HEURISTIC EXPLANATION OF JOURNAL BEARING INSTABILITY

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SUMMARY

A fluid-filled journal bearing is viewed as a powerful pump circulating fluid around the annular space between the journal and the bearing. A small whirling motion of the journal generates a wave of thickness variation progressing around the channel. The hypothesis that the fluid flow drives the whirl whenever the mean of the pumped fluid velocity is greater than the peripheral speed of the thickness-variation wave is discussed and compared with other simple explanations of journal-bearing instability. It is shown that for non-cavitating long bearings the hypothesis predicts instability onset correctly for unloaded bearings but gradually overpredicts the onset speed as the load is increased.

INTRODUCTION

One of the important causes for high speed rotor instability is the oil whip phenomenon in hydrodynamically lubricated journal bearings. It was first reported in 1925 by Newkirk and Taylor (ref. 1) who described several experiments and gave a simple explanation of why a lightly loaded journal whirls at half the frequency of rotation. In the present paper Newkirk and Taylor's argument is reexamined and compared with a heuristic hypothesis briefly suggested (ref. 2) in the first Workshop on Rotordynamic Instability in High Performance Turbomachinery in 1980. When compared with conventional dynamic stability analysis both arguments are incomplete. Nevertheless, both arguments predict the instability onset speed correctly for unloaded full circular bearings. The heuristic hypothesis can additionally be applied to loaded bearings for which it makes useful approximate predictions of instability onset speeds for moderate loads. For simplicity the discussion is centered on the case of a full circular bearing with two-dimensional non-cavitating flow. To set the stage, the classical Sommerfeld analysis (ref. 3) for equilibrium under a constant load is reviewed. After this the problem of whirling stability is discussed, first for an unloaded bearing, and then for a loaded bearing.

SOMMERFELD'S ANALYSIS

The idealized case treated by Sommerfeld (ref. 3) in 1904 is sketched in Fig. 1. A journal of radius R rotates at a fixed angular rate Ω within a full circular bearing of radius $R + h_0$ where the radial clearance h_0 is very small in comparison to R. The annular space between journal and bearing is filled with an incompressible fluid lubricant with uniform viscosity μ. The fluid flow is taken to be two-dimensional; i.e., axial flow is assumed to be negligible. In addition it is assumed
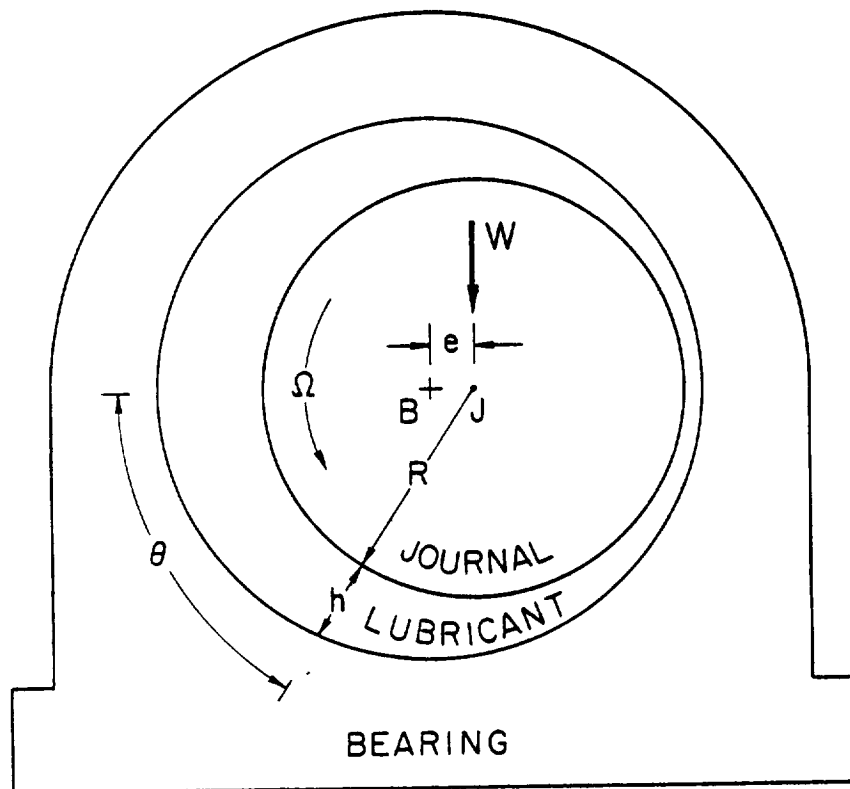
Figure 1. - Equilibrium configuration of journal rotating at rate Ω under load W. Journal center J displaced distance e with respect to bearing center B.

that cavitation does not occur. The width of the bearing normal to the plane of the figure is b.

