FORMATION OF STANDING SHOCKS IN STELLAR WINDS AND RELATED ASTROPHYSICAL FLOWS.

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ABSTRACT

Stellar winds and other analogous astrophysical flows can be described, to lowest order, by the familiar one-dimensional hydrodynamic equations which, being nonlinear, admit in some instances discontinuous as well as continuous transonic solutions for identical inner boundary conditions. We elaborate on the physics underlying the formation of the discontinuous transonic solutions; discuss the *characteristics* of the time-dependent differential equations of motion to show how a perturbation changes profile in time and, under well-defined conditions, develops into a stationary shock discontinuity; and show how the formation of standing shocks in wind-type astrophysical flows depends on the fulfillment of appropriate *necessary* conditions, which are determined by the conservation of mass, momentum and energy across the discontinuity, and certain *sufficient* conditions, which are determined by the flow's "history".

1. Introduction

The present paper continues the study of the solution topologies of the time-dependent isothermal wind equations presented by Habbal, Rosner & Tsinganos (1983; Paper I) in these Proceedings; these two studies are motivated by the desire to understand how the simplest hydrodynamic equations describing spherically symmetric solar-wind type astrophysical flows (Parker 1963) respond to perturbations which disturb the radial symmetry of the outflow (Kopp & Holzer 1976; Munro & Jackson 1977) or, equivalently (cf. Habbal & Tsinganos 1983), perturb the outflow by means of a localized non-thermal momentum deposition in the thermal gas (Holzer 1977). In this paper, we consider the consequences of a localized phenomenological momentum perturbation term of the form (Paper I)

$$D_{[D_o,R_p,\sigma,\tau]}(R,t) = D_o\left[1-e^{-t/\tau}\right] e^{-(R-R_p)^2/\sigma^2} \quad \text{dyn/gm} \quad , \tag{1}$$

where the free parameters D_o, R_p, σ, and τ characterize the amplitude, location, spatial and temporal width, respectively, of the function $D(R,t)$. The detailed dependence of the steady solutions of the governing hydrostatic equations on the parameters D_o, R_p, and σ for $\tau = 0$ can be found in Habbal & Tsinganos (1983). The specific purpose of this note is to further elaborate on the physics underlying the response of the governing hydrodynamic equations to perturbations of the form (1) above (discussed in Paper I), and, in particular, to discuss qualitatively the conditions leading to the formation of shocks in the flow. We shall show that in certain limiting cases much of the relatively complex behavior can be understood on the basis of the straightforward theory of characteristics.

2. Nature of the Equations of Motion

In the following we discuss briefly the nature of the equations of motion and how they allow the development of shocks in the flow. First, it is convenient to write the continuity and momentum balance equations in the following form

$$A\frac{\partial\rho}{\partial t} + Av\frac{\partial\rho}{\partial R} + A\rho\frac{\partial v}{\partial R} = -\rho\frac{\partial A}{\partial t} - \frac{\partial A}{\partial R} \quad , \tag{2a}$$

$$c^2\frac{\partial\rho}{\partial R} + \rho\frac{\partial v}{\partial t} + \rho v\frac{\partial v}{\partial R} = \rho D - \rho\frac{GM}{R^2} \quad , \tag{2b}$$

where $c = [dp/d\rho]^{1/2}$ is the isothermal sound speed and A(R,t) and D(R,t) are the *given* functions of the flow tube's cross section and non-thermal momentum deposition, respectively.

