THE VELOCITY FIELD OF A SYSTEM OF UNSTEADY CYCLOIDAL VORTICIES

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ABSTRACT

An essential difference between two-dimensional and three-dimensional models of cycloidal rotors is the presence of unsteady trailing cycloidal vorticies in the wake. The velocity induced by these vorticies is the primary mechanism producing flow retardation for low span/radius ratio, finite blade number rotors. An idealized rigid wake model of finite blade cycloidal rotors is used to investigate some cycloidal rotor problems.

INTRODUCTION

During the past decade a rekindled interest in energy conversion has stimulated the development of aerodynamic theories and numerical methods for analyzing rotors whose blades make cycloidal paths through the incident stream. Momentum theories and vortex theories appear in the literature. In the case of vortex theories, some noteworthy models of the wake have been presented by Holme and by Wilson in References 1 and 2.

Missing from these developments is information about the velocity seen by the blades of a three-dimensional, finite blade number cycloidal rotor as it moves about its orbit. Also missing is the answer to the question: "What is the optimum orbital load distribution?" There is good reason the latter question has gone unanswered: the problem is complicated. In spite of research spanning seven decades, no theory or method having engineering usefulness has emerged for cycloidal propulsors. The power of present computers may provide the means for pursuing this question further; it is hoped that this work may be a step in that direction.

DISCUSSION OF THE PROBLEM

The load distribution on the blades of a cycloidal rotor has components due to:

- flat plate angle of attack
- camber
- unsteady angle of attack (pitch)
- unsteady motion normal to stream (heave)
- circulation induced by bound vorticity from other blades
- circulation induced from shed vorticity in the wake.

Two-dimensional theory can be used to determine the load due to these components once the velocity at each orbital position and the character of the wake are determined.

An essential difference between the two-dimensional and three-dimensional models of cycloidal rotors is the presence of unsteady cycloidal vorticies in the wake. The velocity induced by these vorticies is the primary mechanism producing flow retardation for finite and low span/radius ratio, finite blade number rotors. It is for this reason that the author has chosen to investigate the velocity field of unsteady rigid cycloidal vorticies. The presence of unsteady bound and shed vorticity in the field between these vorticies is recognized. The computation of the velocity induced by unsteady bound and shed vorticity is not within the scope of this paper. A comparison of the results of this paper with the results of the two-dimensional method of Mendenhall and Spangler (Ref. 3) shows that the velocity induced by trailing cycloidal vorticies is several times that of the bound and shed vorticies.

The wake is assumed to be convected downstream at a constant velocity. It is further assumed that it does not deform. While such assumptions are not as defensible for cycloidal rotors as they are for helicoidal rotors, it can be argued that the induced velocity seen by the blade is primarily influenced by the portion of the cycloidal vortex filament which is nearest the blade.

The validity of the model investigated may not prove useful for the complete range of tipspeeds and rotor loadings. One would expect it to be valid for very low tipspeed ratios where the model reduces to a multielement airfoil system in rectilinear flow. One would also expect it to be valid for lightly loaded rotors. It should be less satisfactory for a cycloidal propulsor where a contracting wake will cause blades in the downstream half of the orbit to cut rolled vortex sheets originating in the upstream half of the orbit. The expanding wake of the turbine should minimize such effects. As with many other developments in the theory of fluid mechanics, the ultimate usefulness can only be established when the complete induced velocity field is computed (including bound and shed vorticity effects) and when correlation with experimental data has been acheived.

THE INDUCED VELOCITY

We have in mind a discrete vortex lifting line model of a straight blade cycloidal turbine. We seek to evaluate the velocity induced at the blades of the rotor by a system of cycloidal vorticies originating at different spanwise locations on the blades. One such vortex filament ant the coordinate system for this discussion are shown in Figure 1. The cycloidal curves followed by vorticies trailing from the i-th blade under rigid wake assumptions are parameterised in time by:

$$x_i = R\cos(\theta-\Psi_i+\omega t) + ut$$

$$y_i = R\sin(\theta-\Psi_i+\omega t)$$

$$z_i = z$$

where the scalar quantity R is the radius of the blade orbit; u is a characteristic velocity through the rotor;

$$\Psi_i = (i-1)2\pi/N$$

is the angular position of the i-th blade of an N-blade rotor when blade 1 is at position $\theta = 0$; ω is the rotor angular velocity; and t is time (less than zero) representing conditions which orinigated in the past.

