Electrohydrodynamic (EHD) Stimulation of Jet Breakup

Joseph M. Crowley

Dept. of Elec. Eng., Univ. of Illinois, Urbana, Illinois
and Xerox Corporation, Rochester, New York

Abstract

Electrohydrodyanmic (EHD) excitation of liquid jets offers an alternative to piezoelectric excitation without the complex frequency response caused by piezoelectric and mechanical resonances. In an EHD exciter, an electrode near the nozzle applies an alternating Coulomb force to the jet surface, generating a disturbance which grows until a drop breaks off downstream. This interaction can be modelled quite well by a linear, long wave model of the jet together with a cylindrical electric field. The breakup length, measured on a 33 μm jet, agrees quite well with that predicted by the theory, and increases with the square of the applied voltage, as expected. In addition, the frequency response is very smooth, with pronounced nulls occurring only at frequencies related to the time which the jet spends inside the exciter.

Introduction

Most of the existing ink jet printers based on jet breakup use acoustic excitation of the jet to introduce the disturbance which eventually grows to form ink drops. With a single jet, this method works very well, since the breakup point can be controlled through a feedback loop which alters the amplitude of the excitation. The extension of this technique to multiple jet printers presents some difficulties, however, since the jets interact with the acoustic waves set up in the exciter, causing variations in the breakup length.

Excitation based on electrostatic forces appears to offer more promise for multiple jet printers, since the excitation is applied via electrodes which are placed downstream of the nozzle, allowing individual control of individual jets. Although droplet production by electric excitation has been known for some time, there has been no work reported which offers a theoretical model of this process, or which describes systematic measurements of the breakup length with EHD excitation. The present paper is intended to fill this gap.

A Simple Theory of EHD Excitation

A detailed analysis of the EHD exciter has been undertaken, and will be presented elsewhere[1]. In the course of that work, a simple model was developed, which will be used here to describe the exciter used in the experiments. The geometry of the jet is defined by Figure 1, which shows a liquid jet entering an electrode. The jet has a radius R, and moves with a velocity U_o.

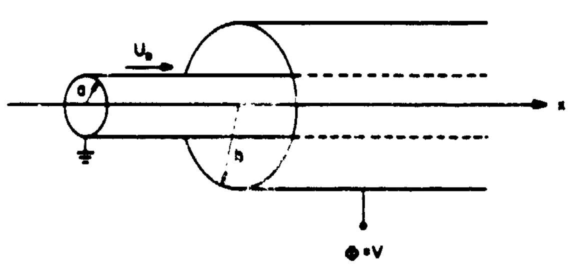

Figure 1
A Liquid jet entering an electrode

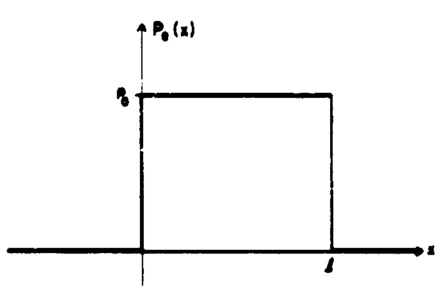

Figure 2
Electric pressure in the step exciter

This particular electrode is shown as a cylinder, but it can have any shape, since the theory assumes only that the electric field at the surface of the jet can be calculated.

Theoretically, the jet is modelled by the long wave approximation[2], in which the disturbance is assumed to be much longer than the jet radius, and The axial velocity and pressure are assumed to be constant across any section of the jet. With these assumptions, the momentum equation for motion in the axial direction can be written as

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{1}$$

while Conservation of Mass for a round jet gives

$$\frac{\partial R}{\partial t} + U\frac{\partial R}{\partial x} + \frac{R}{2}\frac{\partial U}{\partial x} = 0 \tag{2}$$

The pressure in the momentum equation consists of contributions from the surface tension of the jet and also from the electric field at its surface.

