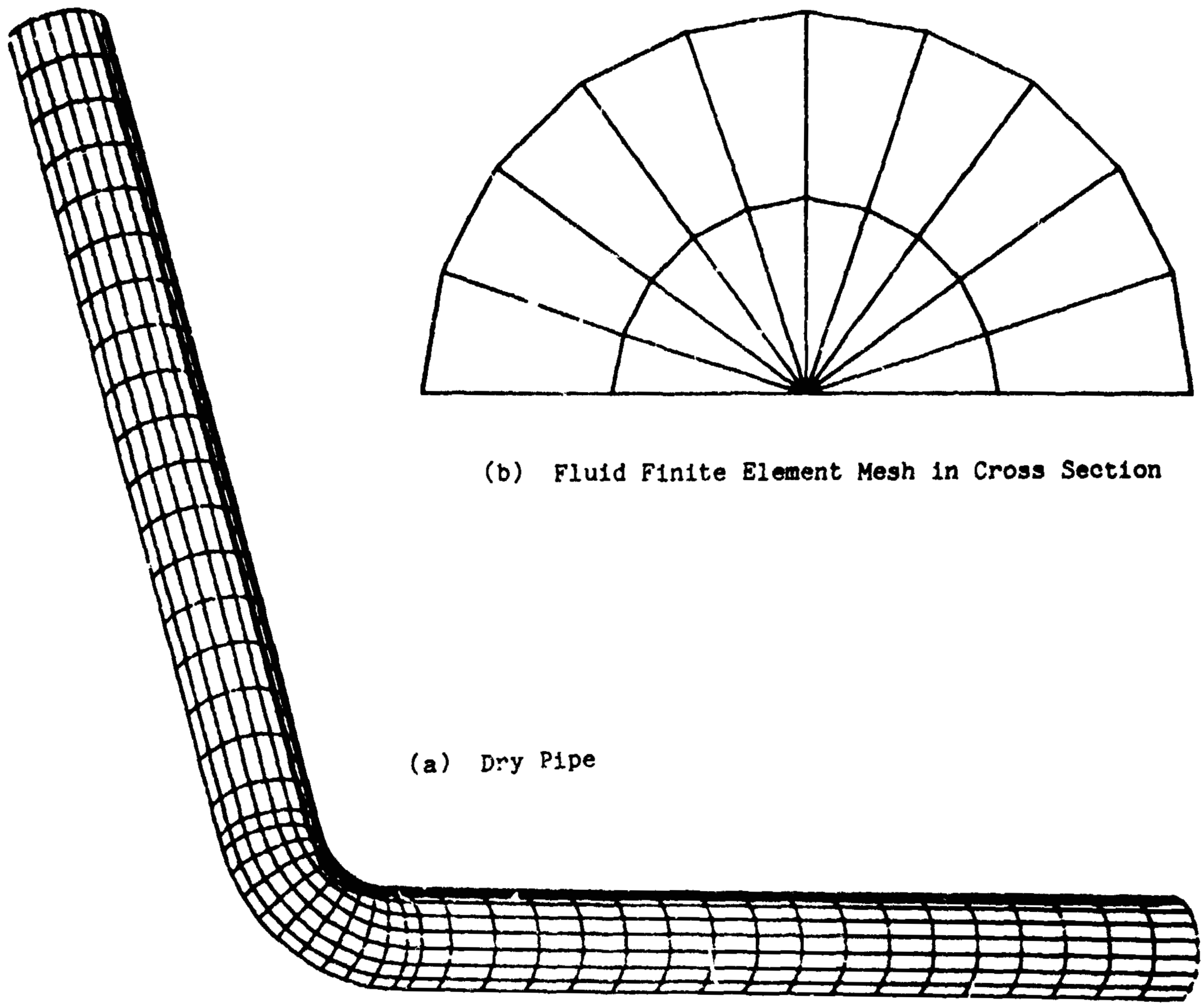
Pipe (4-inch 70-30 Cu-Ni)	
outside diameter (O.D.)	4.5 in
minimum wall thickness	0.203 in
nominal wall thickness	0.232 in
Young's modulus	22,000,000 psi
Poisson's ratio	0.294
weight density	0.323 lb/in^3
Elbow	
bend radius (R)	4 in
bend angle	90 degrees
Fluid (2190 TEP oil)	
actual bulk modulus	282,000 psi
effective bulk modulus in pipe	228,000 psi
weight density	0.0315 lb/in^3

As seen in Figure 2, one end of the pipe was fixed, and the fluid was driven by a piston designed to excite only the fluid. The other end of the system was free. Measurements (Ref. 6) included the fluid pressure at the piston and the inplane components of structural velocity at the free end.