Eqs. (2) form a system of two quasilinear partial differential equations of the first order for the functions ρ and v of the two independent variables R and t, for which solutions in closed form are not known. However, the *qualitative* behavior of their solutions and the associated physical effects can be simply understood by exploiting the hyperbolic nature of these equations; the formal theory of hyperbolic partial differential equations shows then that there exist two characteristic directions at every point of the (R,t)-plane which define the *characteristics* of Eqs. (2) (Courant & Friedrichs 1948),

$$\frac{dR}{dt} = v \pm c \ . \tag{3}$$

The *physical significance* of these characteristics can be understood as follows. Expressing the velocity in terms of the potential $\Phi(R,t)$,

$$\frac{\partial\Phi(R,t)}{\partial R} \equiv v \ , \quad -\frac{\partial\Phi(R,t)}{\partial t} = \frac{v^2}{2} + \int \frac{c^2\, d\rho}{\rho} - \frac{GM}{R} - \int D(\tilde{R})\, d\tilde{R} \ , \tag{4}$$

we obtain a single differential equation for $\Phi(R,t)$,

$$\frac{\partial^2\Phi}{\partial t^2} + 2v\frac{\partial^2\Phi}{\partial R\partial t} + (v^2 - c^2)\frac{\partial^2\Phi}{\partial R^2} = v\left[c^2\frac{\partial \ln A}{\partial R} + D - \frac{GM}{R^2}\right] + c^2\frac{\partial \ln A}{\partial t} + \frac{\partial}{\partial t}\int_{R_o}^{R} D(\tilde{R},t)d\tilde{R} \ . \tag{5}$$

Consider then a *small* perturbation $\delta\Phi(R,t)$ superimposed on some solution $\Phi_o(R,t)$ of Eq. (5). Following Landau & Lifshitz (1975), we substitute $\Phi = \Phi_o + \delta\Phi$ in (5) and obtain to lowest order the following equation for $\delta\Phi$,

$$\frac{\partial^2\delta\Phi}{\partial t^2} + 2v_o\frac{\partial^2\delta\Phi}{\partial R\partial t} + (v_o^2 - c^2)\frac{\partial^2\delta\Phi}{\partial R^2} = 0 \ , \tag{6}$$

where $v_o = \partial\Phi_o/\partial R$ is the "background" flow field (see below). We write $\delta\Phi = Be^{i\Psi}$, where B is a slowly varying function of R and t and the "eikonal" Ψ is almost linear in R and t; the "group velocity" of the disturbance is then $dR/dt = d\omega/dk$, where $k = \partial\Psi/\partial R$ and $\omega = \partial\Psi/\partial t$. We obtain from (6) for this "group velocity" of the perturbation,

$$\frac{dR}{dt} = v_o(R,t) \pm c \ , \tag{7}$$

a relation which coincides with the slope (3) of the characteristic directions on the plane R-t. From the physical point of view Eq. (7) gives the velocity of propagation of sound waves in the moving gas relative to some fixed coordinate system; and since a disturbance of finite amplitude and duration can be regarded as a superposition of a sequence of small amplitude and duration perturbations, Eq. (7) describes the propagation of finite perturbations in the atmosphere as well (Courant & Friedrichs 1948, Landau & Lifshitz 1975). It is evident then that the characteristics represent the paths of all possible disturbances in the (R-t)-plane. The inevitable result of having a non-uniform velocity field, $dv_o/dR \neq 0$, is that the profile of some initial perturbation changes as it propagates in the atmosphere. For example, if $dv_o/dR < 0$ somewhere in the flow, an outward-propagating wave finds its leading segment travelling slower than its trailing edge; the result is that its amplitude profile "steepens". The perturbation then may "break," i.e., it may steepen enough so that the total density and velocity are no longer single-valued; in that case the characteristics intersect (Landau & Lifshitz 1975), and a shock has been formed in the flow. Whether this indeed leads to an equilibrium solution with a discontinuity depends on two facts: first, on the existence of such equilibrium solutions involving shocks for the given set of spatial parameters in the steady state equations (Habbal & Tsinganos 1983); and second, on the temporal history of the flow, as discussed below.

