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The stress system generated by an electromagnetic field in a suspension of drops

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Abstract

This paper deals with the calculation of the stress generated in a suspension of drops in the presence of a uniform electric field and a pure straining motion, taking into account that the magnetohydrodynamic effects are dominant. It is found that the stress generated in the suspension depends on the direction of the applied electric field, the dielectric constants, the viscosity coefficients, the conductivities and the permeabilities of fluids inside and outside the drops. The expression of the particle stress shows that for fluids which are good conductors and poor dielectrics, especially for larger drops, magnetohydrodynamic effects tend to reduce the dependence on the direction of the applied electric field.

Introduction

The study of a flow system in which the electric field and the velocity field effect each other is called electrohydrodynamics. The applications of electrohydrodynamics are numereous: cryogenic fluid management in the zero-gravity environment of space, formation and coelecence of solid and liquid particles, electrogasdynamic high voltage and power generation insulation research, physicochemical hydrodynamic, heat, mass and momentum transfer, electro-fluid dynamics of biological systems, and atmospheric and cloud plysics.[1-5] In nature and industry we are quite often concerned with the properties of a fluid in which small particles are suspended and carried about by the motion of the ambient fluid. If the average distance betweer the particles is small compared with the characteristic length of the motion of the suspension, one can regard the suspension as a homogeneous fluid. The problem is to determine the rheological properties of this equivalent fluid from the knowledge of the properties of the ambient fluid. The macroscopic properties of the suspension will be referred to as bulk properties. When the suspension is dilute, the suspending fluid Newtonian the particles rigid spheres, and the particle Reynolds number sufficiently small, the suspension can be described in bulk as a Newtonian fluid with an effective viscosity $\mu^* = \mu(1+2.5\,c)$, provided only that the particles are not subjected to an externally applied force or couple. For the more general case, the non-isotropic structure of the suspension usually results in a non-Newtonian form for the bulk stress tensor.[6] The formulation of the problem of determining the stress in a suspension of particles is not straight-forward, partly due to the fact that the bulk stress in a suspension is not obvious. Bulk stress and other bulk properties are defined in terms of ensemble averages of the actual quantities. This is shown by Batchelor[6] to be equivalent to defining bulk properties in terms of volume averages provided that the averaging volume is chosen to contain many particles and is such that the statist.cal properties of the suspension are uniform over it.

In the case of a dilute suspension, which means that the flow near any one particle is independent of all the other particles, the contributions to the bulk stress from the various particles are linearly additive. The contributions may be classified in three groups: : The first is a purely isotropic contribution, the second is the contribution of the deviatoric stress and the third represents the contribution to the bulk stress due to the presence of the particles. The stress in third type of contribution is termed as "particle stress" and only the deviatoric part of it is significant.

Non-Newtonian behavior of a suspension can occur in general in the following cases:[7] (i) Non-spherical particles can cause some directional properties.[6,8] (ii) The effect of weak Brownian motion.[9] (iii) The effect of a couple on a particle is to rotate and to generate an antisymmetric part of the particle stress tensor.[8,10] (iv) The effect of the shape of a deformable particle gives rise to non-linear stress.[11] (v) Surface tension at the boundary of a fluid particle or elasticity of a solid particle can cause time-dependent effects and the suspension exhibits visco-elastic properties.[11,12] (vi) In the case of sufficiently large size, the inertia forces cannot be neglected in the relative motion near one particle and the particle stress depends non-linearly on the bulk velocity gradient.[13]

In this paper an expression is found for the particle stress tensor of a suspension of drops in an electric field. It is assumed that the suspension is dilute, suspending fluid Newtonian, the drops spheres and the particle Reynolds number sufficiently small. The explicit form of the stress tensor depends on the electromagnetic properties of the drops and their surroundings ; therefore, the flow due to a single drop is needed in determining the

particle stress of a suspension. For this reason, consideration is given first to the flow due to a drop in the presence of an electric field in a pure straining motion.

