A wind tunnel investigation of the shape of uncharged raindrops in the presence of an external, vertical electric field

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Abstract

Results are presented of a recent wind tunnel experiment in which electrically uncharged water drops of 500 to 3000 μm equivalent radius are freely suspended in the vertical air stream of the UCLA Cloud Tunnel. During this suspension the drops were exposed to external, vertical electric fields of 500 to 8,000 Volts/cm. The change in drop shape with drop size and electric field strength was noted and is discussed in the light of theoretical work cited in the literature which unfortunately does not take into account the effects of air flow past the drop. The wind tunnel study is documented here by stills from a 16 MM film record that demonstrates the shape of water drops in response to both hydrodynamic and electric forces.

Introduction

Cloud precipitation strongly affects the propagation of electromagnetic radiation. Much attention has been given to the dependence of the scattering of electromagnetic radiation of radar frequency on the size of the precipitation particle. Particle shape, however, also appears to be an important parameter. In particular, non-sphericity introduces asymmetry in the backscatter radiation field. This distortion forms the basis of new techniques for hail/rain discrimination, for the determination of raindrop size distribution, and for the determination of rainfall rates from radar observations.

To evaluate the radar data, the above mentioned techniques use the drop shapes experimentally derived from the wind tunnel measurements of Pruppacher and Beard[1] and from the theoretical model of Pruppacher and Pitter[2]. These drop shapes apply to equilibrium conditions only. It is further assumed that the drop is neither electrically charged nor embedded in an external electric field. In actuality, however, drops do not fall with a constant shape but rather undergo complex oscillations around their equilibrium shape. In addition, many clouds are electrically charged even in their early stages of development and, consequently, exhibit vertical and horizontal electric fields. Such electric fields cause the drops to be electrically polarized and therefore affect the equilibrium shape, as well as both the oscillation frequency and the oscillation amplitude of the drop. The electrically induced change in shape also affects the fall velocity of a drop. In addition, changes in the oscillatory motions may cause changes in the drop break-up behavior. All these effects, in turn, influence the drop size distribution and thus the rate of formation of precipitation inside a cloud.

In this report we concern ourselves exclusively with the effects of external, vertical electric field on drop shape. Several recent reports are available on this topic. Dawson and Warrender[3] studied experimentally the effect of a vertical, external electric field E_o on the terminal velocity V_∞ of electrically uncharged water drops in air which suffer a shape deformation due to the electric field. Generally, V_∞ was found to increase with the electric field strength. However, the velocity increase found was quite small. For drops of between 3 and 4 mm equivalent radius a_o (which is the radius of a sphere having the same volume as the distorted drop), V_∞ changed from 8.9 m/sec at $E_o = 0$ to 9.9 m sec^{-1} at $E_o = 9$ kV/cm. These changes are smaller than those due to changes caused by the variation in air density and air viscosity which a drop experiences as it falls in the atmosphere[4]. Unfortunately, these velocity changes were not correlated to the corresponding shape changes of the drop, and only a few drop sizes were studied.

Billings and Holland[5] and Brazier-Smith et al.[6] studied experimentally and theoretically the effect of an horizontal, external electric field on the oscillation frequency of a water drop falling in air. For a given drop size a_o, the vibrational frequency was found to decrease with increasing E_o in a manner closely predictable by theory. Once again, only a few drop sizes were studied, and the turbulence in the wind tunnel used for characterizing the vibration of the drops was not controlled. No studies of the frequency of drop oscillations were made for vertical electric fields.