Sommerfeld's principal result is that in the equilibrium position under a vertical load W the journal is eccentrically displaced by a distance e in the horizontal direction. Because of the eccentricity e the film thickness h varies with position θ around the annular space. For $h_0 \ll R$ the approximate relation is

$$h = h_o + e \cos \theta = h_o (1 + a \cos \theta) \tag{1}$$

where $a = e/h_0$ is the eccentricity ratio. Under the assumptions of laminar viscous flow with no pressure variation across the thickness of the film, the only possible flow patterns that satisfy the requirements of fluid mechanics in a uniform channel of thickness h are combinations of the two basic patterns shown in Fig. 2. Here x is distance along the channel and y is distance across the channel with $0 < y < h$. The fluid velocity (in the x-direction at position y) is denoted by u. The volume flow rate across a section of the channel of width b normal to the plane of the figure is denoted by Q and $\partial p/\partial x$ is the pressure gradient along the channel. For the linear profile at the left of Fig. 2 the pattern depends on the parameter U which is the velocity of the upper channel wall (the velocity of the lower channel wall is taken to be zero). For the parabolic profile at the right of Fig. 2 the pattern depends on the parameter A which is a velocity whose magnitude is four times the peak velocity in the profile or six times the average velocity. While the

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$$u = U\, y/h$$

$$Q = U\, bh/2$$

$$\frac{\partial p}{\partial x} = 0$$

$$u = A(y/h - y^2/h^2)$$

$$Q = A\, bh/6$$
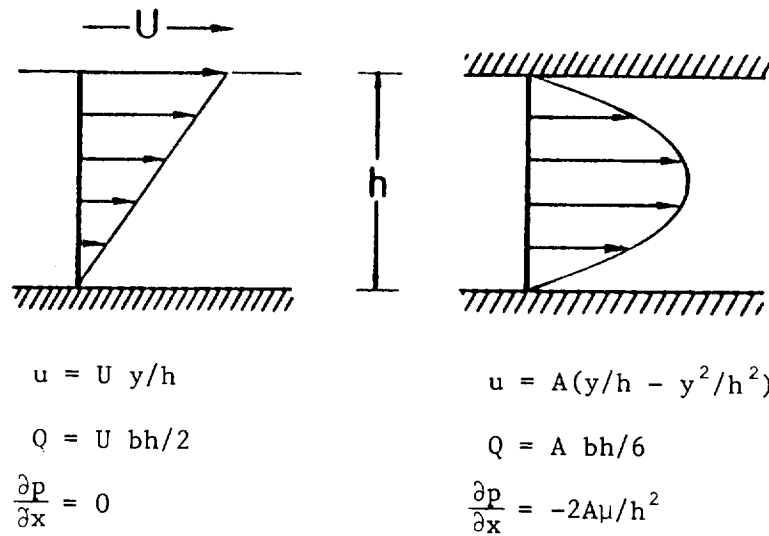
$$\frac{\partial p}{\partial x} = -2A\mu/h^2$$

Figure 2. - Component laminar flows in narrow channel with uniform pressure across channel. Either linear or parabolic velocity profiles.

relations in Fig. 2 are strictly correct only for steady state flows with constant values of the parameters h, U, and A, the Reynold's theory of lubrication extends them to apply to slow variations, both in time t and space x, of these parameters.