According to the Biot-Savart law the velocity induced by a vortex filament is given by the following integral:

$$\hat{V} = 1/4\pi \int \Gamma \frac{\hat{R} \times d\hat{l}}{|\hat{R}|^3}$$

where Γ is the circulation along the vortex filament; $\hat{R}$ is the vector from the point (x_o, y_o, z_o) where the velocity is computed to the point (x_i, y_i, z_i) on the vortex filament; $d\hat{l}$ is the unit tangent vector along the vortex filament directed away from the bound vortex.

The circulation is a function of the spanwise position z, orbital position θ, and blade number i:

$$\Gamma_i = A(z)B_n \cos(n\theta - \Psi_i + \omega t + \phi_n)$$

where B_n and ϕ_n give the variation in circulation around the orbit and A(z) is a spanwise load function.

The velocity induced by this system has streamwise and sidewise components given by the following integrals:

$$u^*(\theta) = \sum_{i=1}^{N} -A(z)R\omega/4\pi \int_{-\infty}^{0} \frac{\cos(\theta-\Psi_i+\omega t+\phi)(z-z_o)\cos(\theta-\Psi_i+\omega t)\,dt}{\{(x-x_o)^2 + (y-y_o)^2 + (z-z_o)^2\}^{3/2}}$$

$$v^*(\theta) = \sum_{i=1}^{N} -A(z)R\omega/4\pi \int_{-\infty}^{0} \frac{\cos(\theta-\Psi_i+\omega t+\phi)(z-z_o)(\sin(\theta-\Psi_i+\omega t)-u/R\omega)\,dt}{\{(x-x_o)^2 + (y-y_o)^2 + (z-z_o)^2\}^{3/2}}$$

For a straight blade rotor parallel to the z-axis we are not interested in the spanwise velocity component. These integrals have been evaluated by numerical integration.

APPLICATION OF THE MODEL

A straight bladed cycloidal rotor having blade span/radius of 4/3 was divided into ten equal segments. A trailing cycloidal vortex system was modeled by eleven cycloidal vortex filaments originating at the ends of these segments. The velocity induced at the midpoints of these segments was computed for a local tipspeed ratio $R\omega/u = 2.85$. The circulation on the segments was taken as the integrated average over the length of the segment for an elliptical spanwise load distribution. The orbital load distribution was cosinusoidal (B_n, $\phi_n = 0$ for $n>1$).

The velocity induced at the midpoints of the two segments nearest the midspan of a one blade cycloidal rotor is shown in Figure 2. For the middle 60% os the span the induced velocity is nearly constant. Near the tips it is not for two reasons. First, the discrete vortex lifting line model with equal segments does not yield the correct induced velocity near the tips. In steady, rectilinear flow models the spacing is ususlly modified near the tips. Secondly, there is no apriori guarantee that the circulation specified will result in the same pitch for the cycloidal vorticies that was used in computing the induced velocities. In fact, one must modify the pitch of the cycloidal vorticies and the repeat the calculation. Such iteration is also required in solving the helicoidal rotor problem. The second iteration has not been caried out in the work reported here.

Another question arising from these results is: "How do we determine the characteristic velocity at which the wake moves downstream?" We want, of course, to make a choice which gives the most accurate results. This question remains open, but in this work the velocity at $\theta = 180°$ has been used. This is done because we will eventually want to use more terms in the cosine series for orbital circulation in the attempt to produce a more uniform induced velocity in the wake and the a more optimal rotor.