Experimentally and theoretically it has been shown by Taylor[14] that a circulation can occur in a drop and its surrounding in the presence of a uniform electric field. This flow field set up is due to the surface charge and the tangential electric field stress over the surface of the drop. The flow field outside the drop given in[14] is very similar to a system in which the field carries a uniform current.[15,16] In such a case the flow field is produced by the rotational Lorentz force due to the distorted electric current and the associated magnetic field. For fluids which are poor conductors the magnetic effects are negligible, and for fluids which are good conductors and poor dielectrics, or for larger drops the magneto-hydrodyamic effect may be dominant.[17]

Following the general arguments given in[18] and [19,20] the pressure and the velocity in the fluid outside the drop and inside the drop which is embedded in a pure straining motion are determined. Since the governing equations are linear in terms of velocity and the electric field, the effects in the cases of a drop in an electric field in the absence of a pure straining motion and a drop in a pure straining motion without an electric field can be superimposed.

Since it is assumed that the suspension is dilute, the flow near one drop is independent of other drops and so we may use the results obtained for one drop to evaluate the integral in the particle stress. The expression of the particle stress has two terms, one with an applied electric field and the other without electric field. The term which depends on electric field presents a directional effect; and this makes the suspension a non-Newtonian fluid. This shows that the suspension of drops in an electric field cannot be represented by a viscosity that is independent of the rate-of-strain. The viscosity depends on the direction of the applied electric field.

The first term of the particle stress shows the effect in the absence of an applied electric field, and the second term denotes the additional effect due to the applied electric field. The second term contains two parts: one is due to the absence of the magnetic effect in the fluid outside the drop and the other is due to the presence of the magnetic effect. When the magnetohydrodynamic effect is absent, the expression of the particle stress reduces to that of given in.[21]

The expression of the bulk stress

In order to establish the relation of the bulk stress to the velocity and stress distributions in the fluid near individual particles, the expression for the bulk stress in the suspension as a volume integral is used. The average volume V is chosen to contain many particles and is such that the statistical properties of the suspension are uniform over it. The hydromechanical bulk stress in the suspension is

$$\Sigma_{ij} = \frac{1}{V} \int \sigma_{ij} \, dV \; ,$$

since the effect of inertia forces in the relative motion near a particle is neglected. The Maxwell bulk stress tensor may be defined in a similar manner. The bulk velocity gradient is given by

$$\frac{\partial U_i}{\partial x_j} = \frac{1}{V} \int \frac{\partial u_i}{\partial x_j} \, dV \; ,$$

where σ_{ij} and u_i are the actual values of the hydromechanical stress and velocity at any point x in the suspension, whether it be in the ambient fluid or inside a particle. The surface and the volume of a typical particle in V will be denoted by A_o and V_o respectively.

Assuming that the ambient fluid is Newtonian with the viscosity μ the hydromechanical bulk stress may be written as

$$\Sigma_{ij} = \frac{1}{V} \int_{V - \Sigma V_o} \left\{ -p \delta_{ij} + \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right\} dV + \frac{1}{V} \Sigma \int_{V_o} \sigma_{ij} \, dV \; , \tag{1}$$

where the summation is over all the particles in V. The bulk velocity gradient becomes

$$\frac{\partial U_i}{\partial x_j} = \frac{1}{V} \int_{V - \Sigma V_o} \frac{\partial u_i}{\partial x_j} \, dV + \frac{1}{V} \Sigma \int_{A_o} u_i n_j \, dA \tag{2}$$

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where n is an outward unit normal to A_o. Neglecting inertia effects, the following can be written :

$$\int_{V_o} \sigma_{ij} \, dV = \int_{A_o} \sigma_{ik} x_j n_k \, dA \qquad (3)$$

With the addition (2) and (3), the expression (1) becomes

$$\Sigma_{ij} = - \delta_{ij} \frac{1}{V} \int_{V - \Sigma V_o} p \, dV + \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$
$$+ \frac{1}{V} \Sigma \int_{A_o} \left\{ \sigma_{ik} x_j n_k - \mu (u_i n_j + u_j n_i) \right\} dA \qquad (4)$$