Theoretical studies of the effects of an external electric field on the drop shape b/a (where a is the semi-axis perpendicular to the drop's fall axis, and b is the semi-axis along the drop's fall axis) were carried out by O'Konski and Thatcher[7], Taylor[8], Brazier-Smith[9,10], Abbas and Latham[11], and Richards and Dawson[12]. These studies showed that a water drop originally spherical becomes strongly prolate-spheroidally deformed in an external, vertical electric field. The deformations predicted by these theories appear to agree quite well with each other, in particular regarding the maximum stable drop deformation before break-up. The critical axis ratios predicted for break-up vary between b/a = 1.86 (Taylor[8]) and b/a = 1.83 (Brazier-Smith[10]). These estimates also agree with the value theoretically predicted by Abbas and Latham[11]. The various theories also indicate that the deformation parameter b/a can uniquely be expressed as an increasing function of the non-dimensional quantity $X = E_0(a_0/\sigma)^{\frac{1}{2}}$, where σ is the surface tension of water in air (Fig. 1). Break-up of electrically uncharged drops was predicted for X = 1.625 (Taylor[8]) and for X = 1.603 (Brazier-Smith[10]), in agreement with the critical value of X computed by Abbas and Latham[11]. This correlation suggests that, for a given drop size and air pressure, the electric field required for a certain drop deformation, for drop break-up, and for the onset of corona discharge increases as the temperature decreases, i.e., as the surface tension increases. This trend has been experimentally verified by Ausman and Brook[13]. A different trend has been found with regard to air pressure. Griffiths and Latham[14] showed that the electric field strength required for onset of corona discharge on drops decreased with decreasing air pressure.

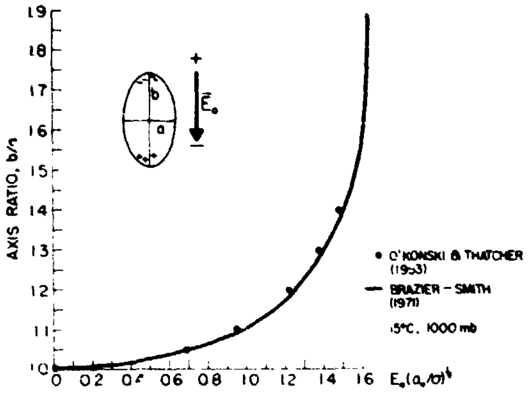

Figure 1. Variation of the theoretically predicted shape of a water drop in air with $X = E_0(a_0/\sigma)^{\frac{1}{2}}$, assuming that the drop's original shape is spherical, and that no hydrodynamic flow is present around the drop.

Electric charge appears to have a considerable effect on the electric field strength required for drop deformation. Abbas and Latham[11] showed that the correlation of b/a with $X = E_0(a_0/\sigma)^{\frac{1}{2}}$ varies with the electric charge Q on a drop, shifting to smaller values as Q increases.

For electrically uncharged drops the corelation b/a vs. $X = E_0(a_0/\sigma)^{\frac{1}{2}}$ suggests that the electric field strength E_0 to achieve a certain drop deformation b/a decreases with increasing drop size a_0. This behavior was verified by Wilson and Taylor[15], Nolan[16], Macky[17], Ausman and Brook[12], Dawson and Richards[18], and by Griffiths and Latham[14]. In none of these studies, however, was a quantitative investigation made on the variation of b/a with E_0 for various a_0. Instead, emphasis was placed on determining the critical electric field strength beyond which the drop would break up or show evidence of corona discharge. It was found that $X = E_0(a_0/\sigma)^{\frac{1}{2}}$ had a value of 1.61 (Wilson and Taylor[15]), 1.51 (Macky[17]), 1.56 (Ausman and Brook[13]), and 1.81 (Griffiths and Latham[14]), for break-up or corona discharge. All experiments suffered from two serious deficiencies since, under the experimental conditions reported, (1) the drops did not reach terminal velocity and thus did not assume the hydrodynamically determined shape, and (2) the drops were forced to abruptly enter an electric field region created between two electrode plates causing non-equilibrium conditions between the drop and the environment.

A review and new wind tunnel results on the shape of water drops falling at terminal velocity in air in the absence of an external electric field have been given by Pruppacher and Beard[1] and Pruppacher and Pitter[2]. Their studies show that drops smaller than about 500 μm radius can be considered as spheres. On the other hand, drops larger than 500 μm are oblate-spheroidally deformed with an increasingly flattened lower side as the drop becomes larger. This causes the axis ratio b/a to be less than unity, with b/a decreasing as a_0 increases. This behavior was found to be the result of an interaction between hydrodynamic forces, surface tension forces and hydrostatic forces. Thus, the actual drop shape in the absence of an external electric field is quite different from the spherical shape assumed by all previous theories on the effect of a vertical, external electric field on the drop shape.

