

## CONSTITUTIVE MODELS BASED ON COMPRESSIBLE PLASTIC FLOWS

A. M. Rajendran  
University of Dayton Research Institute  
Dayton, Ohio 45409

### 1. INTRODUCTION

The need for describing materials under time or cycle dependent loading conditions has been emphasized in recent years by several investigators (Ref. 1 through 4). In response to the need, various constitutive models describing the nonlinear behavior of materials under creep, fatigue, or other complex loading conditions were developed. The developed models for describing the fully dense (non-porous) materials were mostly based on uncoupled plasticity theory. The improved characterization of materials provides a better understanding of the structural response under complex loading conditions. However, the constitutive models describing the fully dense materials will be inadequate for characterizing the regions of the material where voids (porosity) develop due to various complex micromechanisms. For instance, voids may nucleate under high temperature loading conditions due to intergranular cavity formation around the second phase particles (Ref. 5). The necked portion of a tensile specimen and the ductile material at the crack tip are the few examples where the initially non-porous material becomes a porous aggregate due to debonding of the hard particles from the matrix. In these regions, the stress-strain relationship of the porous aggregate starts deviating from the matrix material behavior.

Several authors considered this aspect of the problem. Among them, Gurson (Ref. 6) presented a continuum theory of ductile rupture by void nucleation and growth and he came up with a constitutive equation for void containing materials, which explicitly considered the void volume fraction and the matrix stresses.

The constitutive models for compressible porous materials based on Gurson's yield criterion, was employed by Yamamoto (Ref. 7) and also by Needleman and Triantafyllidis (Ref. 8) in a study of shearband localization in metal sheets and the influence of void growth on forming limit diagrams, respectively. These authors, while describing the porous aggregate, used an idealized simple rate-independent power-law type constitutive model to describe the incompressible matrix material. Their main purpose was to

predict the onset of localized necking or shearband localization through an approximate description of the porous aggregate and the matrix material. However, it is important to describe the matrix material behavior more accurately in order to properly characterize the porous region of the solid material under complex loading conditions.

The present paper provides a simple methodology to introduce void nucleation and its growth into the nonlinear incompressible constitutive equation through Gurson's yield criterion which is based on compressible plastic flow. This yield criterion is combined with the state variable flow theory of Bodner and Partom (Ref. 4), for the incompressible solid. Stouffer and Bodner (Ref. 9), have demonstrated the predictive ability of the state variable theory by applying it to high temperature nickel base super-alloys, such as IN100 and Rene' 95. Since the matrix material behavior is well characterized, this will result in an improved description of the porous material under complex loading conditions.

The usefulness of the present approach is its capability for establishing meaningful stress-strain behavior of a localized damage zone in which void initiation and growth is occurring and also of the surrounding zone of void free material.

## 2. CONSTITUTIVE MODEL FOR COMPRESSIBLE SOLID

To describe the void containing aggregate, the slightly modified version of Gurson's yield criterion as proposed by Tvergaard is considered (Ref. 10). The corresponding yield criterion used in the present paper is

$$\Phi = \frac{3J_2}{Y_m^2} + 2q_1 f \cosh\left(\frac{q_2 I_1}{2Y_m}\right) - q_3 f^2 - 1 = 0 \quad (1)$$

where  $J_2$  is the second invariant of the stress deviator,  $I_1$  is the first stress invariant,  $Y_m$  is the equivalent stress of the matrix material,  $f$  is the current void volume fraction, and  $q_1$ ,  $q_2$ , and  $q_3$  are the void shape factors. The yield function, based on the spherically symmetric deformation of a rigid perfectly plastic body around a spherical void, as derived by Gurson (Ref. 6) can be retrieved by setting  $q_1 = q_2 = q_3 = 1$  in equation 1.

Since the plastic work done by the aggregate is equal to the plastic work done by the matrix material, the plastic strain-rates in the aggregate ( $\dot{\epsilon}_{ij}^P$ ) and the matrix ( $\dot{D}_m^P$ ) are related by the following expression

$$\sigma_{ij} \dot{\epsilon}_{ij}^P = (1 - f) Y_m \dot{D}_m^P \quad (2)$$

where  $\sigma_{ij}$  is the aggregate stress and the dot represents the time derivative.

The plastic strain-rates of the aggregate can be expressed in terms of the flow rule of the yield function as,

$$\dot{\epsilon}_{ij}^P = \Lambda \frac{\partial \Phi}{\partial \sigma_{ij}} \quad (3)$$

where  $\partial \Phi / \partial \sigma_{ij}$  is the partial derivative of the yield function with respect to the aggregate stresses. The proportionality factor,  $\Lambda$ , can be obtained by combining the equations (2) and (3) and the plastic strain-rates of the porous aggregate can be shown as

$$\dot{\epsilon}_{ij}^P = \frac{(1 - f) Y_m \dot{D}_m^P}{\frac{\partial \Phi}{\partial \sigma_{kl}} \sigma_{kl}} \frac{\partial \Phi}{\partial \sigma_{ij}} \quad (4)$$

where repeated indices  $k$  and  $l$  mean summation.