Both beam and 3-D models were prepared for this piping system using the finite element approaches described in the preceding section. The beam model consisted of ten beam elements in each straight section and eight elements in the elbow. For this 2-D problem, each fluid and structural grid point had, respectively, one and three degrees of freedom (DOF). The beam model thus had 112 DOF. The elbow flexibility factor used for the beam analysis was 8.14, which was computed by the ELBOW computer program (Ref. 17). For inplane moment loads on elbows with long straight sections, ELBOW has been shown to be satisfactory, although it does overestimate the flexibility factors slightly (Ref. 14).

The mesh used for the 3-D model is shown in Figure 3. The structural element used is a low-order four-node quadrilateral plate (NASTRAN'S QUAD2). Because of symmetry, only half of the circumference (180 degrees) was modeled. The model had ten elements in the circumferential direction, 19 elements longitudinally in each straight section, and nine elements in the elbow. The dry pipe thus had about 2800 DOF. As shown in Figure 3, the fluid finite element mesh had two elements (a constant strain wedge and an eight-node isoparametric hexahedron) in the radial direction between the center of the pipe and the shell. With the fluid added, the size of the 3-D model increased to about 3900 DOF.

The assumed uniform loss factor used for all calculations for this piping system was 0.0262, independent of frequency. This value was selected on the basis of previous experimental experience with similar systems.



(b) Fluid Finite Element Mesh in Cross Section

(a) Dry Pipe

Figure 3 - Finite Element Model of Planar Piping System

The results of the analyses of this system are shown in Figures 4 and 5 over the frequency range 10 Hz to 10,000 Hz. Mobility responses (the ratio of velocity response to driving force) are shown for both analytical models and the Davidson-Smith experimental data (Ref. 6).

The two analytical solutions shown in Figures 4 and 5 are in reasonably good agreement even for frequencies above the first lobar ($n=2$) frequency, where the modal density is high. The $n=2$ lobar frequency for a long pipe can be estimated from the

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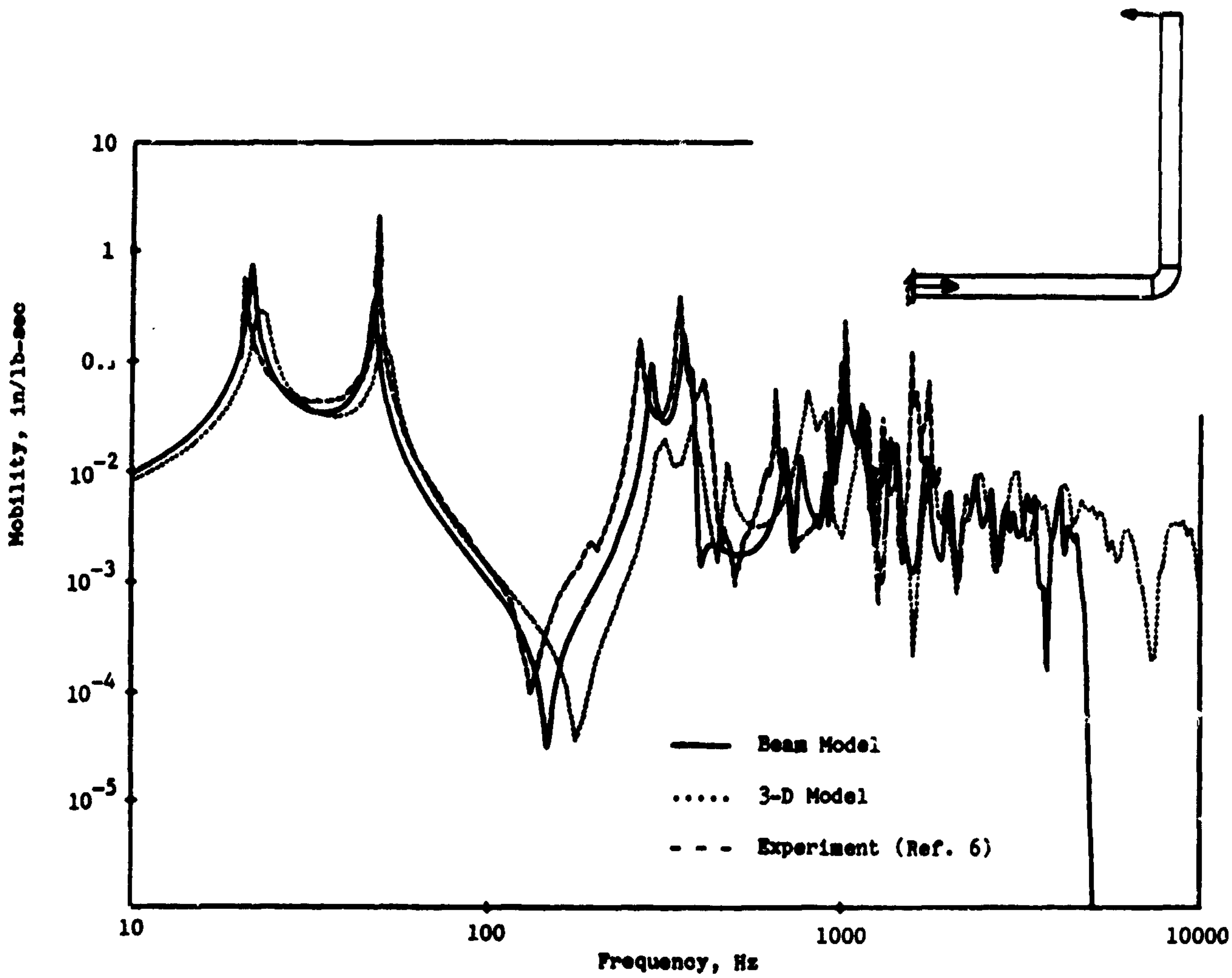