The results in Figure 2 form the basis on which the strength of the circulation and/or the incident stream may be varied to produce the local velocity at the rotor (u(180°)) which matches the assumed local tipspeed ratio. If both are varied, a range of power coefficients is obtained. The largest then would be the optimum power coefficient for that local tipspeed ratio, or pitch, for that orbital and spanwise circulation distribution.

Figure 3 shows 1 and 3 blade rotor results of varying the circulation and adjusting the inflow velocity until a local tipspeed ratio of 2.85 was acheived. These power coefficients are not quantitatively exact since they do not account for induced velocities from shed and bound vorticity, and are not derived from a completed iteration of spanwise loading and wake pitch. They do exhibit qualitative results showing an increase in obtainable power coefficient with increased blade number and show that there is an optimal loading for each blade number.

CONCLUDING REMARKS

This method would be enhanced if a fast means of evaluating the induced velocity integral were found. Asymptotic series and a regression analysis of computed values seem reasonable approaches.

The induced velocity components for bound vorticity and shed vorticity must be included in the analysis.

The performance computation requires modifying the pitch of the trailing vorticies so that the wake motion is consistant with the prescribed inflow conditions and the induced velocities at the appropriate location. An iteration is required.

The results of the inviscid theory must be reckonned with the reality of the viscous fluid effects. Correlation with experimental data is the ultimate test of a fluid dynamic theory or method.

The method should be exercised in an attempt at the solution of the optimal orbital load distribution problem by including further terms in the circulation series. Assuming we can find the optimal load distribution, we must then see if it can be obtained by a technically feasible lifting surface.

With minor extensions, this method could be applied to curved blade rotors of the Darrieus type. This is appealing because of the greater number of units which exist and the abundant theoretical and experimental date which is available for comparison.

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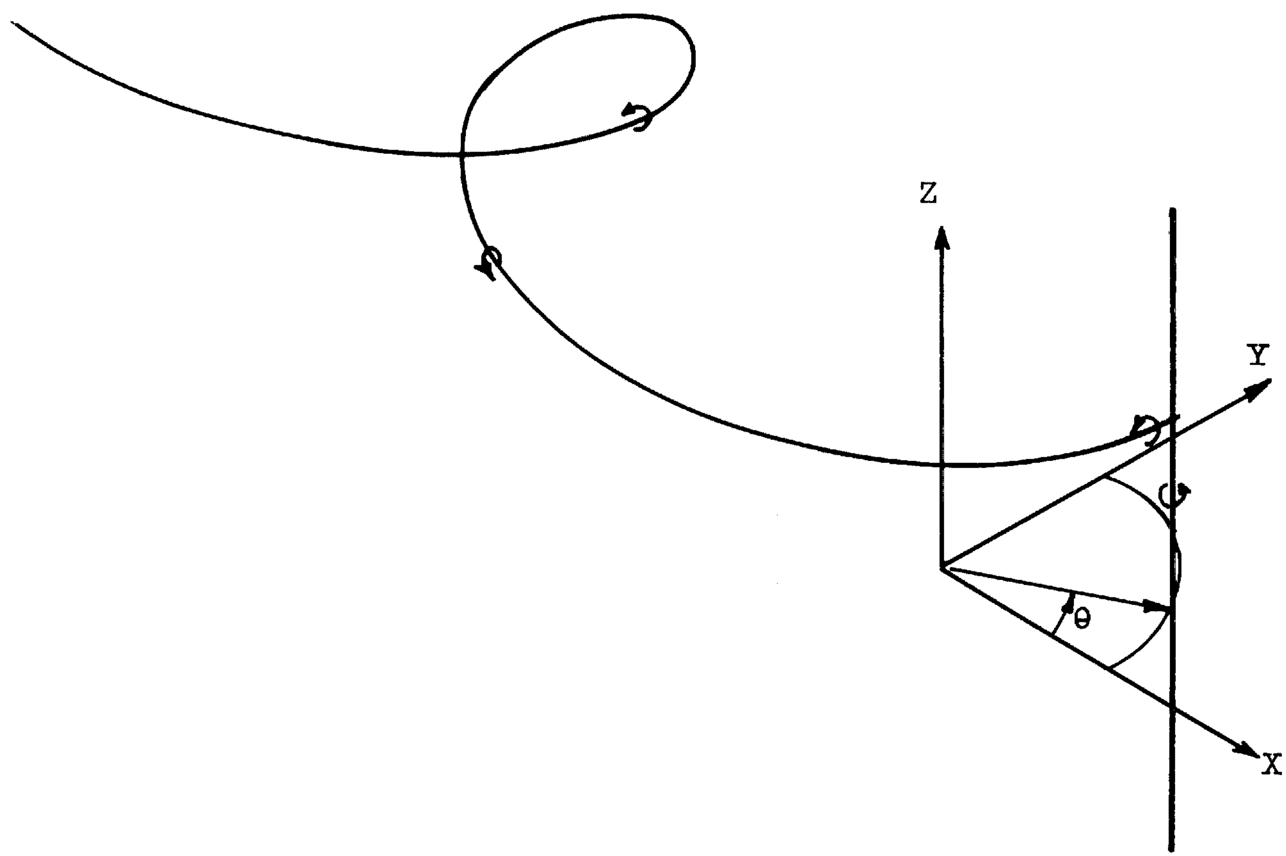