3. Evolution of the Perturbation and Shock Formation

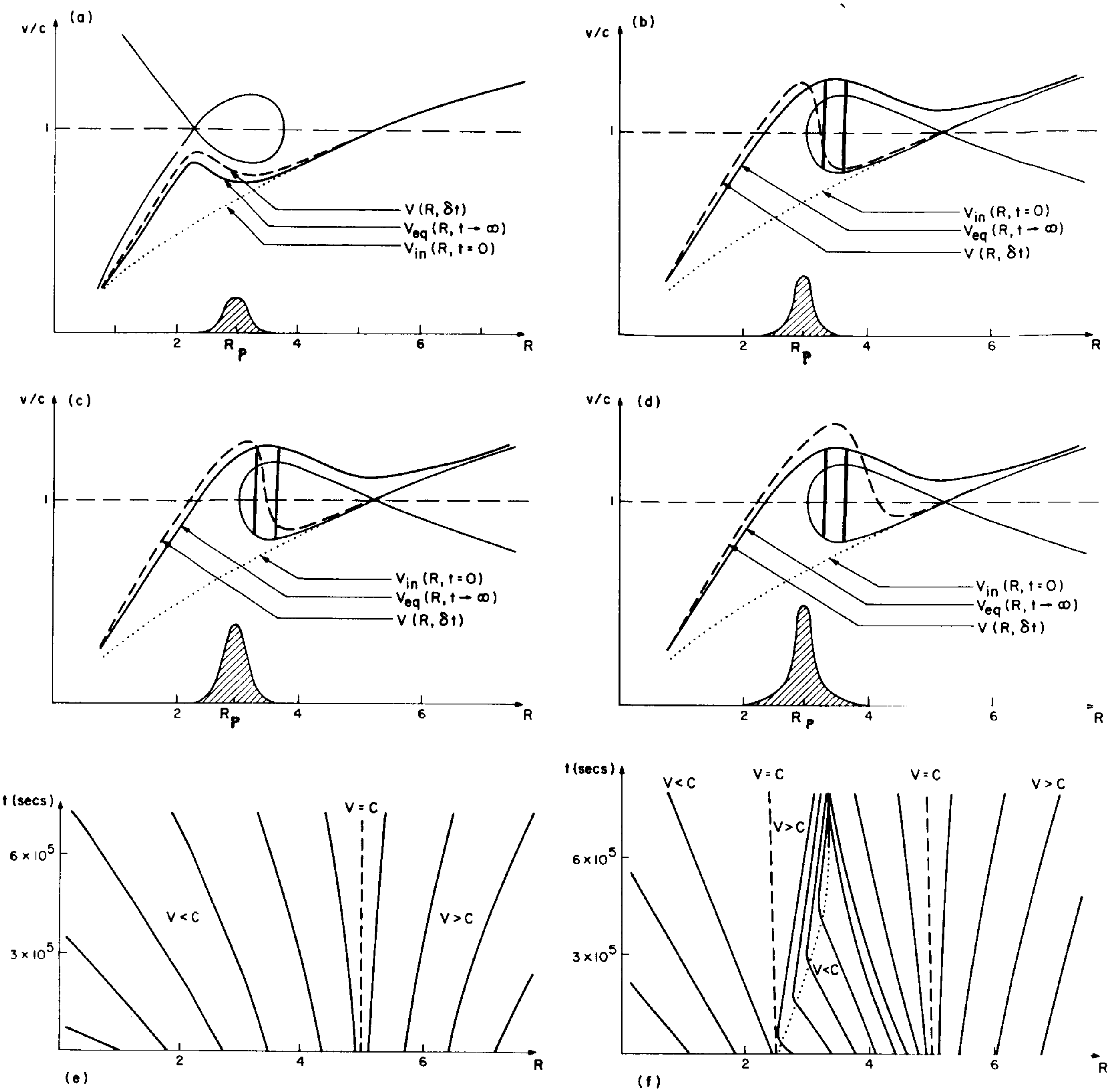
We would now like to see whether the above discussion of characteristics can shed any light on an understanding of the behavior of a perturbed isothermal wind flow. We begin with a steady and isothermal (single critical point, or Parker) wind flow at t=0, and apply a perturbation of the type given in Eq. (1). As shown in Paper I, which of the (maximally three in the present, simplest case) distinct steady solutions the perturbed flow finally approaches depends upon the time scale parameter τ of the perturbation; whereas the character of these possible alternative steady states is determined by the remaining parameters of the perturbation [D_o, R_p, and σ], which fix the spatial form and strength of the perturbation (Habbal & Tsinganos 1983). How is one of these steady states selected by virtue of the time scale of the perturbation?