Figure 4 - Mobility of Planar Piping System: Transverse Response at Free End

classical formula (Ref. 29) for the plane strain vibrations of a ring. For a dry ring, the lowest n=2 frequency is

$$f = (1/2\pi)\ \sqrt{3E_s t^2/5\rho_s r^4(1-\nu^2)} \qquad (10)$$

where E_s, ρ_s, and ν are, respectively, the Young's modulus, mass density, and Poisson's ratio for the pipe material, t is the wall thickness, and r is the mean radius. Thus, from Equation (10), for 4-inch Cu-Ni pipe, the lowest n=2 lobar mode occurs at about 1066 Hz for dry pipe.

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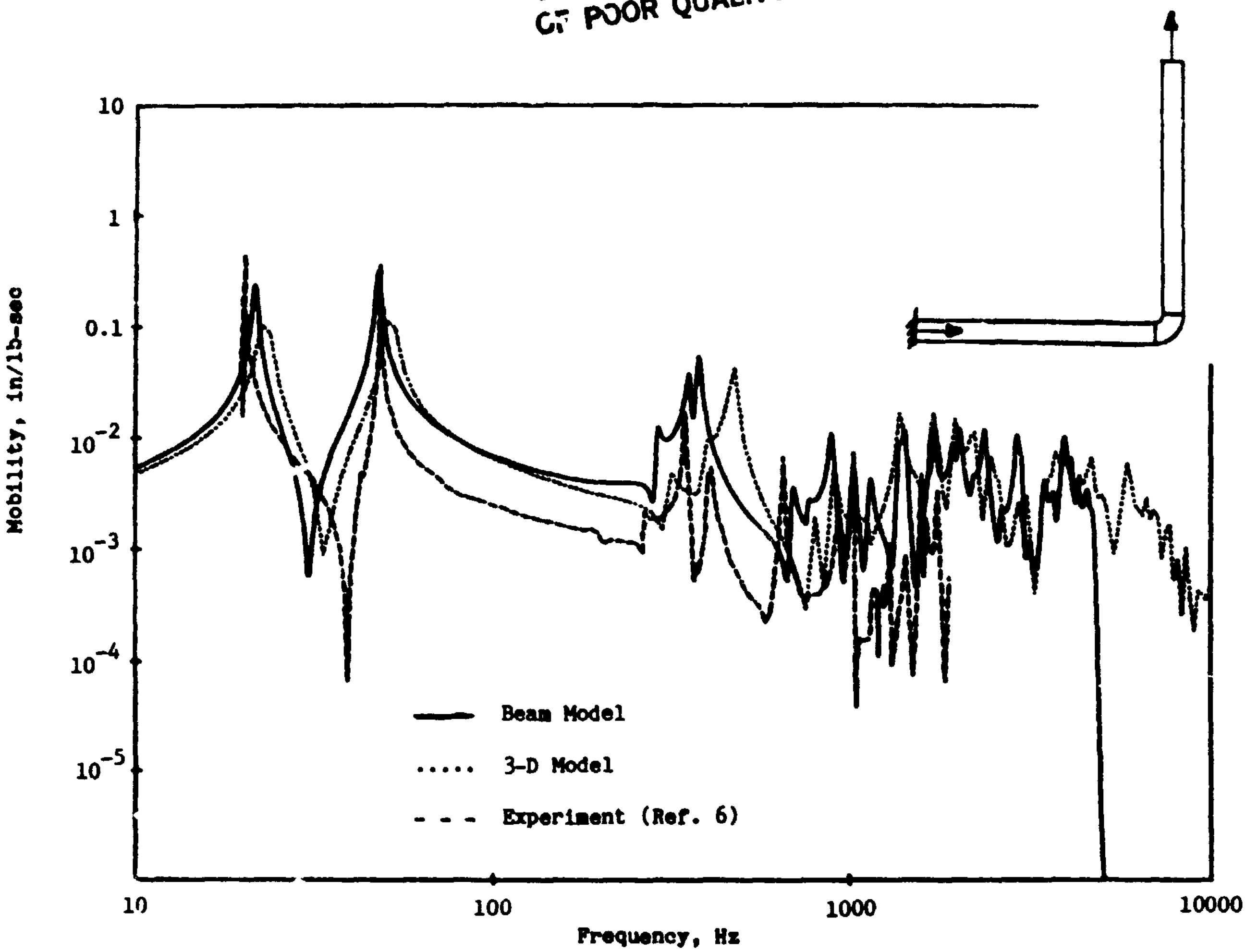