The volume $V - \Sigma V_o$ is wholly occupied by ambient fluid and the volume V_o must be regarded as including the interfacial layer ; and the surface A_o will be regarded as a surface just outside of the interfacial layer. The first term in the expression for Σ_{ij} in (4) is a purely isotropic contribution, the second is the deviatoric stress and the third represents the contribution to the bulk stress due to the presence of the particles. The third term in (4) is called the "particle stress" and is denoted with $\Sigma_{ij}^{(p)}$.

It is assumed that the suspension of particles is force-free and couple-free. Since the exertion of a couple on the particles by external means generates an antisymmetric contributions to the bulk stress, the bulk stress in the absence of a couple exerted on particles thus becomes a symmetrical tensor.

When the ratio of the convection of charge to the conduction current, which is referred to as the electric Reynolds number, is much smaller than unity, the influence of the electric stresses on the fluid is included in the model, but there is no reciprocal effect of the motion on the fields. A similar situation for the magnetic field can be considered with no effect of the motion on the magnetic field ; and the electric current density is thus given by its electrostatic form. For this reason, in this paper, in view of the absence of the reciprocal effect of the motion on the electromagnetic field, the hydromechanical bulk stress in the suspension is considered alone.

Governing equations

The magnetic induction in the fluid in and out of the drop is not negligible because of dynamic currents are not small enough. It is assumed that the influence of the electric stresses on the fluid is included in the model, but there is no reciprocal effect of the motion on the fields. Therefore, the appropriate laws of electrodynamics are essentially those of electrostatics for the electric field and the electric current density, except for the magnetic induction field. Under the conditions considered here, the governing equations of electrohydrodynamics are[22]:

$$\nabla \cdot E = 0 , \qquad (5)$$

$$\nabla \cdot E = 0 , \qquad (6)$$

$$J = \sigma E \qquad (7)$$

$$\nabla \times H = J \qquad (8)$$

$$\nabla \cdot H = 0 \qquad (9)$$

$$\nabla p = \mu \nabla^2 u + \chi \sigma E \times H \qquad (10)$$

$$\nabla \cdot u = 0 \qquad (11)$$

where E , is the electric field intensity, J the electric current density, H the magnetic field, σ the electric conductivity, u the velocity, p the pressure, μ the viscosity, χ the permeability. Throughout the paper MKS units are used. Because of the absence of fluid motions the term $\sigma \chi u \times H$ in (7) and the term $\sigma \chi (u \times H) \times H$ in (10) are omitted.

The boundary conditions to be applied at the interface of a drop in an electric field are the following[22]

$$n \times \{E\} = 0 \tag{12}$$

$$n \cdot \{\sigma E\} = 0 \tag{13}$$

$$n \cdot \{u\} = 0 \tag{14}$$

$$n \times \{u\} = 0 \tag{15}$$

$$n \times \{\sigma + t + h\} = 0 \tag{16}$$

$$n \cdot \{\sigma + t + h\} + T \left(\frac{1}{R_1} + \frac{1}{R_2}\right) = 0 \tag{17}$$

where σ is the viscous stress, t and h are the electric part and magnetic part of the Maxwell stress tensor, respectively, and $\{A\}$ denotes the jump of A across the interface. T is the surface tension, and R_1 and R_2 are the radii of curvature of the surface; these radii are taken as positive when the corresponding center of curvature lies on the side of the interface to which n points.

Under the conditions considered here, the electric field E and H and the velocity field u can be determined independently by equations (5)-(11) and then, they can be related by the boundary conditions (12)-(17).