In application to the bearing of Fig. 1, the component flows of Fig. 2 are superposed, with $x = R\theta$ and $U = R\Omega$, with h given by (1), and the undetermined parameter A to be fixed by the requirements of continuity $[\partial Q/\partial \theta = 0]$ and uniqueness of pressure $[p(\theta) = p(\theta + 2\pi)]$. The value of A so determined is

$$A_1 = R\Omega \left[\frac{6(1 - a^2)}{2 + a^2} \frac{1}{1 + a\cos\theta} - 3 \right] \tag{2}$$

and the total volume flow rate is

$$Q_1 = R\Omega b h_o \frac{1 - a^2}{2 + a^2} \tag{3}$$

The pressure gradients of the component flows in Fig. 2 are superposed and integrated to obtain the pressure distribution $p(\theta)$ acting on the journal. The resultant of the pressures acting on width b of the journal is a force, acting vertically upwards through the journal center J, of magnitude

$$W_1 = \frac{12\pi\mu b R^3 \Omega}{h_o^2} \frac{a}{(2 + a^2)(1 - a^2)^{\frac{1}{2}}} \tag{4}$$

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The viscous shearing stresses acting on the boundary of the journal also produce a resultant force on the journal but its magnitude is smaller than the pressure resultant (4) by a factor of order h_0/R, and thus may be neglected. The force (4) must then be equal and opposite to the applied load W for the journal to be in equilibrium.

The remarkable property of the Sommerfeld bearing model is that the equilibrium displacement is at right angles to the applied load. This characteristic implies that an unloaded bearing is unstable with respect to slow forward whirling motions of the journal. To see this, imagine that an external agent moves the center of the journal J of Fig. 1 in a circular path of radius e in the counterclockwise sense about the bearing center B. When the journal passes through the position shown in Fig. 1 if the motion is slow enough the lubricant flow and pressure distribution will be very nearly the same as for the equilibrium configuration shown there, which means that the fluid will be exerting a force very nearly equal and opposite to W on the journal. This force, in the same direction as the velocity of the journal center J in its circular path does positive work on the whirling motion. The Sommerfeld bearing thus promotes whirling instability for very slow forward whirling rates. To consider more rapid whirling rates it is necessary to extend the Sommerfeld analysis to include motion of the journal center J.

WHIRLING STABILITY OF UNLOADED BEARING

According to (4) the equilibrium position for an unloaded bearing (W = 0) has zero eccentricity (a = 0, e = 0). In this configuration the parameter A_1 of Eq. (2) vanishes and the lubricant flow pattern is simply the linear profile shown on the left of Fig. 2. The stability of this configuration is discussed, first by describing Newkirk and Taylor's explanation of half-speed whirl (ref. 1) and the
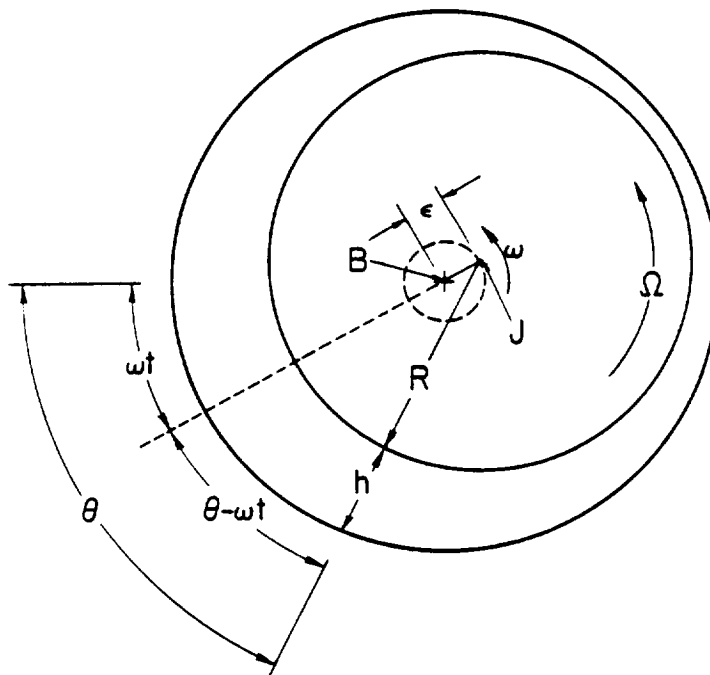

Figure 3. - Traveling wave of lubricant film thickness variation circling channel when center of rotating journal J whirls at frequency ω around circle of radius ϵ.