Figure 5 - Mobility of Planar Piping System: Axial Response at Free End

The corresponding fluid-filled frequency can be estimated if ρ_s in Equation (10) is replaced by the effective density ρ_{eff} for the structure-fluid combination:

$$\rho_{eff} = \rho_s + \rho r/2t \qquad (11)$$

where ρ is the fluid mass density. With this correction, the lowest n=2 lobar frequency for oil-filled 4-inch Cu-Ni pipe about 888 Hz.

EXAMPLE 2: A NON-PLANAR PIPING SYSTEM

Although the results for the planar system show good agreement between the beam results and both the 3-D results and the experimental data, this planar system does not provide an adequate test of a beam model. One difficulty which arises with non-planar systems is that the flexibility factor k is hard to calculate. Although idealized approaches for calculating k do not distinguish between inplane and out-of-plane cases, it has been shown (Ref. 14) that, for 90-degree elbows with straight pipe extensions, the out-of-plane flexibility factor differs considerably from the inplane factor. In addition, many piping systems of practical interest have straight sections which are not so long (relative to the elbow arc length) as in the planar system of Example 1. We would expect the elbow flexibilities to become more important to the overall dynamic response for systems with shorter straight sections.

Here we consider a complex 3-D piping system also built with 4-inch Cu-Ni pipe. As shown in Figure 6, the system consists of four straight sections and three elbows (two 90-degree elbows and one 45-degree elbow). The pipe is filled completely with