A drop in an electric field

We consider a drop or bubble, assuming that its shape is spherical with radius a. The distance between electrodes and the drop measures to many radii thus causing the electric field far from the drop to be uniform. An appropriate spherical polar coordinates are defined with the origin at the center of the drop and the symmetry axis in the direction of the applied field. There are four boundary conditions for the electric field intensity : (i) E is finite inside the drop ; (ii) the tangential component of the electric field is continuous across the surface of the drop ; (iii) there is no surface current ; and (iv) E tends to E_0 as $|x|$ tends to infinity. Subject to these boundary conditions, equations (5)-(7), (12) and (13) give for the electric field outside the drop

$$E = E_0 \left(1 + \frac{1-\alpha}{2+\alpha} \frac{a^3}{r^3}\right) - 3 \frac{1-\alpha}{2+\alpha} \frac{a^3 (E_0 \cdot X) X}{r^5} \tag{18}$$

where $r = |x|$, and for that inside the drop

$$\bar{E} = \frac{3}{2+\alpha} E_0 \tag{19}$$

where $\alpha = \bar{\sigma}/\sigma$. A symbol with a bar is used for the quantities of the medium inside the drop and a symbol without a bar is used for those of the medium outside the drop. The expression in equation (19) shows that the electric field inside the drop is uniform.

Since there is no applied magnetic field, equations (8) and (9) give for the magnetic field outside the drop

$$H = -\frac{1}{2}\sigma \left(1 - 2 \frac{1-\alpha}{2+\alpha} \frac{a^3}{r^3}\right) X \times E_0 \tag{20}$$

and for that inside the drop

$$\bar{H} = -\frac{3\bar{\sigma}}{2(2+\alpha)} (X \times E_0) \tag{21}$$

The circulation in and round the drop is responsible for the electric surface force density and the magnetic force density which are related to the Maxwell stress tensor. Expression of t and h over the surface of the drop are needed. The tangential and normal component differences of t across the surface of the drop are

$$n \times \{t\} = \frac{9\bar{E}}{(2+\alpha)^2} (\alpha\beta - 2)(n \times E_0)(n \cdot E_0) \quad ,$$

$$n \cdot \{t\} = \frac{9\bar{E}}{2(2+\alpha)^2} \left[E_0^2 (1-\beta) + (E_0 \cdot n)^2 (\beta + \alpha^2\beta - 2)\right] \quad ,$$

where $\beta = \mathcal{E}/\bar{\mathcal{E}}$ is the ratio of the permittivities. The tangential and normal component differences of h across the surface of the drop are

$$n \times \{h\} = 0 \quad ,$$

$$n \cdot \{h\} = \frac{9 a^2 \sigma^{-2}}{8(2+\alpha)^2} \ (\bar{\chi} - \chi)(n \times E_0)^2 \quad .$$

The flow considered in this paper is governed by equations (10) and (11). The boundary conditions for the velocity are: (i) $\underline{u}$ is finite inside the drop and tends to zero as $|x|$ tends to infinity , (ii) u.n = 0 and ū.n = 0 at the interface ; (iii) the tangential component of the velocity across the drop is continuous ; (iv) tangential electric stress and tangential magnetic stress and tangential viscous stress are in balance at the interface.

Following the general arguments given in[18] and[19] the pressure and the velocity in the fluid outside the drop[23] can be written as

$$\frac{p-p_0}{\mu} = b_{jj} R(r) + b_{kj} x_k x_j Q(r) \quad , \tag{22}$$

$$u_i = b_{jj} x_i f(r) + b_{ij} x_j g(r) + b_{kj} x_k x_j x_i h(r) \tag{23}$$

where

$$b_{ij} = E_{oi} E_{oj} \quad , \qquad b_{ij} = b_{ji} \quad , \qquad b_{ii} = E_{oi}^2 \quad ,$$

and the expressions for R,Q,f,g and h are given in the appendix. Using the same reasoning as for outside the drop, the pressure and the velocity inside the drop[23] can be written as

$$\frac{\bar{p}-\bar{p}_0}{\bar{\mu}} = b_{jj} \bar{R}(r) + b_{kj} x_k x_j \bar{Q}(r) \quad , \tag{24}$$

$$\bar{u}_i = b_{jj} x_i \bar{f}(r) + b_{ij} x_j \bar{g}(r) + b_{kj} x_k x_j x_i \bar{h}(r) \quad , \tag{25}$$

where the expression for $\bar{R},\bar{Q},\bar{f},\bar{g}$ and $\bar{h}$ are given in the appendix.