Figure 12- Coordinate System for the Analysis of a Cycloidal Vortex Filament

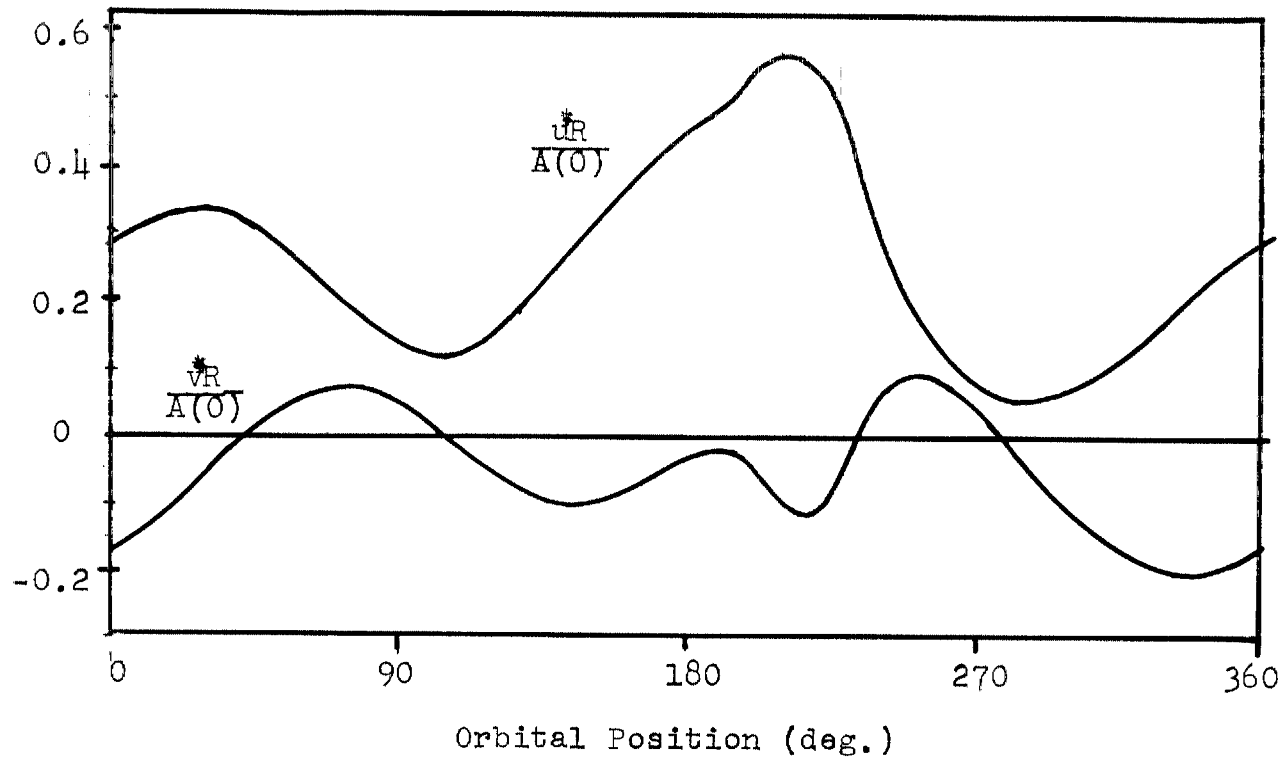

Figure 2- Streamwise and Spanwise Velocity Induced at Blade