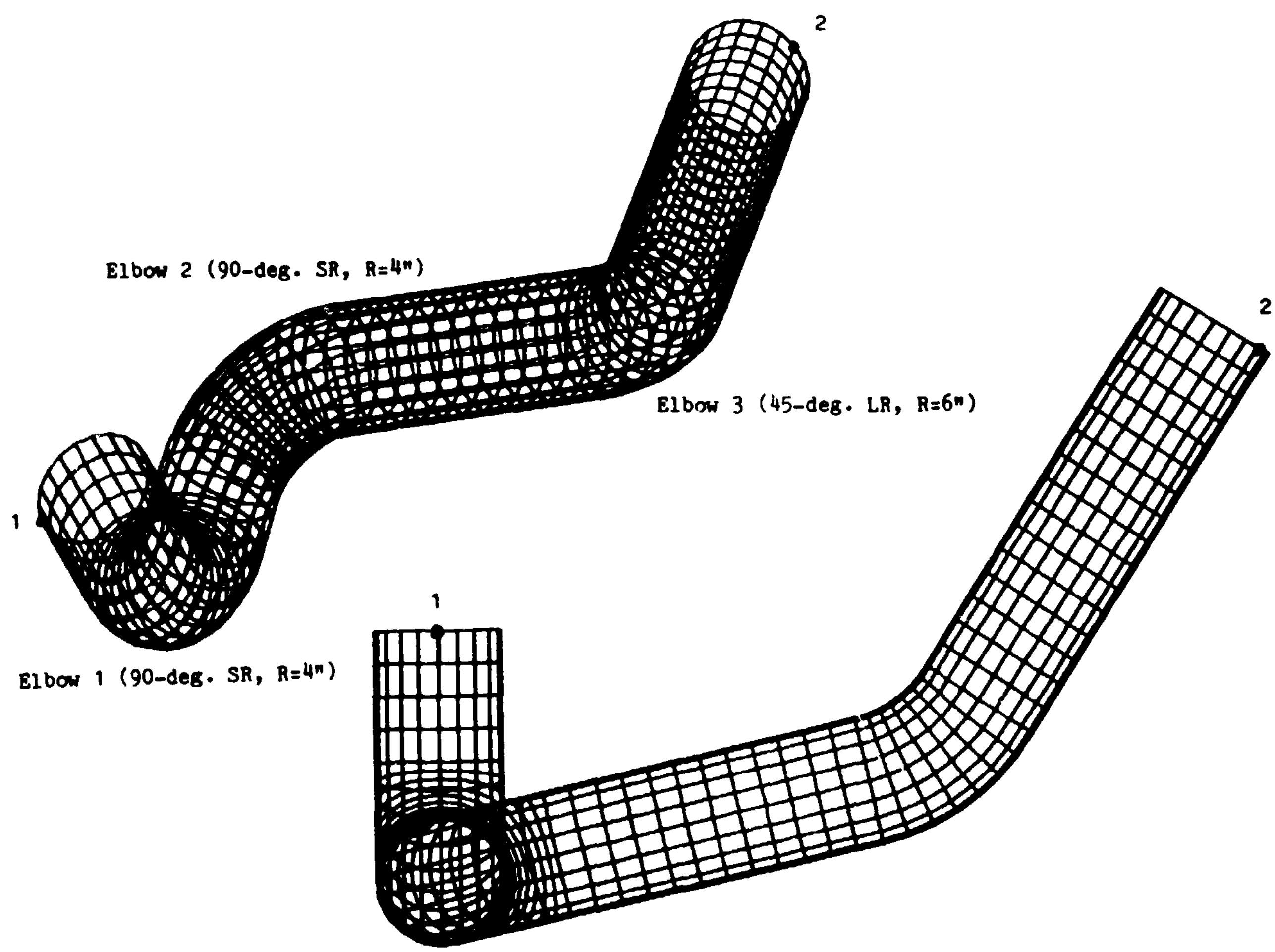

Figure 6 - Finite Element Model of Non-Planar Piping System (Two Views)

fresh water. Fluid free surfaces are assumed at both ends of the system. (Experimentally, such conditions could be simulated approximately by capping the ends with flexible rubber membranes.) The loading for this system is a time-harmonic force applied to the structure in the axial direction at point 1. The entire system is assumed to be freely suspended. No experimental data are available for this system.

Both beam and 3-D finite element models were prepared for this piping system. The beam model, which consisted of 21 beam elements, had 154 DOF. The 3-D model (Figure 6) had about 7500 DOF (dry) and 9900 DOF (fluid-filled). The assumed loss factor used for both analyses was 0.02, independent of frequency.

The response predictions for this system are shown in Figures 7 and 8. Since the same pipe size is used as in the planar system, the lowest n=2 dry lobar mode occurs at the same frequency, 1066 Hz. This pipe, however, is filled with water rather than oil, so the lowest fluid-filled n=2 lobar frequency is slightly lower: 867 Hz.