The balance of the normal stresses on the interface of the drop is given by equation (17). Since it is assumed that the interface of the drop is to be spherical the last term in equation (17) is replaced by -2T/a. In order to find out whether the drop will become oblate or prolate under conditions where equation (17) is not quite satisfied, the Taylor technique[14] is employed and it is assumed that a stress $F_o \cos^2\theta [= F_o (E_o \cdot n)^2 / E_o^2]$ is applied normally to the surface of the drop, which is necessary to keep it spherical. Replacing T in the modified form of equation (17) it is found that

$$p_0 - \bar{p}_0 = \frac{\bar{\mu} U(3+2\gamma)}{a} + \frac{9\bar{\mathcal{E}} E_0^2 (1-\beta)}{2(2+\alpha)^2} - \frac{2T}{a} - \frac{3\sigma^2 a^2 E_0^2}{2(2+\alpha)^2} - \frac{9\bar{\sigma}^2 a^2 E_0^2}{2(2+\alpha)^2} (\chi + \bar{\chi}) \quad ,$$

$$F_0 = \frac{\bar{\mathcal{E}} E_0^2}{(2+\alpha)^2} \left\{ \frac{9}{2} \left[\beta(1+\alpha)^2 - 2 + \frac{2}{5} \ \frac{(3+2\gamma)(\alpha\beta-1)}{\gamma+1} \right] \right.$$

$$\left. - \frac{3\chi\sigma^2 a^2}{40 \bar{\mathcal{E}}} \left[\frac{7-2\alpha-5\alpha^2}{\gamma+1} + 14 - 4\alpha - (25 + 15 \frac{\bar{\chi}}{\chi})\alpha^2 \right] \right\}$$

where $\gamma = \mu/\bar{\mu}$. The expression of F_O has been given in[17] (an arithmetic error in equation (30) in[17] is corrected, and 44 is replaced by 14). The equilibrium geometry depends on F_O , namely the functional relation which is given by $\alpha,\beta,\gamma,\chi,\bar{\mathcal{E}},\sigma$ and a. For a detailed discussion the reader may be referred to.[17]

A drop in an electric field in a pure straining motion

Since the governing equations are linear in terms of the velocity and electric field, the effects in the case of a drop in an electric field without a linear velocity at infinity and that of a drop embedded in a pure straining motion in the absence of a uniform electric field at infinity can be superimposed. Following the general arguments given in the previous paragraph the pressure and the velocity in the fluid outside the drop can be written as

$$\frac{P-P_o}{\mu} = b_{jj} R(r) + h_{kj} x_k x_j Q(r) + B \frac{e_{kj} x_k x_j}{r^3} \, , \tag{26}$$

$$u_i = b_{jj} x_i f(r) + b_{ij} x_j g(r) + b_{kj} x_k x_j x_i h(r) + e_{ij} x_j G(r) + e_{kj} x_k x_j x_i H(r) \tag{27}$$

and inside the drop as

$$\frac{\bar{P}-\bar{P}_o}{\mu} = b_{jj} \bar{R}(r) + b_{kj} x_k x_j \bar{Q}(r) + \bar{B} e_{kj} x_k x_j \, , \tag{28}$$

$$\bar{u}_i = b_{jj} x_i \bar{f}(r) + b_{ij} x_j \bar{g}(r) + b_{kj} x_k x_j x_i \bar{h}(r) + e_{ij} \bar{G}(r) + e_{kj} x_k x_j x_i \bar{H}(r) \, , \tag{29}$$

where $R, Q, f, g, h, \bar{R}, \bar{Q}, \bar{f}, \bar{g}, \bar{h}, B, G, H, \bar{B}, \bar{H}$ are given in the appendix. Although a similar discussion to that given in the previous paragraph for the shape of the drop can be done, this is beyond the scope of this paper.