The nonlinear constitutive relationship for the porous aggregate can be expressed in terms of total strain-rate as sum of the elastic and plastic components. The corresponding relationship is given by,

$$\dot{\epsilon}_{ij} = \frac{(1+\nu)}{E} \dot{\sigma}_{ij} - \frac{\nu}{E} \delta_{ij} \dot{\sigma}_{kk} + \frac{(1-f) Y_m \dot{D}_m^P}{\frac{\partial \Phi}{\partial \sigma_{kl}} \sigma_{kl}} \frac{\partial \Phi}{\partial \sigma_{ij}} \quad (5)$$

where  $\nu$  and  $E$  are poisson's ratio and elastic modulus, respectively. The above equation describes the porous aggregate for a given stress-strain behavior of the matrix material.

The void volume fraction rate ( $\dot{f}$ ) of the aggregate consists of two parts. The nucleation rate of voids ( $\dot{f}_n$ ) at various stages of the deforming solid contribute to the current void volume fraction rate as the first part. The second part is due to the growth ( $\dot{f}_g$ ) of the already nucleated voids. The growth law is easily obtained by equating the volume change of the voids to the dilation as

$$\dot{f}_g = (1 - f) (\dot{\epsilon}_{11}^P + \dot{\epsilon}_{22}^P + \dot{\epsilon}_{33}^P) \quad (6)$$

There are few models at present, available in the literature to approximately model the nucleation rate of the voids at room temperatures. However, for high temperature applications, it is important to consider a nucleation model based on an appropriate micromechanism, such as the intergranular cavitation around an inclusion (Ref. 5). For completion, in the present work, the plastic strain controlled void nucleation model as proposed by Goods and Brown, (Ref. 11) is arbitrarily considered. The particular form used by Chu and Needleman (Ref. 12) is given by

$$\dot{f}_n = \frac{\Psi}{s\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{D_m^p - e_n}{s}\right)^2} \cdot \dot{D}_m^p \quad (7)$$

where  $s$  is the standard deviation of the distribution and  $\Psi$  is determined so that the total void of volume nucleated is consistent with the volume fraction of second phase particles.  $e_n$  is a mean equivalent plastic strain for nucleation.

The total void volume fraction rate is then expressed as

$$\dot{f} = (1-f)(\dot{\epsilon}_{11}^p + \dot{\epsilon}_{22}^p + \dot{\epsilon}_{33}^p) + \frac{\Psi}{s\sqrt{2\pi}} \dot{D}_m^p e^{-\frac{1}{2}\left(\frac{D_m^p - e_n}{s}\right)^2} \quad (8)$$

To complete the description of the voided aggregate, it is now necessary to describe the matrix material with an appropriate constitutive model. For this purpose, the model developed by Bodner and Partom (Ref. 4) based on state variable theory is considered. The main advantage of this theory is its ability to describe the material response under various loading conditions. The following equation describes the constitutive relationships in terms of second invariant of the strain-rate ( $D_2^p$ ) to the second invariant of the stress deviator ( $J_2$ ), as

$$D_2^p = D_0^2 \exp\left[-\left(\frac{Z}{3J_2}\right)^n \left(\frac{n+1}{n}\right)\right] \quad (9)$$

where  $D_2^p = 1/2 \dot{\epsilon}_{ij}^p \dot{\epsilon}_{ij}^p$ . Here,  $\dot{\epsilon}_{ij}^p$  are the plastic strain-rate tensors of the matrix material,  $n$  is the strain-rate sensitivity parameter,  $Z$  is the inelastic state variable, and  $D_0$  is the limiting value of the plastic strain-rate in shear.

The evolution equation for Z is given by Bodner as

$$\frac{\dot{Z}}{Z} = 2m \left(1 - \frac{Z}{Z_1}\right) (D_2^P J_2)^{\frac{1}{2}} - A \left(\frac{Z - Z_2}{Z_1}\right)^2 \quad (10)$$

where m,  $Z_1$ ,  $Z_2$ , r, and A are material constants. M is a parameter that controls the rate of work hardening,  $Z_1$  and  $Z_2$  are saturation values of Z, and the value of Z corresponds to the complete non-work hardened condition, respectively. The constants A and r are needed to describe the recovery process of the material.

The main equation (9) can be written in terms of equivalent stress and strain of the matrix material and it is given by

$$\dot{D}_m^P = \frac{4}{3} D_o^2 \exp\left[-\left(\frac{Z}{Y_m}\right)^{2n} \left(\frac{n+1}{n}\right)\right] \quad (11)$$

The equations (10) and (11) together complete the description of the matrix material.

The nonlinear constitutive relationships for the compressible (porous) material are described by equations (5) through (11), along with the consistency condition for plastic loading ( $\dot{\phi} = 0$ ). However, to demonstrate the stress-strain behavior of the aggregate, uniaxial stress-strain relations can be obtained from the already derived governing equations. The following section describes the aggregate and the matrix stress-strain relations explicitly under uniaxial stress state.