Midspan (Local Tipspeed Ratio R/u = 2.85) vs. Orbital Position

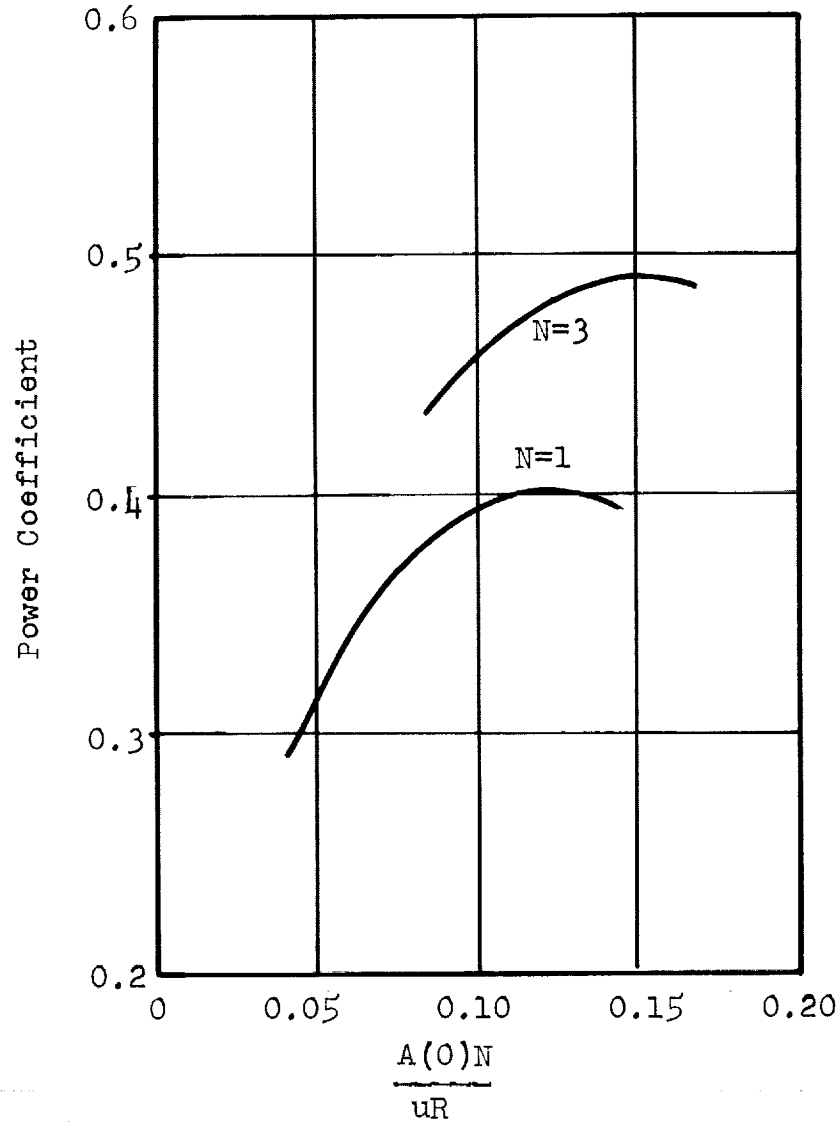

Figure 3- Power Coefficient vs. Circulation Parameter for 1 and 3 Blades