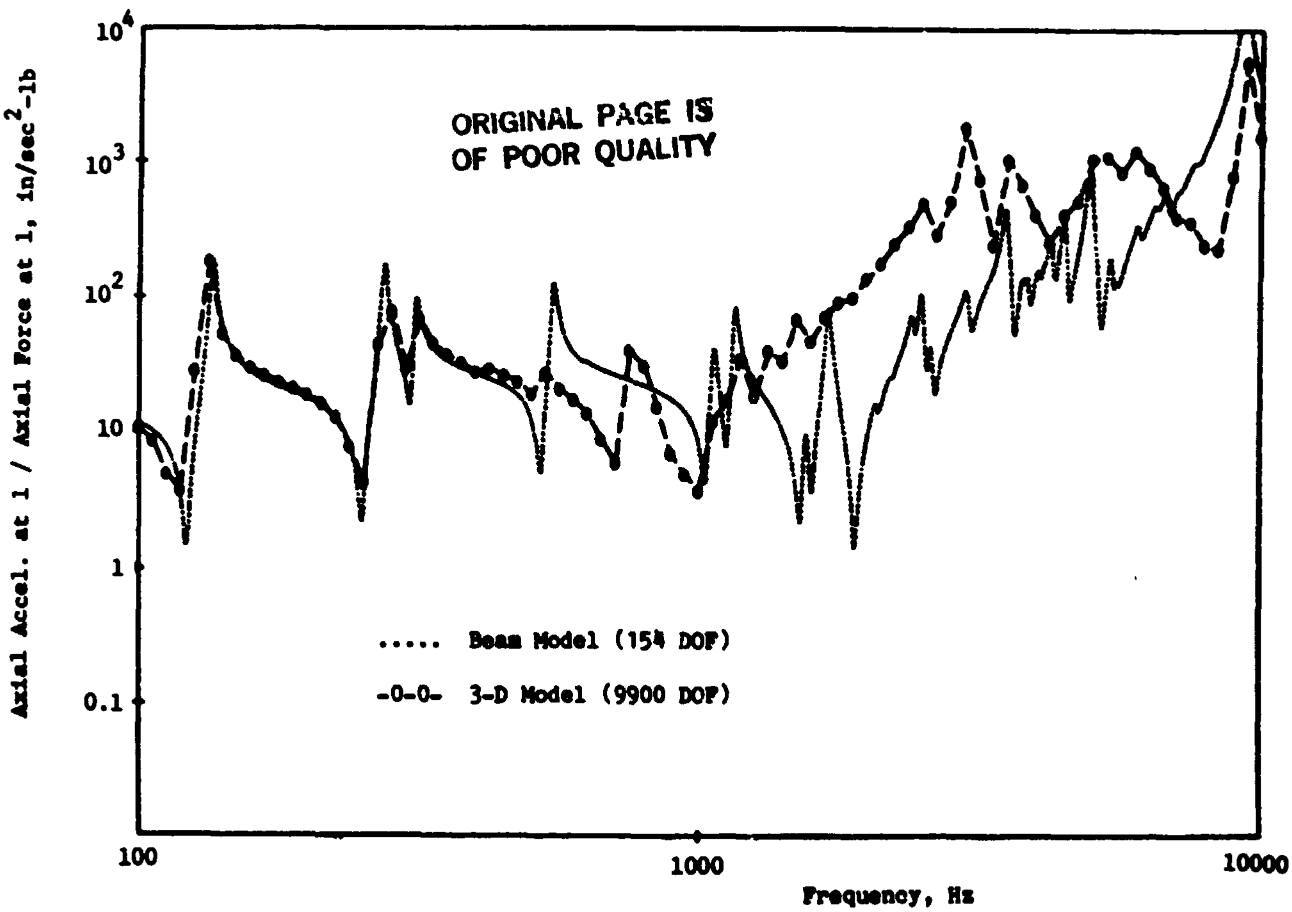

Figure 7 - Acceleration Response of Non-Planar Piping System: Drive Point Accelerance