### 3. UNIAXIAL CASE

The necessary equations to describe the voided aggregate under uniaxial stress state condition can be deduced from the governing equations (equations (1) through (11)). Uniaxial matrix plastic strain-rate ( $\dot{\epsilon}_m^P$ ) can be obtained from equation (11) as

$$\dot{\epsilon}_m^P = \frac{2D_o}{\sqrt{3}} \exp\left[-\frac{1}{2}\left(\frac{Z}{Y_m}\right)^{2n} \left(\frac{n}{n+1}\right)\right] \quad (12)$$

The corresponding matrix stress-rate can be obtained from the definition of total strain-rate as sum of the elastic and plastic components and it is given by

$$\dot{Y}_m = E(\dot{e}_m - \dot{e}_m^P) \quad (13)$$

The aggregate plastic strain-rates in the principal directions can be written using equation (5) as

$$\dot{e}_1^P = \frac{(1-f) Y_m \dot{e}_m^P}{\sigma} \quad (14)$$

$$\dot{e}_2^P = \frac{(1-f) Y_m \dot{e}_m^P (H-\sigma)}{\sigma(2\sigma+H)} \quad (15)$$

$$\dot{e}_3^P = \dot{e}_2^P \quad (16)$$

where  $H = q_1 q_2 f Y_m \sinh \xi$  and  $\xi = q_2 \sigma / 2Y_m$ . (17)

Here,  $\sigma$  represents the uniaxial aggregate stress.

An expression for the void volume fraction rate can be obtained by combining equations (14) through (17) with equation (8) as

$$\dot{f} = \frac{3(1-f)^2 H Y_m \dot{e}_m^P}{\sigma(2\sigma+H)} + \frac{\psi}{s\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{e_m^P - e_n}{s} \right)^2} \dot{e}_m^P \quad (18)$$

The aggregate stress-rate ( $\dot{\sigma}$ ) can be obtained from the consistency condition ( $\dot{\phi} = 0$ ) for loading and it is expressed as

$$\dot{\sigma} = \frac{-2Y_m^3 (q_1 \cosh \xi - f q_3) \dot{f} + \sigma(2\sigma+H) \dot{Y}_m}{(2\sigma+H)Y_m} \quad (19)$$

The uniaxial stress-strain relationships for the aggregate can be expressed through the total strain-rate as the sum of elastic and plastic components and they are given by

$$\dot{e}_1 = \frac{\dot{\sigma}}{E} + \frac{(1-f) Y_m \dot{e}_m^P}{\sigma} \quad (20)$$

$$\dot{e}_2 = -\frac{\nu \dot{\sigma}}{E} + \frac{(1-f) Y_m \dot{e}_m^P (H-\sigma)}{\sigma(2\sigma+H)} \quad (21)$$

$$\dot{e}_3 = \dot{e}_2 \quad (22)$$

The equations (12) through (22) can be simultaneously solved through numerical integration and the aggregate stress-strain response can be computed for various matrix stress or strain-rate conditions.

#### 4. RESULTS

The stress-strain behavior of the aggregate with voids is computed by simultaneously solving the uniaxial equations through an appropriate numerical integration. The computations are made for an imposed matrix under constant stress or strain-rate conditions to facilitate comparing the reduced strength (or stiffness) of the aggregate to that of the fully dense matrix material.

Since the material constants for describing the materials Rene' 95 and IN100 at 650°C and 730°C, respectively, under complex loading conditions are readily available, the stress-strain response of the porous aggregate is calculated assuming that these materials represent the matrix materials in this study. Apart from the arbitrarily chosen nucleation model as explained in Section 2, for illustrative purposes, a simple nucleation criterion based on voids being nucleated at the onset of plastic deformation, is assumed in these calculations. The assumed value for the void volume fraction represents the initial void constant of the aggregate.

The void shape factors  $q_1$ ,  $q_2$ , and  $q_3$  that appear in the yield function described by equation 1, can be determined based on (a) the values already available in the literature and (b) the results obtained from the experiments on sintered materials. The effect of various values of these constants on the yield function is shown in Figure 1, for  $f = 0.15$ . As can be seen in the figure, the yield function is shown as the variation of  $\sqrt{3J_2}/Y_m$  with respect to  $(I_1/Y_m)$  for a given value of  $f$  and the other constants. For  $f = 0$ , the yield criterion becomes obviously independent of the hydrostatic pressure ( $I_1$ ) and represents the von-mises yield criterion for an incompressible solid. Whereas for  $f \neq 0$ , the yield functions represented by the curves A or B show the dependency on the level of void contents. The curve A represents the Gurson's yield function ( $q_1 = q_2 = q_3 = 1$ ) while the curve B is the yield function used by Tvergaard (Ref. 10) with  $q_1 = 1.5$ ,  $q_2 = 1.0$ , and  $q_3 = 2$ . The experimentally obtained single point\* as shown in the

---

\*Experiments Conducted in Air Force Materials Laboratory, Wright-Patterson Air Force Base, Dayton, Ohio.

figure, represents the results of an uniaxial compression test on a sintered material of void volume fraction,  $f$ , equals 0.15. It can be seen from these results that the yield criterion based on the values of  $q_1$ ,  $q_2$ , and  $q_3$  which are available in the literature, is significantly off from the experimental result. However, when improved values for  $q_1$ ,  $q_2$ , and  $q_3$ , obtained by trial and error, were used, the theoretical predictions for  $f = 0.15$  and  $f = 0.25$  were quite compatible with the experimental values at least for the case of uniaxial stress state as shown in Figure 2.

To demonstrate the effect of the various  $q_1$ ,  $q_2$ , and  $q_3$  on flow stress, the uniaxial stress-strain curves of the porous aggregate for a constant matrix strain-rate of  $1.4 \times 10^{-3} \text{ sec}^{-1}$  are shown in Figure 3. These curves clearly show the differences in the predictive stress levels by the three sets of values chosen for these constants. However, in the present calculations, the improved values of  $q_1$ ,  $q_2$ , and  $q_3$  are used to describe the porous material behavior under uniaxial stress-state.