The first point to note is that the flow is characterized by three distinct time scales: the advection time scale τ_a=R/v, the downstream signal propagation time scale τ_+ = R/(v+c) [corresponding to propagation along the C^+ characteristics], and the upstream signal propagation time scale τ_- = R/(v-c) [corresponding to propagation along the C^- characteristics]; the crucial element is that although τ_a and τ_+ are comparable in the region of interest (i.e., of order of 3 x 10^3 sec near the sonic point), τ_- may be very large; the question is then how τ compares with these three flow time scales.

Consider then the evolution of the solutions sketched in Figs. (a)-(d) [see also Fig. 2 of Paper I], which qualitatively reproduce the essential features of the time-dependent solutions. In each case, we have sketched the velocity profiles of the initial Parker-type flow field, v_{in}(R,t=0), the final equilibrium flow field v_{eq}(R,t→∞) consistent with some given set of the spatial parameters [D_o, R_p, σ] characterizing D(R) and, finally, an intermediate velocity field v(R,δt) which results after, say, a time δt of the order 10^4 secs of momentum deposition. The question we would like to address then is: given the initial state v_{in}(R,t=0) and the parameters [D_o, R_p, σ, τ] of our momentum deposition, what is the final state towards which the intermediate state v(R,δt) evolves? That is, which of the three possible states -- the continuous transonic solution, or any one of the allowed (if any) discontinuous transonic solutions -- the system chooses to relax to after a sufficiently long evolution?

In the first case (Fig. a), no difficulty arises: in this case, the asymptotic value of D is sufficiently small that the steady perturbed solution (corresponding to t →∞) is unique and continuous, and always passes through the outer critical point; the eventual evolution of the intermediate solution is hence unambiguous. This is, however, not the case for the three other examples given, for which the asymptotic value of D is sufficiently large to result asymptotically in three distinct steady transonic solutions. In the first of the latter cases (Fig. b), τ is large (≈40,000 secs) when compared to τ_+ and τ_a everywhere, and when compared to τ_- almost everywhere. Hence the flow profile evolves slowly, and the propagation of disturbances along characteristics may be viewed as occurring on a background flow field which is relatively steady almost everywhere; the exceptional location is near the sonic point(s), where the C^- characteristic becomes vertical (as can be seen from the sketch of the C^- characteristics in Fig. f, derived from the numerical solutions given in Paper I). Thus, the flow is near equilibrium essentially during the entire course of its history; but which sequence of equilibria is followed? The key point is that this sequence of equilibria (corresponding to a gradually increasing amplitude for D_o, cf. Habbal & Tsinganos 1983) is characterized by always having an outer sonic point downstream from the (eventual) inner critical point, and hence always has a region of subsonic flow between these two "x-type" critical points. Hence, once the flow becomes supersonic near the (eventual) inner critical point, the C^- characteristics are "trapped" between the two critical points, and the downstream-facing C^- characteristics emanating just downstream from the inner critical point must intersect the upstream-facing C^- characteristics emanating from the subsonic region just upstream from the outer critical point; a shock results (Fig. f), and the subsequent sequence of quasi-steady solutions corresponds to the discontinuous solution branch of the steady equations (for this choice of τ, that corresponding to the upstream shock). Note that from the characteristics one may readily obtain the time scale for, viz., formation of shocks by locating their first intersection.