The particle stress in a dilute suspension

By the expression dilute suspension it is meant that the flow near one particle is independent of all the other particles. However, a simple model[24] illustrates how surprisingly close the spheres are for concentrations which are numerically quite small. The relation (4) shows that for a dilute suspension the different particles in the volume V of suspension make linearly additive contributions to the particle stress and the particle stress obtained under these conditions is correct to the order of c (where c is the concentration of particles by volume). Thus, the results obtained in the previous section may be used to evaluate the third term in equation (4). However, some necessary modifications must be made. The velocity, pressure and stress in the fluid will be written as

$$u_i = e_{ij} x_j + u'_i \quad , \quad p = P + p' \quad , \quad \sigma_{ik} = -P \delta_{ik} + 2\mu e_{ik} + \sigma'_{ik} \, ,$$

where P is a constant and u'_i, p', σ'_{ik} are the disturbance quantities resulting from the presence of the particle. The particle stress becomes

$$\Sigma^{(P)}_{ij} = \frac{1}{V} \int_{A_o} \left\{ \sigma'_{ik} x_j n_k - \mu (u'_i n_j + u'_j n_i) \right\} dA \, , \tag{30}$$

since only the deviatoric part of the particle stress is significant, the term $-P \delta_{ij}$ and similar terms are omitted.

Inserting equations (26) and (27) in equations (30) and using the well-known identities

$$\int n_i n_j dA = \frac{4\pi a^2}{3} \delta_{ij} \quad ,$$

$$\int n_i n_j n_k n_l dA = \frac{4\pi a^4}{15} (\delta_{ij} \delta_{kl} + \delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) \quad ,$$

in which the integration is carried over the surface of a sphere of radius a, the following equation is obtained

$$\Sigma^{(P)}_{ij} = c \left[\frac{\gamma(2\gamma+5)}{\gamma+1} \bar{\mu} e_{ij} + \frac{27(1-\alpha\beta)\gamma}{5(2+\alpha)^2(1+\gamma)} (1-\lambda\phi) \bar{\varepsilon} E_{\alpha i} E_{\alpha j} \right] \, , \tag{31}$$

in this equation

$$\phi = \frac{\gamma+1}{9\gamma(1-\alpha\beta)} \left[\frac{3}{2} + \frac{(1-\alpha)(7+5\alpha)}{4(1+\gamma)} \right] \, , \quad \lambda = \frac{\chi \bar{\varepsilon}^2 a^2}{\bar{\varepsilon}} \, , \quad \alpha = \frac{\bar{\sigma}}{\sigma} \, , \quad \beta = \frac{\bar{\varepsilon}}{\varepsilon} \, , \quad \gamma = \frac{\mu}{\bar{\mu}} \, , \quad c = \frac{\Sigma V}{V}$$

Since only deviatoric part of the particle stress is significant, an isotropic term in the expression for the particle stress is not considered. If λ goes to zero equation (31) reduces to that given in.[21] The first term of the particle stress, given by equation (31), shows the effect in the absence of an applied electric field, and the second term denotes the additional effect due to the applied electric field. The second term of the particle stress contains two parts : one is due to the absence of the magnetic effect in the fluid outside the drop and the other is due to the presence of the magnetic effect. Since it is assumed that the fluid outside the drop is more conductive than inside the drop, the conductivity ratio α is very much smaller than unity, and the fluid outside the drop is poor dielectric in comparison to that inside the drop, the dielectric ratio β is very much smaller than unity, then $\phi > 0$ and the magnetic effect works to reduce the dependence of the particle stress on the direction of the applied electric field.