The various levels of flow stress of a porous aggregate with IN100 as the matrix material for different void contents are shown in Figure 4 using the simple nucleation criterion. The dotted line corresponds to the flow stress level of fully dense matrix material. The reduced strength of the material due to the presence of ten percent void content can be seen from this figure. The initially nucleated voids grow during the plastic deformation according to the growth law represented by equation (6). The increasing void volume fraction ( $f$ ) normalized by the initial value  $f_0$  is shown in Figure 5 for the three values of  $f_0$ , corresponding to the earlier Figure 4. It can be seen from the figure that the growth levels are almost the same in these cases. Since the plastic strain levels under uniaxial stress conditions are of the same order, the plastic strain-rate based growth law predict the same order of growth.

To demonstrate the effect of void nucleation model on the flow stress, solutions were obtained for the nucleation model discussed in Section 2. The results for various nucleation strains are shown in Figure 6. For  $\Psi = 0.05$  and  $s = 0.01$  (narrow range of nucleation strain), the curve corresponds to  $e_n = 0.01$  shows the entire nucleation to occur between A and B. The rapidly reducing strength of the material due to the entire void nucleation occurring in the narrow range, stabilizes beyond point B. The



steady drop in the flow stress level later on depends mainly on the void growth in the material.

For  $e_n = 0.05$ , the nucleation process starts at point C and the stress-strain behavior of the aggregate is the same as that of the matrix due to the absence of any void up to point C. The stress-strain curve for the aggregate and the matrix material are identical for the case  $e_n = 0.1$  due to the absence of void nucleation up to the strain corresponding to point D. The variations of void volume fraction with respect to the aggregate strain for  $e_n = 0.01$  and  $e_n = 0.05$  are shown in Figure 7. The rapid increase in  $f$  as shown by the curves between AB and CD are due to the nucleation of new voids and the growth of existing voids. When the nucleation process is completed over the narrow range of strain, the increase in void volume fraction, later is due to the growth of the nucleated voids alone. The rate of increase stabilizes beyond the points B and D as shown in the figure.

As an additional exercise, the effect of the standard deviation,  $s$ , of the nucleation strain distribution on the flow stress for  $e_n = 0.01$  and  $\Psi = 0.05$  is shown in Figure 8. It can be seen from the figure as the distribution takes place over a broad range of strain ( $s = 0.05$  and  $0.1$ ) the decay in strength due to void nucleation and growth is less pronounced with a steady decline. The corresponding increase in the void volume fractions are shown in Figure 9.

The effect of strain-rate on the stress-strain response using the simple nucleation model is shown in Figure 10. The response of the fully dense matrix material (IN100) and also of the porous aggregate are obtained through the numerical solutions for various matrix strain-rates ( $\dot{D}_m$ ). The reduced strength of the material due to the presence of a low void content (two percent) can be seen from this figure.

As an additional description of the modeling procedure the creep response of the voided aggregate with a Rene' 95 matrix is demonstrated in Figure 11, when the matrix material creeps at different stress levels ( $Y_m = 1206$  and  $903$  MPA). The aggregate stresses are calculated for two different creep stress levels applied to the matrix material. For the higher matrix stress level, the stress in the aggregate is reduced due to five percent voids in the material. However, for the lower stress ( $Y_m = 903$  MPA), since the plastic flow has not yet initiated within the time shown in

Figure 11 (1200 seconds), voids are not nucleated. Thus, both the matrix and the aggregate with no voids creep at the same stress. Also, the corresponding strain responses are obviously the same as shown in Figure 12. Whereas the responses corresponding to higher stress, show the distinct difference for two different values of void volume fraction ( $f = 0.05$  and  $0.10$ ).

Thus, it can be seen from these results that the response of a porous aggregate to rate or time dependent loading conditions can depend on various material parameters that appear in the yield function and as well as in the nucleation model.

## 5. SUMMARY

The accuracy of modeling the porous aggregate behavior depends mostly on the (a) yield function which characterizes the compressible yield behavior, (b) description of the matrix material, and (c) nucleation model. It is demonstrated in the present studies that an approximate yield function to describe the porous aggregate can predict significantly different stress levels which may be inaccurate. It is important to test the yield function and its validity through carefully designed experiments under various stress conditions. As an example, it is shown that the values of the shape factors which appear in the yield function can be improved based on the experimentally obtained stress state at yielding. However, the values selected in this report based on two experiments may not be unique. Nevertheless, under uniaxial stress state conditions, these values may better characterize the yield function.

The improved characterization of the matrix material behavior under complex loading conditions through various nonlinear constitutive theories has been successfully achieved by several investigators. If the well defined and accurately described matrix material models are appropriately built into the constitutive models for the porous aggregate, that would substantially improve the characterization of the porous solid as demonstrated in the present studies.