Present experimental set-up

A series of experiments in the UCLA Cloud Tunnel were undertaken to improve on previous experimental studies and to extend our present knowledge of the effect of an external electric field on the shape of water drops in air. The UCLA facility is a vertical wind tunnel which allows the free suspension of drops of equivalent radius a_0 between 20 μm and 3 mm in a vertical, low-turbulence air stream for long periods of time. In the present study, drops of equivalent radius between 500 μm and 3000 μm were suspended in the air stream between two metal screens acting as electrodes to create a vertical electric field. The screens, 6 cm x 6 cm, were separated by a distance of about 5 cm. The upper screen was charged to a positive electric potential; the lower to a negative electric potential. Electric fields between 500 Volts cm^{-1} and 8,000 Volts cm^{-1} were created. Computations based on the theoretical considerations of Smythe[19] indicate that, for the given electrode arrangement in our experimental set-up, the electric field is uniform and computable from $E_0 = \Delta V/d$, where d is the distance between the two electrodes and ΔV is the potential difference between the two electrodes, with an error of less than 1% if the drop is kept inside an area around the wind tunnel axis of 10 mm x 10 mm. In most cases the drop was confined to this area. The drops studied were made from doubly distilled water and their size and shape documented photographically (Fig. 2). In each experiment, it was insured that the drops were electrically uncharged. For zero electric field, comparison was made with our previous studies on drop shape[1,2]. All experiments were carried out in an atmosphere of about 1000 mb air pressure and 15°C air temperature.

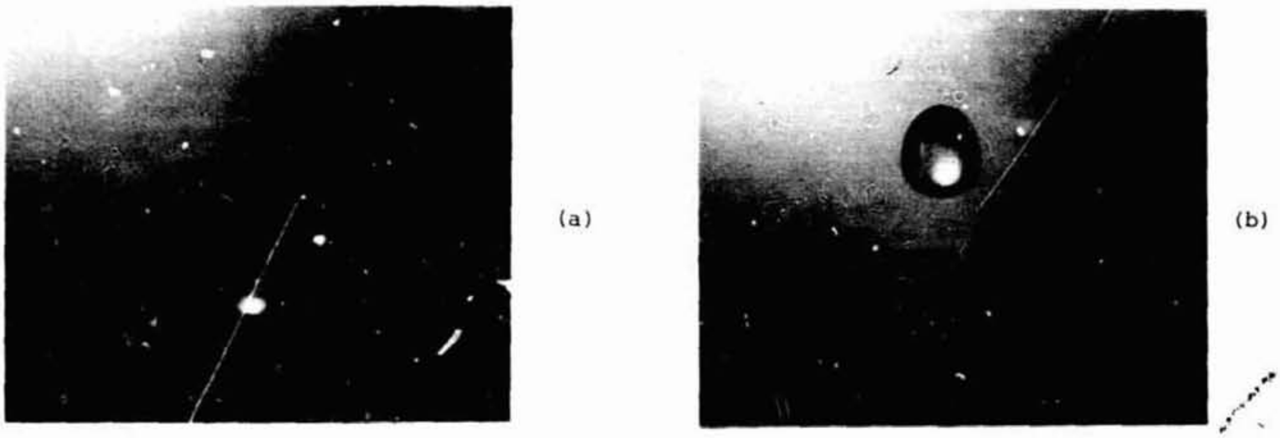

Figure 2. Water drops freely suspended in the vertical air stream of the wind tunnel; $a_0 = 2.3$ mm. (a) No external electric field. (b) Vertical, external electric field; $E_0 = 7.75$ kV cm^{-1}.