new heuristic hypothesis (ref. 2), and then by performing a conventional whirling stability analysis which sheds light on the preceding arguments. The bearing kinematics are displayed in Fig. 3 for the case where the journal center J whirls at the steady angular rate ω in a circle of radius ε about the equilibrium position in which J and B coincide. Note that the diametrally opposite sections of maximum and minimum film thickness advance around the bearing at the rate ω. At the location θ the film thickness is

$$h(\theta,t) = h_o + \varepsilon \cos (\theta - \omega t) \qquad (5)$$

Note that the dependence of h on space θ and time t is that of a <u>progressive wave</u> circling the annular channel with an angular phase velocity of ω or a linear phase velocity of $R\omega$.

The Newkirk and Taylor explanation (ref. 1) is based on an application of the continuity requirement to the flow in a channel whose thickness varies according to (5). The continuity requirement applied to a differential arc of length $Rd\theta$ is

$$\frac{\partial Q}{R\partial \theta} + b \frac{\partial h}{\partial t} = 0 \qquad (6)$$

Newkirk and Taylor assumed that the lubricant flow retains the linear profile of Fig. 2 so that $Q = R\Omega\, b\, h/2$ which reduces (6) to

$$\frac{\Omega}{2} \frac{\partial h}{\partial \theta} + \frac{\partial h}{\partial t} = 0 \qquad (7)$$

When $h(\theta,t)$ from (5) is substituted in (7) the result is that continuity cannot be satisfied unless

$$\omega = \frac{\Omega}{2} \qquad (8)$$

Newkirk and Taylor took this to provide analytical verification of the half-frequency whirl phenomenon which they observed in a vertical shaft running in a bearing with plentiful oil supply. It also appeared to explain the "Oil Resonance" peak in response when the rotation rate was twice the natural frequency of the system. The simple result (8) was less satisfactory in explaining the oil-whip phenonenon where the whirling frequency remains at the natural frequency as the rotation rate Ω is increased, although it was noted that the onset of whipping always occured at speeds equal to or greater than twice the natural frequency.

In the heuristic hypothesis (ref. 2) the rotating journal is considered to be a pump impeller which maintains the lubricant flow pattern with linear profile when the journal is unloaded and centered. The fluid velocity varies linearly from u = 0 at the bearing to $u = R\Omega$ at the journal. This flow can be decomposed into a mean flow with uniform velocity $u = R\Omega/2$ and no vorticity plus a residual flow with zero mean velocity and large vorticity. It is assumed that the mean flow is available to encourage (or discourage) small whirling perturbations of the centered configuration. If a whirl involving a thickness variation wave of the form (5) is imposed the heuristic hypothesis (ref. 2) postulates that energy will be pumped into the whirl if the mean flow velocity $R\Omega/2$ is greater than the phase velocity $R\omega$ of the whirl around the periphery of oil film. Conversely energy will be removed from the whirl if the phase velocity of the whirl is greater than the mean velocity of the lubricant. Neutral stability occurs when these velocities are equal; i.e., when $\omega = \Omega/2$. This

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hypothesis thus "explains" half-frequency whirls and oil whip for a system which has an unloaded bearing with plentiful oil supply. If the system provides little restraint on the journal its motion will be primarily determined by the fluid film forces acting on it. When a low frequency whirl ($\omega < \Omega/2$) is accidently started energy will be pumped into the whirl, accelerating the whirl until ω gets sufficiently close to $\Omega/2$ that the energy pumped into the whirl just balances the system energy losses in a steady "half-frequency" whirl. If the system provides considerable restraint on the journal and only permits appreciable whirling motion at a natural frequency the fluid film in the bearing will remove energy from accidental whirls at a natural frequency ω whenever $\Omega < 2\omega$. However, when $\Omega > 2\omega$ the fluid film will pump energy into any such whirl. Whether or not oil whip occurs depends on whether the energy supplied by the fluid film is sufficient to overcome the system losses.

Both of the previous explanations are incomplete in the sense that they do not make use of all the physical requirements involved. Both arguments use the flow pattern of the underlying steady centered flow and both use the kinematics of a small whirling perturbation. The Newkirk and Taylor argument also makes explicit use of the continuity requirement for the perturbation but neither explanation explicitly invokes any consideration of the pressures developed in the oil film.