The description of the porous material through an improved yield function and the matrix material model, may be accurate for a homogeneous, isotropic material with randomly distributed voids, such as the sintered

materials. However, for materials which were initially non-porous but developed porosity at some stage of the deformation due to various micro-mechanisms operating at the void nucleation site, the improved characterization of the porous aggregate will then also depend on the models for describing the nucleation process. The dependency of the flow stress on the parameters that describe the nucleation process is demonstrated for a model which was arbitrarily selected for illustrative purposes. For high temperature applications, it is important to select a model on a sound fundamental basis. Unfortunately, a continuum mechanics approach in this area is still lacking and needs more rigorous research efforts to model the complex nucleation process. The growth process of the nucleated voids seemed to be more or less temperature independent and also it is reasonably well established as a process which depends on the plastic strains (Ref. 13).

In summary, the present studies demonstrate that the rate or time dependency of the response of a porous aggregate can be incorporated into the nonlinear constitutive behavior of a porous solid by appropriately modeling the incompressible matrix behavior. It is also shown that the yield function which was determined by a continuum mechanics approach must be verified by appropriate experiments on void containing sintered materials in order to obtain meaningful numbers for the constants that appear in the yield function.

#### REFERENCES

1. Allen, D. H., Haisler, W. E., "The Application of Thermal and Creep Effects to Combined Isotropic-Kinematic Hardening Model," Tech. Report No. 3275-79-3, Texas A&M Univ., 31 May 1979.
2. Valanis, K. C. and Wu, H. C., "Endochronic Representation of Cyclic Creep and Relaxation of Metals," J. of Appl. Mech., 1975.
3. Laflen, J. H. and Stouffer, D. C., "An Analysis of High Temperature Metal Creep," Vol. 100, October 1978.
4. Bodner, S. R. and Partom, Y., "Constitutive Equations for Elastic-Viscoplastic Strain-Hardening Materials," J. of Applied Mech., June 1975.
5. Rishi Raj and Ashby, M. F., "Intergranular Fracture at Elevated Temperature," Acta. Met., Vol. 23, June 1975.
6. Gurson, A. L., "Porous Rigid-Plastic Materials Containing Rigid Inclusions-Yield Function, Plastic Potential, and Void Nucleation," Advances in Research on Strength and Fracture of Materials, Vol. 2a, Tablin, D. M. R., ed., Pergamon Press, New York, 1977.

7. Yamamoto, H., "Conditions for Shear Localization in the Ductile Fracture of Void-Containing Materials," International Journal of Fracture, Vol. 14, No. 4, 1978.
8. Needleman, A. and Triantafyllidis, N., "Void Growth and Local Necking in Biaxially Stretched Sheets," ASME Journal of Engr. and Mat. Tech., Vol. 100, April 1978.
9. Stouffer, D. C. and Bodner, S. R., "A Relationship Between Theory and Experiment For a State Variable Constitutive Equation," AFWAL-TR-80-4194, Tech. Report, January 1981.
10. Tvergaard, V., "Influence of Voids on Shear Band Instabilities Under Plane Strain Conditions," Report No. 159, Danish Center for Applied Math and Mech., June 1979.
11. Goods, S. H. and Brown, L. M., "The Nucleation of Cavities by Plastic Deformation," Acta Metallurgica, Vol. 27, 1979.
12. Chu, C. C. and Needleman, A., "Void Nucleation Effects in Biaxially Stretched Sheets," J. of Engr. Mat. and Tech. Vol. 102, July 1980.
13. Lagneborg, R., "Creep of Engineering Materials and Structures," Edited by Bernasconi and Piatti, 1980.

### YIELD FUNCTION (GURSON)

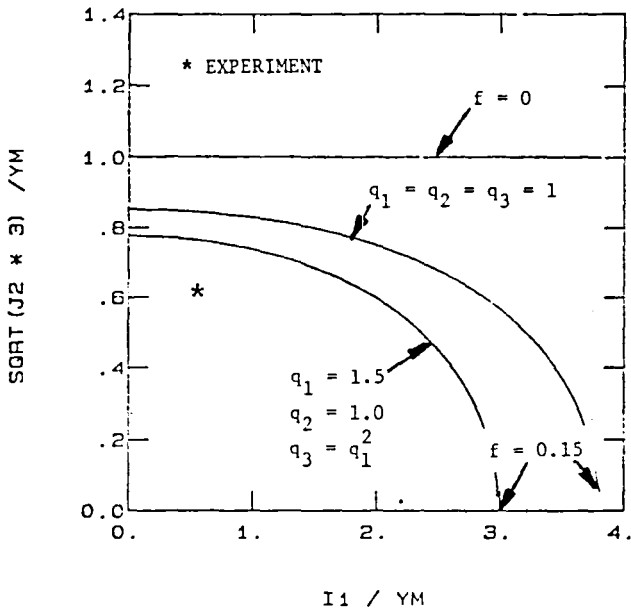


Fig. 1. Effect of the Shape Factors on the Yield Function for a Porous Solid.