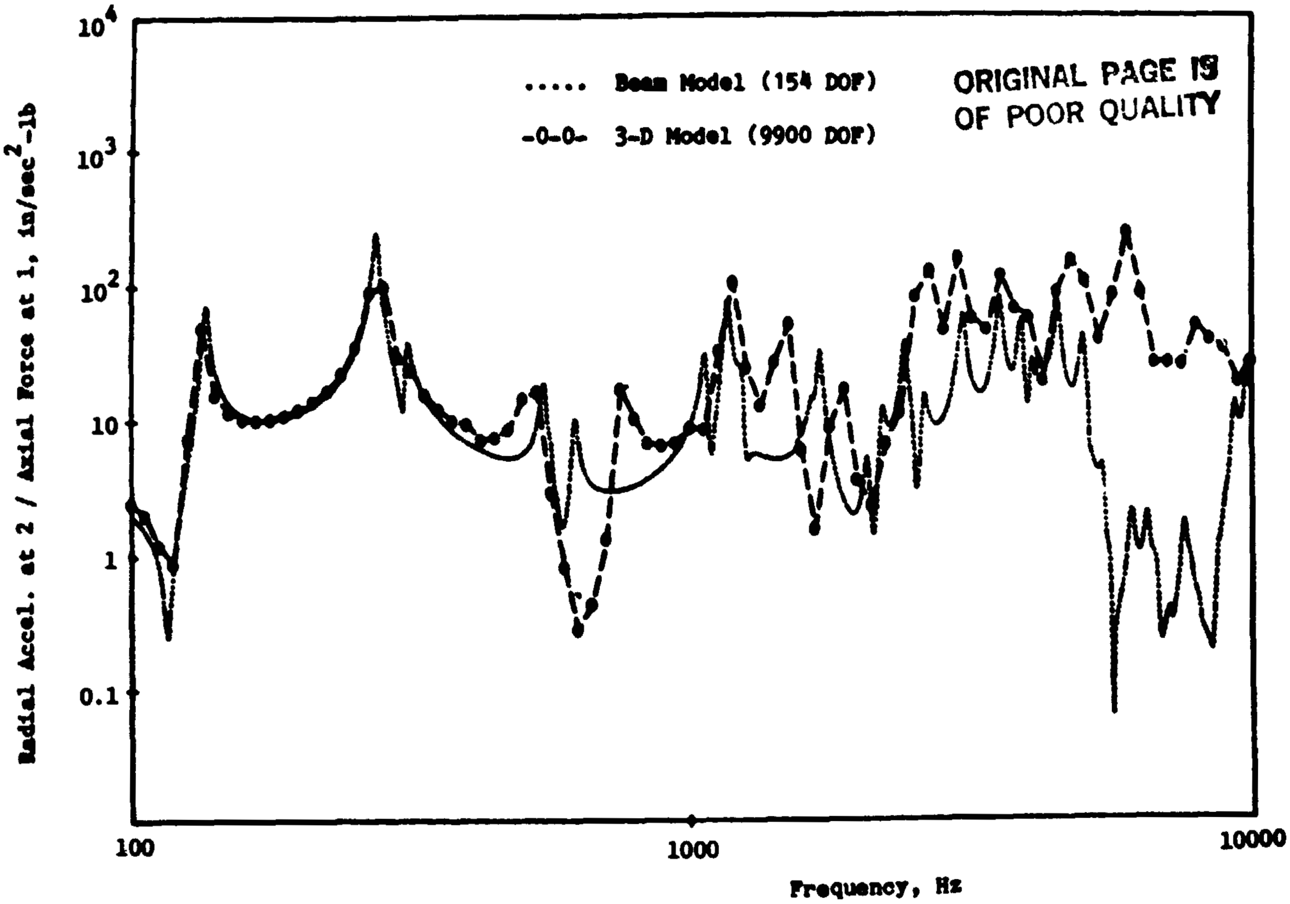

Figure 8 - Acceleration Response of Non-Planar Piping System: Transfer Point Accelerance

The flexibility factors for the 90-degree elbows in the beam model were estimated from the tables published by Quezon and Everstine (Ref. 14) for 90-degree elbows with various lengths of pipe extension. The flexibility factors for the 45-degree elbow in this system were estimated from similar tables nc ' heing compiled for 45-degree elbows. (These tables will be published soon.) The f 'bility factors used in this beam analysis are listed in the second and thirc olumns of Table 2.

As seen in Figures 7 and 8, the two analyses (beam and 3-D) are in very close agreement up to about 550 Hz, which is about 63% of the lowest n=2 fluid-filled lobar mode (867 Hz) of the pipe. This agreement indicates that the beam model is a valid model for low frequencies.

SENSITIVITY OF RESPONSE TO FLEXIBILITY FACTORS

Here we use the beam model of the non-planar piping system to determine the effects of errors in the flexibility factors used in the analysis. As we indicated

TABLE 2 - FLEXIBILITY FACTORS FOR NON-PLANAR PIPING SYSTEM

ELBOW	FLEXIBILITY FACTORS			
	BASED ON REF. 14		ELBOW PROGRAM [17]	EFFECT IGNORED
	INPLANE	OUT-OF-PLANE		
1 (90-deg. SR)	5.0	2.8	7.8	1.0
2 (90-deg. SR)	5.4	2.8	7.8	1.0
3 (45-deg. LR)	4.2	3.7	5.2	1.0

in the preceding section, the flexibility factors used were based on tables for 90-degree elbows published by Quezon and Everstine (Ref. 14) and other (as yet) unpublished tables for 45-degree elbows with various lengths of straight pipe extensions. Because the low frequency beam results using these factors agree well with the 3-D model results in Figures 7 and 8, these flexibility factors are considered reasonably accurate.

For comparison purposes, analyses were also made using two other sets of flexibility factors. The first set was that calculated by the ELBOW computer program (Ref. 17). ELBOW is probably typical of the idealized approaches used by piping designers. ELBOW's flexibility factors are in close agreement (Ref. 17) with the current ASME code (Ref. 30), which, for zero internal static pressure, uses the relation

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$$k = 1.65\, r^2/tR \qquad (12)$$

where k is the flexibility factor, r is the mean radius, t is the wall thickness, and R is the bend radius. The ELBOW program flexibility factors are listed in column 4 of Table 2.

The second additional set of flexibility factors used for an analysis was obtained merely by setting k = 1.0 for all factors. This analysis would show the consequences of ignoring the flexibility factor effect entirely.

The response predictions for all three sets of flexibility factors are shown in Figure 9, in which the drive point accelerances (the ratio of acceleration to force) are plotted over the frequency range 100 Hz to 1000 Hz (the low frequency regime). Figure 9 clearly indicates the importance of using accurate flexibility factors in the analysis.