The Inertial Limit

The surface tension term is well known in the fluid mechanics of jets; it leads to growth of the disturbance into drops. Inside the exciter, which is usually only a fraction of a wavelength long, the droplet growth is very small, so the effects of surface tension will be neglected here. The electric pressure can be most easily obtained by using the Maxwell stress tensor at the surface of the jet. Since the jet is assumed to be a good electrical conductor, the electric field will be normal to the surface of the jet, which implies that no electric shear forces can be exerted[3]. The only electrical force corresponds to a (negative) pressure, given in terms of the field strength at the jet surface as

$$p(r,x,t) = -\frac{1}{2}\epsilon E^2 \tag{3}$$

For example, if the electrode surrounding the jet is assumed to be a circular cylinder of radius b, at a voltage V(x,t), then the normal electric field at the surface of the jet will be

$$E = \frac{V(x,t)}{a\ \ell n(b/a)} \tag{4}$$

so that the electric pressure is given by

$$p = -\frac{\epsilon V^2(x,t)}{2a^2\ \ell n^2(b/a)} \tag{5}$$

Linearization

In most of the growth region the amplitude of the waves is usually quite small, so that the equations may be further simplified by linearization. If we define

$$U = U_o + u\ ,\ u \ll U_o \tag{6}$$

$$R = a + \delta\ ,\ \delta \ll a \tag{7}$$

and neglect all second order terms, the equations of motion for small disturbances on the jet reduce to

$$\frac{\partial u}{\partial t} + U_o \frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{8}$$

$$\frac{\partial \delta}{\partial t} + U_o \frac{\partial \delta}{\partial x} + \frac{a}{2}\frac{\partial u}{\partial x} = 0 \tag{9}$$

These equations have been written in terms of the convective derivative of the surface displacement, defined by

$$\delta' = \frac{\partial \delta}{\partial t} + U_o \frac{\partial \delta}{\partial x} \tag{10}$$

In the inertial approximation, the use of this surface velocity simplifies the resulting equations.

The electric pressure depends on the electric field at the surface of the jet, and this field will usually change as the jet expands and contracts. Thus perturbations in radius will lead to perturbations in the electric pressure, which in turn depend on the detailed structure of the field near the jet. Like the surface tension term, the perturbation electric field mainly influences the growth rate of the droplet disturbance. Its effect is usually much less than that of the surface tension, so it will also be neglected, as the surface tension was. The equation of the jet then becomes

$$\left(\frac{\partial}{\partial t} + U_o \frac{\partial}{\partial x}\right) \delta' = \frac{a}{2\rho}\frac{\partial^2 p}{\partial x^2} \tag{11}$$

The linear response of the jet will generally be expressed in terms of its response to a sinusoidal pressure variation. For each term in the Fourier series, the driving pressure is most conveniently written in terms of complex amplitudes as

$$p = \mathrm{Re}[\tilde{p}(x)\, e^{i\omega t}] \tag{12}$$

The response to this drive can likewise be written as

$$\delta = \mathrm{Re}[\tilde{\delta}(x)\, e^{i\omega t}] \tag{13}$$

so that the equation of motion for the exciter (Equation 12) reduces to an ordinary differential equation for the complex amplitude of the radial velocity

$$\left(i\omega + U_o \frac{d}{dx}\right) \tilde{\delta}' = \frac{a}{2\rho}\frac{d^2 \tilde{p}}{dx^2} \tag{14}$$

There are several ways to solve this equation, but since we will be dealing with singular spatial functions such as the impulse and step, the easiest solution is that which uses the Laplace transform in x. With the definition of the Laplace transform as

$$\tilde{\delta}(x) = \int \hat{\delta}(s)\, e^{sx} dx \tag{15}$$

and

$$\tilde{p} = \int \hat{p}(x)\, e^{sx} dx \tag{16}$$

the equation can be transformed into

$$(i\omega + U_o s)\,\hat{\delta}' = \frac{a}{2\rho} s^2 \hat{p} \tag{17}$$

which can be inverted to find the response as soon as the spatial distribution of electric pressure is known.