### YIELD FUNCTION (GURSON)

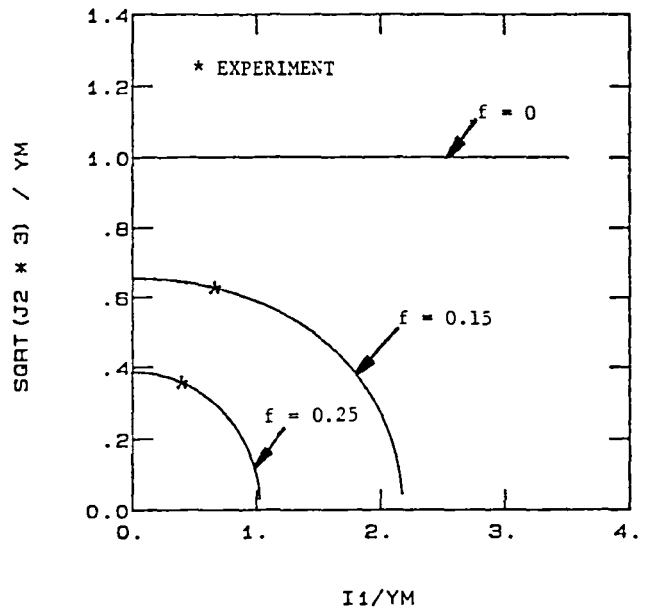


Fig. 2. Gurson's Yield Function With  $q_1 = 2.2$ ,  $q_2 = 1.0$ ,  $q_3 = 4$  For  $f = 0.15$  and  $f = 0.25$ .

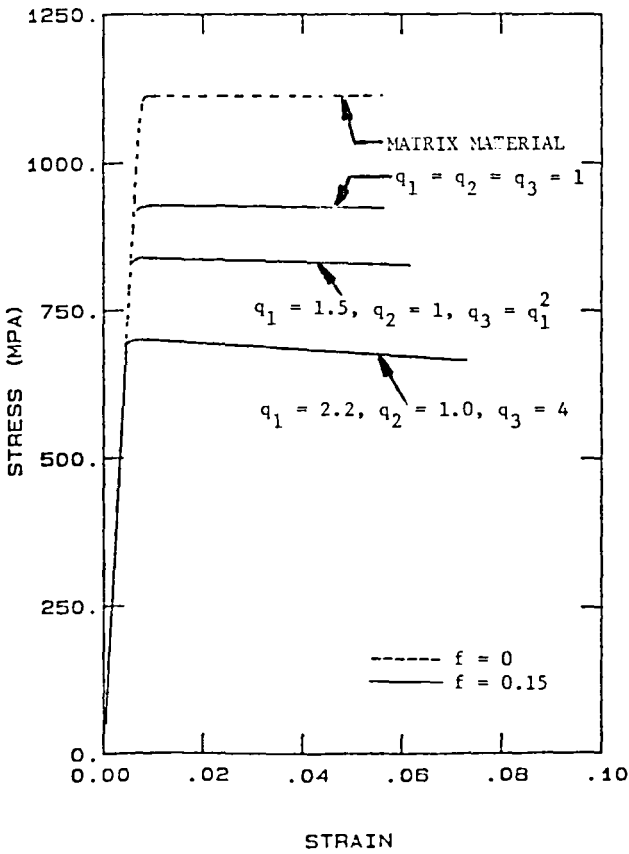


Fig. 3. Effect of the Shape Factors on the Uniaxial Stress-Strain Curves for a Porous Aggregate With IN100 as Matrix Material.

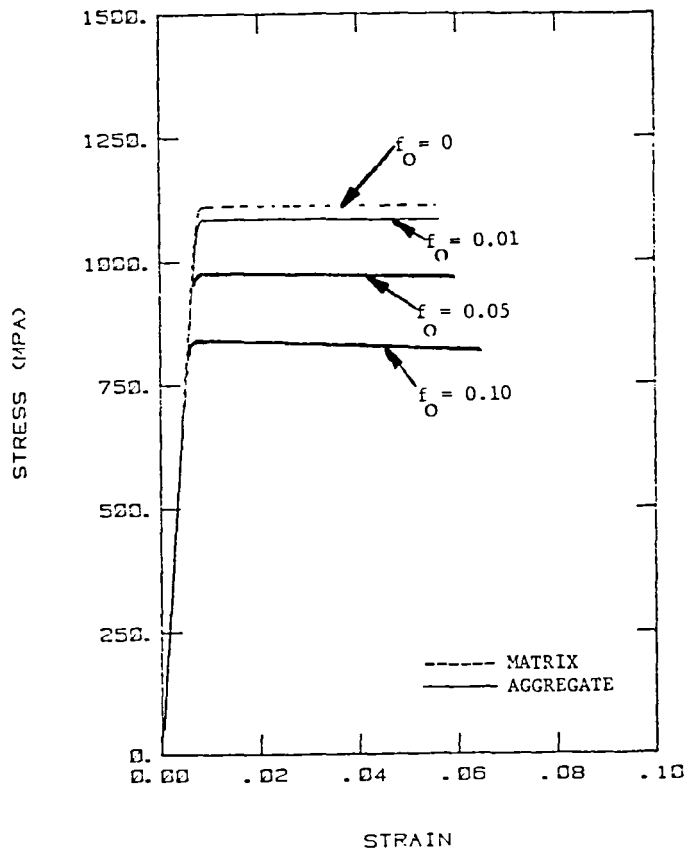


Fig. 4. Effect of Void Volume Fraction on the Flow Stress.  $f$  is the Initial Value of  $f$ .  $\dot{D}_m$  is the Strain-Rate in the IN100 Matrix.