Acknowledgment

Appendix

$$R(r) = -\frac{\chi\sigma^2}{4\mu}r^2 - \frac{A}{3r^3} \quad , \quad Q(r) = \frac{\chi\sigma^2}{4\mu} + \frac{A}{r^3} - \frac{\chi\sigma^2}{\mu}\frac{1-\alpha}{2+\alpha}\frac{a^3}{r^3} - \frac{\chi\sigma^2}{2\mu}\frac{(1-\alpha)^2}{(2+\alpha)^2}\frac{a^6}{r^6} \quad ,$$

$$A = -\frac{6a^2}{E_0^2}U + \frac{3\chi\sigma^2 a^5}{4\mu}\frac{(1-\alpha)(5+\alpha)}{(2+\alpha)^2} \quad , \quad U = \frac{aE_0^2\bar{\epsilon}}{\bar{\mu}(2+\alpha)^2(4+\gamma)}\left[-\frac{9}{40}(\alpha\beta-1)+\frac{\chi\sigma^2 a^2}{\bar{\epsilon}}\frac{(1-\alpha)(7+5\alpha)}{40}\right] \quad ,$$

$$f(r) = \frac{U}{aE_0^2}\left[\left(\frac{a^3}{r^3} - \frac{a^5}{r^5}\right) + \frac{U^*}{U}\left(\frac{a}{r} - \frac{5+\alpha}{2+\alpha}\frac{a^3}{r^3} + 2\frac{1-\alpha}{2+\alpha}\frac{a^4}{r^4} + \frac{1+2\alpha}{2+\alpha}\frac{a^5}{r^5}\right)\right] \quad ,$$

$$g(r) = -\frac{2U}{aE_0^2}\left[\frac{a^5}{r^5} + \frac{U^*}{U}\left(\frac{a}{r} - \frac{1-\alpha}{2+\alpha}\frac{a^4}{r^4} - \frac{1+2\alpha}{2+\alpha}\frac{a^5}{r^5}\right)\right] \quad ,$$

$$h(r) = \frac{U}{a^3 E_0^2}\left[\left(5\frac{a^7}{r^7} - 3\frac{a^5}{r^5}\right) - \frac{U^*}{U}\left(\frac{a^3}{r^3} - 3\frac{5+\alpha}{2+\alpha}\frac{a^5}{r^5} + 8\frac{1-\alpha}{2+\alpha}\frac{a^6}{r^6} + 5\frac{1+2\alpha}{2\alpha}\frac{a^7}{r^7}\right)\right] \quad , \quad U^* = \frac{\chi\sigma^2 a^2 E_0^2}{8\mu}\frac{1-\alpha}{2+\alpha}$$

$$\bar{R}(r) = \left[\frac{7U}{a^3 E_0^2} - \frac{9\bar{\sigma}^2\bar{\chi}}{4\bar{\mu}(2+\alpha)^2}\right]r^2 \quad , \quad \bar{Q}(r) = \frac{21U}{a^3 E_0^2} + \frac{9\bar{\epsilon}^2\bar{\chi}}{4\mu(2+\alpha)^2} \quad ,$$

$$\bar{f}(r) = \frac{U}{aE_0^2}\left(\frac{r^2}{a^2} - 1\right) \quad , \quad \bar{g}(r) = \frac{U}{aE_0^2}\left(3 - 5\frac{r^4}{a^4}\right) \quad , \quad \bar{h}(r) = \frac{2U}{a^3 E_0^2} \quad ,$$

$$B = -\frac{2\gamma+5}{\gamma+1}a^3 \quad , \quad \bar{B} = \frac{21\gamma}{2(\gamma+1)a^2} \quad , \quad G = i\frac{a^6}{(\gamma+1)r^5} \quad , \quad H = \frac{B}{2r^5} + \frac{5a^3}{2(\gamma+1)r^7} \quad , \quad \bar{G} = \frac{3\gamma}{2(\gamma+1)} + \frac{5\bar{B}}{24}r^2 \quad , \quad \bar{H} = \frac{2\bar{B}}{21}$$

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