Historically, the first complete dynamic analysis of the whirling stability of an unloaded bearing was given by Robertson in 1933 (ref. 4). The development which follows is essentially just a linearized version of Robertson's analysis. We consider the whirl defined by the eccentricity ε ($\varepsilon \ll h_o$) and the frequency ω to be a small perturbation on the underlying centered rotation in which the flow distribution is simply

$$u = R\Omega \, y/h_o, \quad 0 < y < h_o \tag{9}$$

When the journal is whirling the flow pattern will be a superposition of the two profiles of Fig. 2. To fit the conditions of Fig. 3 we take

$$u = R\Omega \, y/h + A(y/h - y^2/h^2) \tag{10}$$

where $h(\theta,t)$ is given by (5) and A is to be determined from the continuity requirement (6) and the requirement of single-valued pressure $[p(\theta) = p(\theta + 2\pi)]$. Using a linear perturbation analysis we neglect terms of order $(\varepsilon/h_o)^2$ in comparison with terms of order ε/h_o and find

$$A = -6R \, \frac{\varepsilon}{h_o} \, (\frac{\Omega}{2} - \omega) \cos(\theta - \omega t) \tag{11}$$

The corresponding total volume flow rate for the lubricant film is

$$Q = R\Omega bh/2 - \varepsilon R(\frac{\Omega}{2} - \omega)b \cos(\theta - \omega t)$$

$$= R\Omega bh_o/2 + \varepsilon Rb\omega \cos(\theta - \omega t) \tag{12}$$

The pressure distribution is obtained by integrating the pressure gradient given in Fig. 2. To first order in ε/h_o

$$p(\theta,t) - p_o = 12\mu R^2 \frac{\varepsilon}{h_o^3} (\frac{\Omega}{2} - \omega)\sin(\theta - \omega t) \tag{13}$$

The resultant force acting on the journal due to these pressures is directed at right angles to the journal displacement ε and has the magnitude

$$F = 12\pi\mu\varepsilon b \frac{R^3}{h_o^3} (\frac{\Omega}{2} - \omega) \tag{14}$$

The sense is such that when $\Omega/2 > \omega$, F has the same direction as the instantaneous velocity V_J of the journal center. The rate at which the fluid film forces do work on the journal (i.e., the power flow into the whirl) is

$$FV_J = 12\pi\mu b \frac{R^3}{h_o^3} \varepsilon^2\omega(\frac{\Omega}{2} - \omega) \tag{15}$$

This analysis shows that the amplitude A of the component flow with parabolic profile, the pressure in the fluid film, the resultant force on the journal, and the power flow into the whirl all are proportional to the factor $(\Omega/2 - \omega)$. Low speed whirls are encouraged and high speed whirls are discouraged. The whirling frequency of neutral stability is $\omega = \Omega/2$.

These results can be compared with the two simplified arguments considered previously. The assumption in the Newkirk and Taylor argument that the velocity profile remains linear is equivalent to assuming that the parameter A in Fig. 2 vanishes, which according to (11) implies that ω must equal $\Omega/2$. Furthermore the vanishing of A implies an absence of pressure gradient and consequently an absence of resulting force so that at the particular whirl frequency $\omega = \Omega/2$ the fluid film neither retards or advances the whirl. This is the neutral stability condition. The heuristic hypothesis (based simply on the flow pattern of the underlying centered configuration and the kinematics of the whirl perturbation) that the mean flow drives the whirl whenever the mean fluid velocity is greater than the phase velocity of the whirl is essentially a qualitative statement equivalent to the quantitative statement represented by eqn. (15). It happens that for an unloaded bearing the frequency of neutral whirl is given correctly by the heuristic hypothesis. For loaded bearings it is difficult to see how the Newkirk and Taylor argument can be extended to predict any other frequency of neutral whirl than $\omega = \Omega/2$. The heuristic hypothesis is however easily extended. It no longer predicts the exact frequency of neutral whirl but it provides useful approximations for moderate loads.