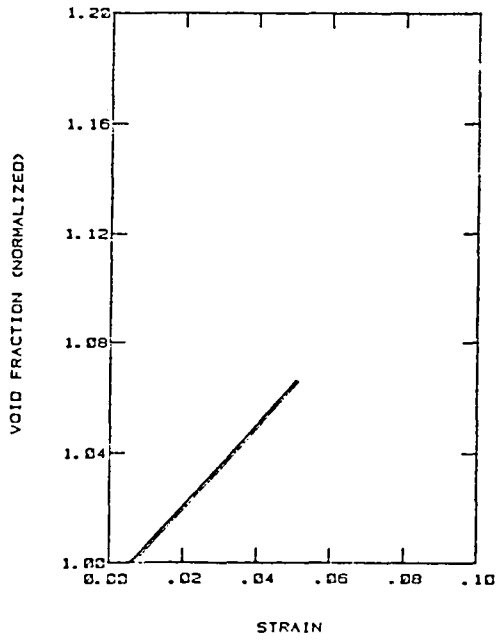


Fig. 5. Variation of Void Growths With Respect to the Strain in the Porous Aggregate for Different Initial Void Volume Fraction Levels.

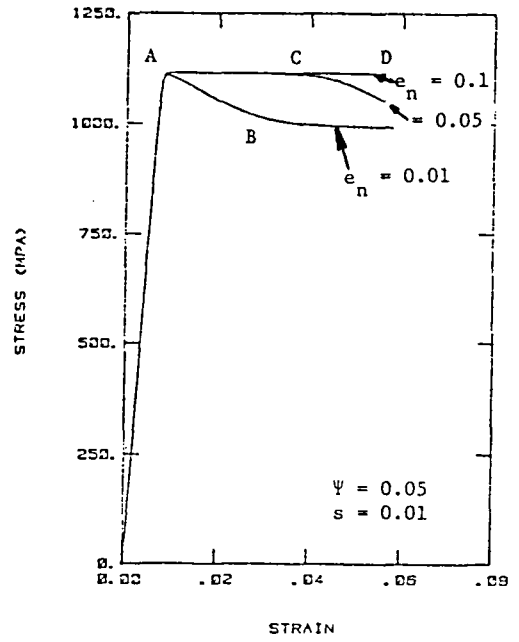


Fig. 6. Effect of Void Nucleation Model on the Flow Behavior of the Voided Aggregate for  $\dot{D}_m = 1.4 \times 10^{-3} \text{ sec}^{-1}$ .

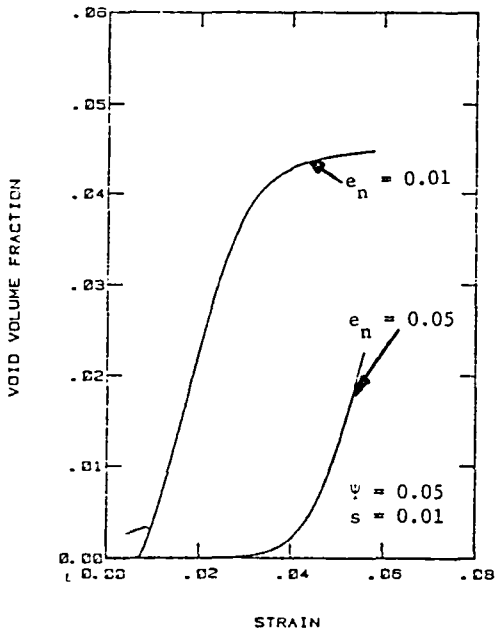


Fig. 7. Effect of Nucleation Strain ( $e_n$ ) on the Growth of Void Volume Fraction with Respect to Strain.

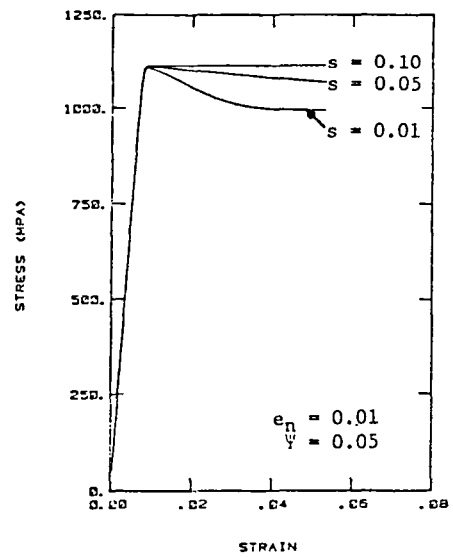


Fig. 8. Effect of Standard Deviation ( $s$ ) of the Nucleation Strain Distribution on the Voided Aggregate Flow Stress for a Constant Matrix Strain-Rate ( $\dot{D}_m = 1.4 \times 10^{-3}/\text{sec}$ ).

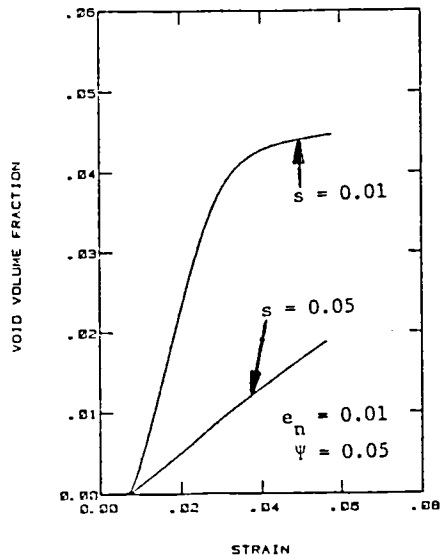


Fig. 9. Effect of 's' on the Void Volume Fraction with Respect to the Strain in the Aggregate.