Results and discussion

The experimental results are summarized in Figs. 2 to 4. Figure 3 gives the variation of the drop shape b/a with equivalent drop radius a_0 for various electric field strengths E_0. Comparison of the values for b/a for $E_0 = 0$ with values previously obtained[1,2] shows satisfactory repeatability of our earlier results. We note further from this figure that the deformation caused by the electric polarization forces due to the electric field is strongly opposed by the hydrodynamic forces. The latter act to influence the drop to become oblate-spheroidally deformed, while the former attempt to deform the drop into a more spherical shape. Also notice that the larger the drop, the stronger the effect of the electric field. Extrapolation of our results to smaller drops suggests that electric field strengths smaller than 6 kV cm^{-1} have a negligible effect on the drop shape if $a_0 \lesssim 1$ mm. With increasing drop size, and with increasing field strength of the external electric field, the hydrodynamically caused deformation is counteracted with increasing efficiency. Thus, an external electric field of 3.2 kV cm^{-1} begins to significantly affect the drop shape if $a_0 > 0.15$ cm.

In Fig. 4 the variation of the drop shape b/a is plotted as a function of the electric field strength for different drop sizes. We note that, for drops of $a_0 \gtrsim 2$ mm, electric fields of $E_0 \gtrsim 1$ kV cm^{-1} affect the drop shape b/a. This electric shape deformation increases with drop size. Since hydrodynamic forces also affect the drop more strongly the larger its size, a "cross-over" of the individual shape curve results. Thus, although large drops are most strongly oblate-spheroidally deformed for $E_0 = 0$, the deformation is efficiently counteracted with increasing field strength, resulting in a spherical and

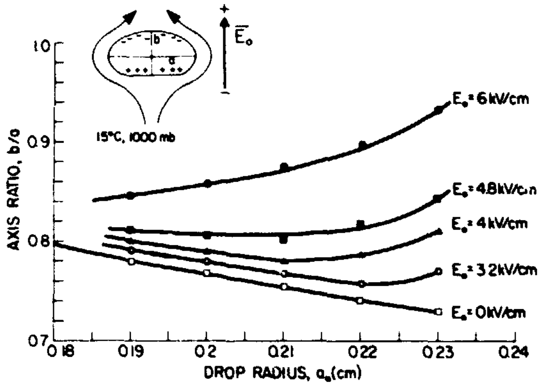

Figure 3. Present experimental results for the variation of the shape of a water drop falling at terminal velocity in air with equivalent drop radius a_o for various external electric field strengths.

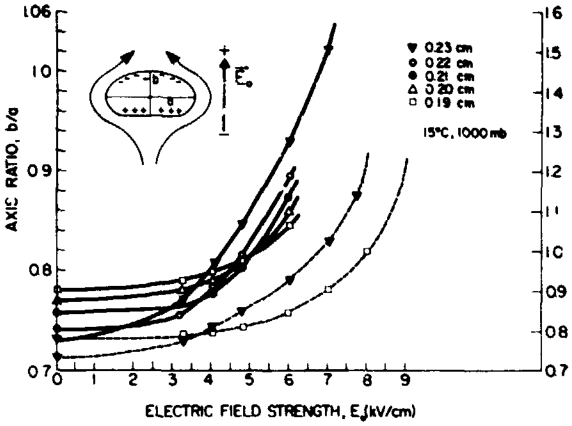

Figure 4. Present experimental results for the variation of the shape of a water drop falling at terminal velocity in air with electric field strength for various drop sizes. Solid line applies to left coordinate; dashed lines to right coordinate.

eventually a prolate-spheroidal drop shape. We further note that, at much lower values of E_o than those required for smaller drops which are originally less oblate-spheroidally deformed, a drop of a_o = 0.23 cm appears to require a limiting value of $E_{o,crit}$ = 8 kV cm^{-1} for break-up; while a drop of a_o = 0.19 cm requires $E_{o,crit} \simeq$ 9 kV cm^{-1}. These latter findings are in good agreement with the observations of Ausman and Brook[13] and of Dawson and Richards[18].