WHIRLING STABILITY OF LOADED BEARINGS

In this section a linear perturbation analysis is made to determine the whirling stability of the equilibrium configuration of Fig. 1. The heuristic hypothesis is then applied to predict an approximate value of the whirling frequency for neutral stability. We consider a small whirling motion of the journal centered about the equilibrium position of Fig. 1 where the journal center whirls with angular velocity ω in a counter-clockwise sense about a circle of radius

ε centered on the equilibrium position which lies a distance e to the right of the bearing center. In the equilibrium configuration the film thickness h, the parameter A_1, the volume flow rate Q_1, and the load W_1 are given by eqns. (1), (2), (3), and (4) respectively. When the whirling motion is added the film thickness becomes

$$h(\theta,t) = h_o + e \cos \theta + \varepsilon \cos (\theta - \omega t) \qquad (16)$$

With this value of h the velocity profile is taken to have the form of eqn. (10) with the parameter A to be determined anew from the continuity requirements (6) and the requirements of single-valued pressure [$p(\theta) = p(\theta + 2\pi)$]. Using a linear perturbation analysis with respect to ε/h_o (but including terms of all order with respect to $a = e/h_o$) we find

$$A = A_1 + \frac{\varepsilon}{h_o} \frac{6R}{1 + a \cos \theta} [\Omega f(\theta,t) + \omega g (\theta,t)] \qquad (17)$$

where

$$f(\theta,t) = - \frac{1 - a^2}{2 + a^2} \frac{\cos(\theta - \omega t)}{1 + a \cos \theta} - \frac{6a}{(2 + a^2)^2} \cos \omega t$$

$$(18)$$

$$g(\theta,t) = \cos(\theta - \omega t) + \frac{3a}{2 + a^2} \cos \omega t$$

Note that (17) reduces to (2) when $\varepsilon = 0$ and to (11) when $a = e/h_o = 0$. The corresponding total volume flow rate is

$$Q = Q_1 - \varepsilon Rb \left[6\Omega \frac{a}{(2 + a^2)} \cos \omega t - \omega g(\theta,t) \right] \qquad (19)$$

which reduces to (3) when $\varepsilon = 0$ and to (12) when $a = 0$. The corresponding pressure gradient from Fig. 2 when integrated gives the pressure distribution in the lubricant film. The resultant force acting on the journal due to these pressures has horizontal and vertical components given by

$$H = - \frac{12\pi\mu bR^3}{h_o^3 (1-a^2)^{3/2}} \left[\frac{1 - a^2}{2 + a^2} \Omega - \omega \right] \varepsilon \sin \omega t$$

$$(20)$$

$$V = W_1 + \frac{12\pi\mu bR^3}{h_o^3 (1-a^2)^{3/2}} \left[\frac{1 - a^2}{2 + a^2} \Omega - \omega + 3a^2 \frac{a^2\Omega + (2+a^2)\omega}{(2 + a^2)^2} \right] \varepsilon \cos \omega t$$

which reduce to (4) and (14) respectively when $\varepsilon = 0$ and when $a = 0$. The forces (20) may be decomposed into three forces: the steady-state load $W_1 = W$, a force proportional to [$(1 - a^2)\Omega/(2 + a^2) - \omega$] which whirls in phase with the velocity of the journal center, and an oscillating vertical force whose magnitude is small when

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a is small. The total work done on the journal by these forces during one whirling cycle; i.e., the energy per cycle ΔE imparted to the whirl is

$$\Delta E = \frac{12\pi^2\mu bR^3\epsilon^2}{h_o^3(2+a^2)(1-a^2)^{3/2}}\left[\frac{4 - 2a^2 + a^4}{2 + a^2}\,\Omega - (4 - a^2)\omega\right] \qquad (21)$$