DISCUSSION AND CONCLUSIONS

We have described two different finite element modeling procedures for predicting the dynamic response of general 3-D fluid-filled elastic piping systems. The beam model, a ˙ ›w frequency procedure, was, for the non-planar system considered,

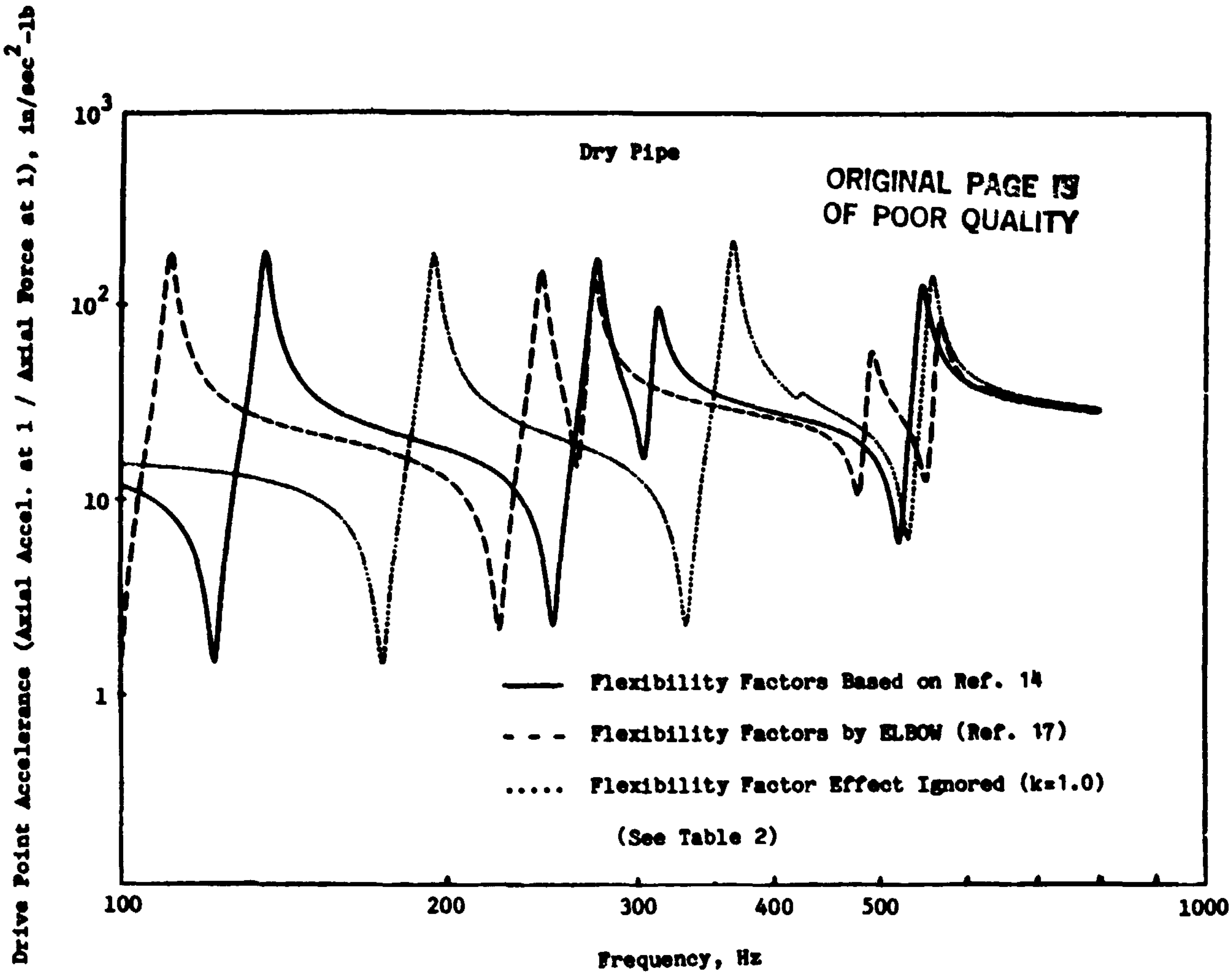

Figure 9 - Sensitivity of Non-Planar Piping System Response to Flexibility Factor

valid for frequencies up to about 63% of the lowest fluid-filled lobar (n=2) pipe mode. For frequencies between that and 100% of the n=2 mode, the general finite element modeling procedure described could be used. For still higher frequencies, where the modal density is high, the finite element approach remains theoretically valid, but the analyst is probably wiser to use statistical energy analysis (S.E.A.) techniques (Ref. 31) instead. The S.E.A. techniques are particularly well suited to the high modal density regime.

It was shown that the accuracy of beam analyses of piping systems depends strongly on the accuracy of the flexibility factors used. Such factors are not required in the 3-D finite element model since they are implicit in a shell model of the pipe.

Finally, the beam analyses presented here used straight beams to model the bends (elbows). It would clearly be preferable to use instead curved beam elements, if available.

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