Figure 5 gives the variation of drop deformation with the quantity $X = E_o(a_o/\sigma)^{\frac{1}{2}}$ for two drop sizes. We note from this figure that b/a is not a unique function of X for all a_o, as had been found on the basis of electrostatic theory cited in the literature which disregarded the flow around a drop. Instead, b/a vs. X is a function of a_o due to the significantly stronger oblate-spheroidal deformation of larger drops. However, we note that the dependence on a_o diminishes rapidly as X approaches the critical value for drop break-up. In fact, our experimental data suggest that for all drops break-up appears to occur as X approaches a value of about 1.6, as had been predicted by theory which does not take into account the flow around the drop.

Break-up of single large drops in the absence of an electric field proceeds via hydrodynamic forces acting on the drop's lower side (i.e., on the drop's upstream side). Our observations show that, in contrast, drop break-up in the presence of an external electric field proceeds via instabilities on the drop's upper side (i.e., on the drop's downstream side).

Our observations suggest that for low external, vertical electric fields the hydrodynamic forces dominate initially; and that the effect of the electric forces is small and confined to the downstream side of the falling drop where the flow field is weak. However, as the electric field grows in intensity, the percentage contribution of the electric field to the drop deformation increases rapidly. Thus, for a drop of a_o = 0.23 cm and for an electric field of E_o = 4 kV cm^{-1} (7.75 kV cm^{-1}), b/a = 1.07 (1.45), if the drop is stagnant and there is assumed to be no flow past it (see Fig. 1). For the case of drop deformation due only to hydrodynamic flow ($E_o \simeq 0$), we find b/a = 0.73, in accordance with our previous results. For the case of both electric field and flow affecting the drop shape b/a = 0.81 (1.14) for E_o = 4 kV cm^{-1} (7.75 kV cm^{-1}) (see Fig. 4). These

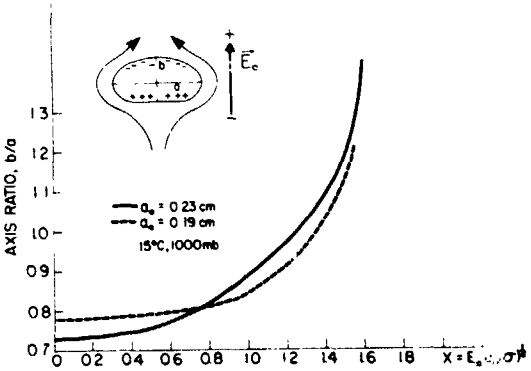

Figure 5. Present experimental results for the variation of the shape of a water drop falling at terminal velocity in air with $X = E_o(a_o/\sigma)^{\frac{1}{2}}$, for different drop sizes.

numbers suggest that the percentage deviation of the shape $(b/a)_{\text{flow only}}$ from the shape $(b/a)_{\text{flow + field}}$ increases from 10% to 36% as the field grows from 4 kV cm^{-1} to 7.75 kV cm^{-1}, thus indicating the rapidly reduced effect of the flow on the final drop shape as the electric field grows. Eventually, as the field approaches breakdown value, the hydrodynamic forces completely lose their effect on the drop's downstream side. The critical value, then, of $X = E_0(a_0/\sigma)^{\frac{1}{2}}$ for drop disintegration is the same as for no flow conditions, i.e., approximately 1.6.

No quantitative studies have yet been undertaken by us to verify results published in the literature on the effect of an external electric field on the oscillation frequency and amplitude of large drops. Qualitatively, we found that the amplitude of oscillation was reduced with increasing strength of an externally applied vertical electric field. Our studies on drop variations also indicate that, during a vibration cycle, the concave impression on the upstream (i.e., the bottom) side of the drop - induced by hydrodynamic forces (see Pruppacher and Pitter[2]) - remains a present feature on the drop's lower side, even when the drop is exposed to an external electric field.

Also, no quantitative studies have yet been undertaken by us to verify the results published in the literature on the effect of temperature, pressure, and the presence of electric charges on the drop deformation in an external electric field. Such studies are planned for future experiments.

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