The lubricant film forces are once more destabilizing for slow whirls and stabilizing for rapid whirls. The neutral stability whirl frequency

$$\omega = \frac{4 - 2a^2 - a^4}{8 + 2a^2 - a^4}\,\Omega = \frac{3 + (1-a^2)^2}{9 - (1-a^2)^2}\,\Omega \qquad (22)$$

varies from $\omega = \Omega/2$ at $a = e/h_o = 0$ for an unloaded bearing to $\omega \to \Omega/3$ when the load approaches infinity and the eccentricity ratio a approaches unity. The variation of ω/Ω according to (22) is represented by the curve labeled A in Fig. 4.
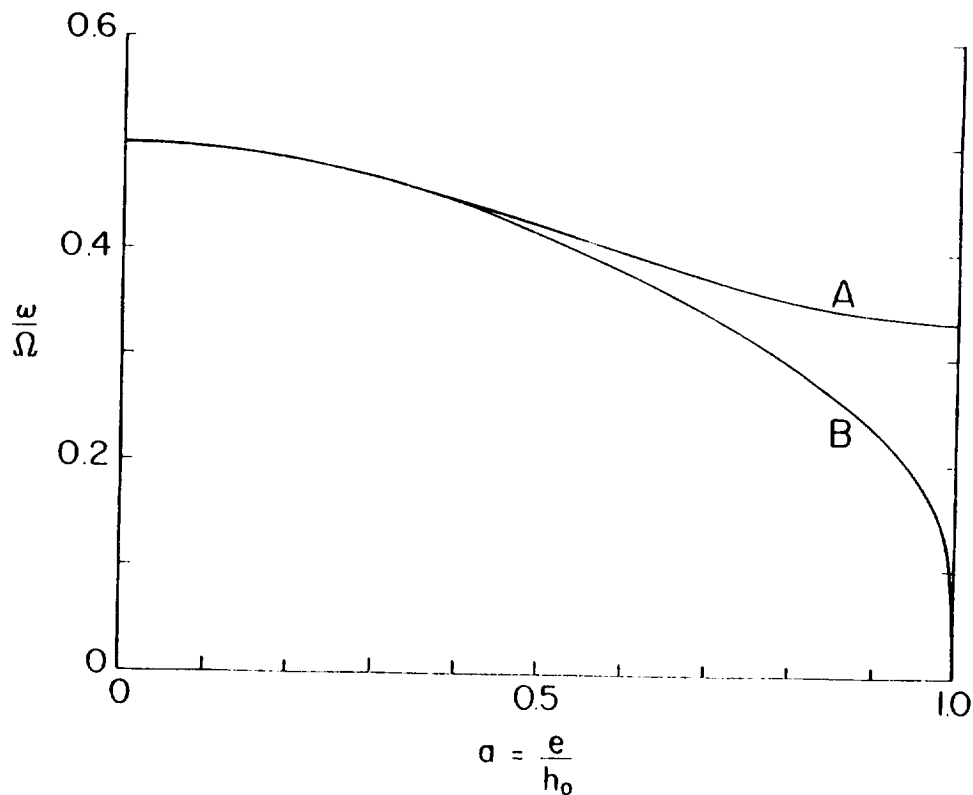

Figure 4. - Frequency ω of neutral stability whirl as function of steady state eccentricity ratio: A, according to complete stability analysis (22); B, according to heuristic proposal (25).

Turning now to the heuristic hypothesis that the lubricant flow pumps energy into the whirl whenever the mean flow velocity is greater than the phase velocity $R\omega$ of the thickness variation wave, we recall that in the equilibrium configuration

of Fig. 1 the film thickness h is given by (1) and the volume flow rate Q_1 is given by (3). The mean velocity

$$\bar{U} = \frac{Q_1}{bh} = \frac{R\Omega}{1 + a\cos\theta} \frac{1 - a^2}{2 + a^2} \tag{23}$$

varies with location around the annulus. To obtain a single velocity to compare with the phase velocity of the thickness variation wave, we propose to take the root-mean-square average of $\bar{U}$. The heuristic criterion for neutral stability then becomes

$$R\omega = \bar{U}_{rms} = \left[\frac{1}{2\pi} \int_0^{2\pi} \bar{U}^2 \, d\theta \right]^{1/2} \tag{24}$$

Evaluating the integral, we obtain

$$\omega = \frac{(1-a^2)^{1/4}}{2 + a^2} \Omega \tag{25}$$

as the neutral stability whirl frequency according to the proposed heuristic hypothesis. The variation of this frequency with eccentricity ratio a is represented by the curve B in Fig. 4. The heuristic prediction (25) is not identical to (22) obtained from the dynamic stability analysis, but it provides a useful approximation for values of the steady state eccentricity ratio that are less than a = 0.5.

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