The opposite extreme is obtained for very small τ (≈10 secs; Fig. d); in this case the time scale for the perturbation is far shorter than any signal propagation time scale in the flow. Hence, the flow is never near any equilibrium solution during the time of significant change in the perturbing momentum deposition term; for 0 < t < τ, the flow's evolution is essentially entirely determined by



Figures (a)-(d) are sketches (taken from the solutions obtained in Paper I) of the solution topologies for the velocity field of the wind equations (2) with a localized momentum deposition function similar to the gaussian given by expression (1), for different values of the parameters D_o, R_p, σ and τ and at three representative times: t=0, t=δt>0 and t→∞. The dotted solution is the initial velocity v_{in}(R,t=0), the dashed solution is the intermediate velocity v(R,δt) and the solid velocity curves, v_{eq} (R,t→∞), represent the solution topologies of the steady state equations (2). All figures (a)-(d) have the same values of the parameters R_p and σ while figures (b)-(d) correspond to the same set of the spatial parameters [D_o, R_p, σ] with τ decreasing from (b) to (d). The two vertical solid lines in figures (b)-(d) indicate the location of the steady state shocks.

Figures (e)-(f) are sketches of the C^- characteristics. In Fig. (e) are plotted the fan-like characteristics of the initial flow v_{in}(R,t=0); and in Fig. (f) the intersecting characteristics of the evolving flow field (Fig. b). The effect of this momentum addition with a large value of τ is to "turn" downstream the characteristics just downstream of the inner critical point such that they intersect the upstream facing characteristics just upstream of the outer critical point.

the local acceleration due to the imposed perturbation. Thus, by the time the perturbation has reached its final amplitude and form, the flow has been locally accelerated (by the external force) to virtually its final (steady) spatial form throughout the region of application of the perturbation. Note that the discontinuity which temporarily forms is advected downstream from the position of the second shock because the C^- characteristics immediately downstream from this position face downstream; the continuous transonic solution is obtained.

Finally, in the intermediate case sketched in Fig. (c), the intermediate solution $v(R,\delta t)$ finds itself close to the equilibrium solution which involves the second (downstream) shock. Subsequent propagation along the C^- characteristics leads to the steepening of the velocity profile and the intersection of the characteristics at a location between the two equilibrium shocks; the intersection is convected downstream until it finds an equilibrium at the position of the second shock, and the system relaxes to the equilibrium solution which involves this second shock.

In all these cases the discontinuity forms because of the dispersive properties of the non-uniform background flow which lead to the gradual intersection of the C^- characteristics. The value of the time constant of the perturbation τ determines how soon and where in the flow, in relation to the position of the equilibrium shocks, this intersection occurs. In other words, τ determines how rapidly the fan-like C^- characteristics of the unperturbed flow (Fig. e) will be "turned" so as to "face" one another, i.e., converge, and inevitably intersect before the changes in the background flow are convected downstream past the positions of the equilibrium shocks (Fig. f). Notice that propagation along the C^+ characteristics does not lead to any interesting effects since the changes are advected away relatively fast due to the smaller value of τ_+.

4. Summary

We have shown that the formation of standing shocks in wind-type astrophysical flows with some effective, localized, non-thermal momentum deposition depending on the spatial parameters D_o, R_p and σ - which determine its amplitude, location and width, respectively - and the temporal parameter τ - which determines its temporal width - requires both *necessary* and *sufficient* conditions. The *necessary* conditions are simply the Rankine-Hugoniot relations for the conservation of mass, momentum and energy - but not entropy - across the shock discontinuity, which in turn depend on the set of the spatial parameters $[D_o, R_p, \sigma]$. The *sufficient* conditions are determined by the detailed temporal evolution of the non-thermal momentum addition and, in particular, on the intersection of the C^- characteristics of the flow upstream of the position of the equilibrium shocks, which in turn depends on the fourth parameter τ. It is expected that such standing shocks might be formed in astrophysical flows such as the solar and stellar winds, or flows in astrophysical jets, when these necessary and sufficient conditions are satisfied.

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