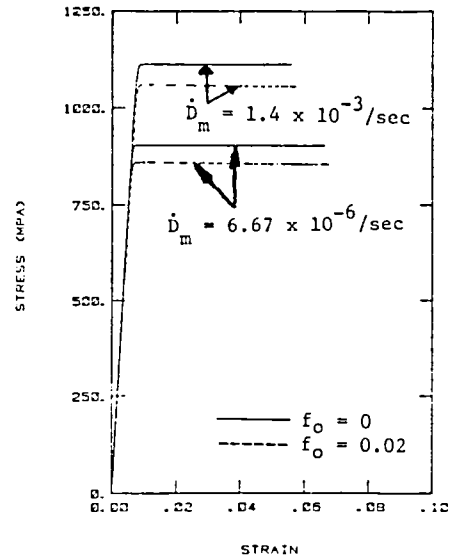


Fig. 10. Effect of Strain-Rate on the Matrix (IN100) and the Voided Aggregate.  $\dot{D}_m$  is the Matrix Strain-Rate.  $f_0$  is the Initial Void Volume Fraction.

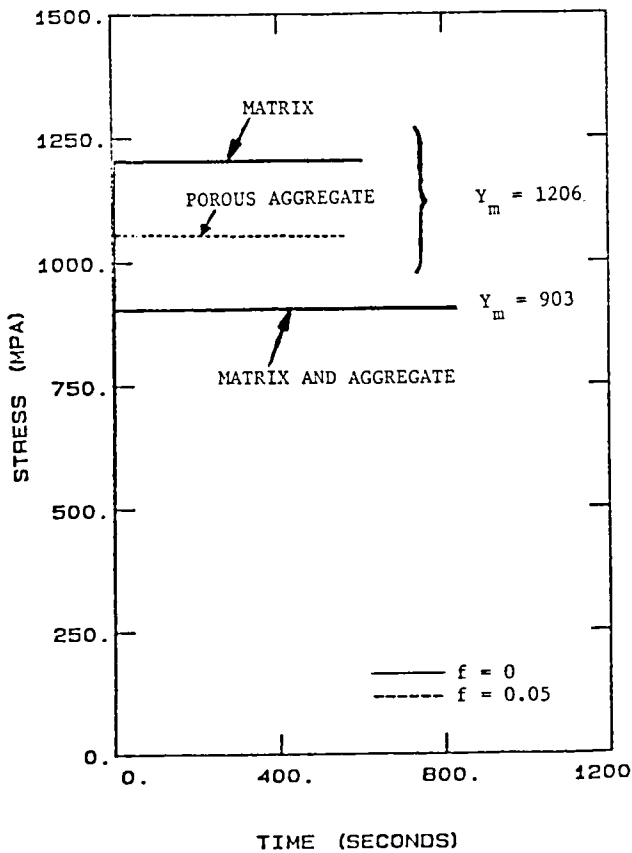


Fig. 11. Effect of Creep in the Aggregate for Various Matrix Creep Stress Levels at 650°C.  $Y_m$  is the Matrix (Rene' 95) Stress.

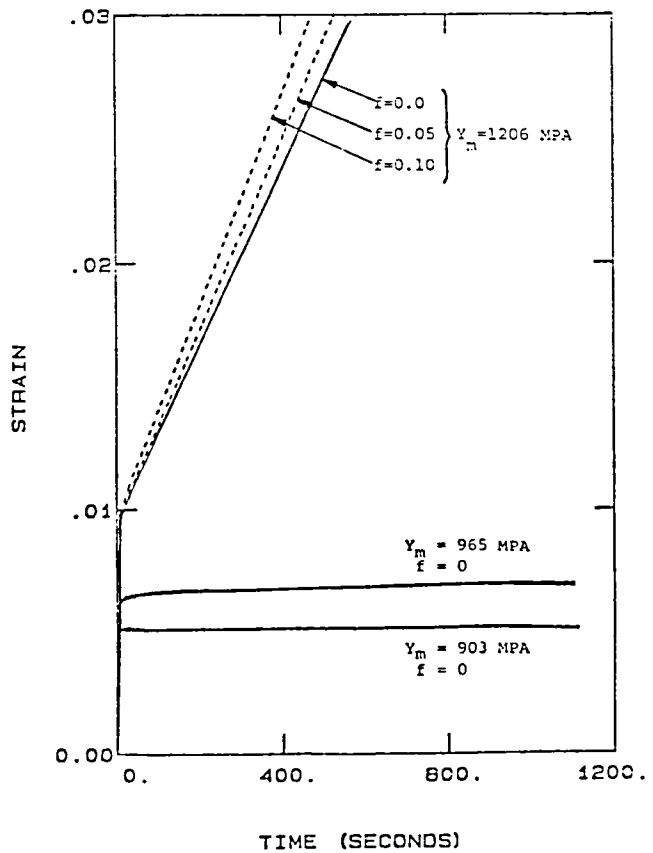


Fig. 12. Creep Response of the Aggregate and the Matrix at 650°C.  $Y_m$  is the Matrix Stress,  $f$  is the Void Volume Fraction.