Constant Pressure Exciter

The simplest practical exciter is a cylindrical electrode of length ℓ. The entire exciter is driven from a single voltage source, for example $V_o+V_1 \sin(\omega t)$, so that the same pressure is applied throughout. For this exciter, p has the spatial form shown in Figure 2. with a time varying magnitude of

$$p(t) = -\frac{\epsilon}{2a^2\,\ell n^2(b/a)}\left[\left(V_o^2 - \frac{V_1^2}{2}\right) + 2V_1V_2 \sin\omega t + \frac{V_2^2}{2}\sin 2\omega t\right] \tag{18}$$

This pressure has components at DC, ω, and at 2ω, but since the equation is linear, each component can be treated separately. For the component at ω, the electric pressure has the form

$$\tilde{p} = p_o = \frac{\epsilon\,V_{eff}^2}{2a^2\,\ell n^2(b/a)} \tag{19}$$

where V_{eff} is the effective (RMS) value of the voltage, which in this case is $2V_0V_1$. For this exciter, the Laplace transform of the drive is

$$\hat{p} = -p_o\left(\frac{1 - e^{-s\ell}}{s}\right) \tag{20}$$

giving the radial velocity at the exit as

$$\tilde{\delta}'(x) = -\frac{ap_o}{2\rho\,U_o}\left[\Delta(x) + \frac{i\omega}{U_o} e^{\frac{-i\omega x}{U_o}} - \Delta(x-\ell) - \frac{i\omega}{U_o} e^{\frac{-i\omega(x-\ell)}{U_o}}\right] \tag{21}$$

The response has an impulse (Δ) in velocity because the jet quickly contracts as it enters the exciter and expands as it leaves. These impulses have no effect downstream, since they vanish as soon as the jet has passed the entrance or exit. For practical purposes, then, the velocity of the jet at the exit of the exciter can be written as

$$\tilde{\delta}' = \frac{i\omega ap_o}{2\rho\,U_o^2}\left(1 - e^{-\frac{i\omega\ell}{U_o}}\right) e^{-\frac{i\omega x}{U_o}} \tag{22}$$

The velocity has an exponential delay term which can be analyzed more conveniently by extracting a factor corresponding to the center of the exciter, using the relation

$$1 - e^{-\frac{i\omega\ell}{2}} = e^{-\frac{i\omega\ell}{2}}\,2i\sin\frac{\omega\ell}{2} \tag{23}$$

The velocity response then has the magnitude

$$|\delta_o'| = \frac{a\omega p_o}{\rho\,U_o^2}\left|\sin\frac{\omega\ell}{2U_o}\right| \tag{24}$$

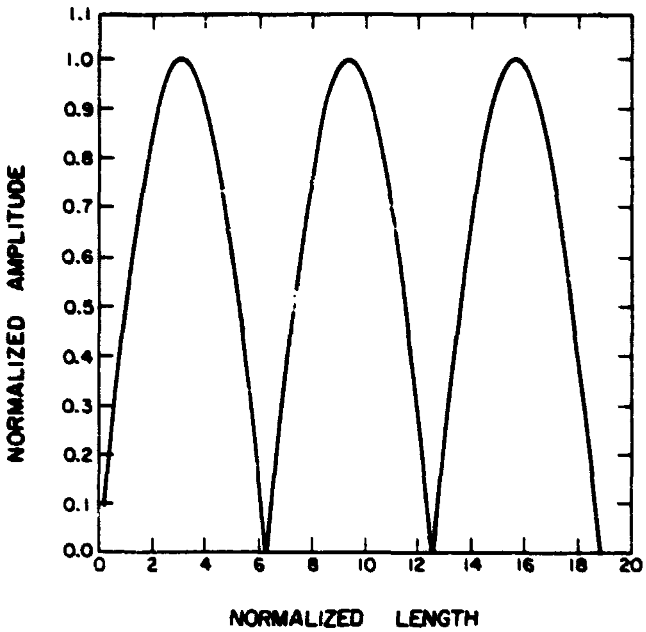

Figure 3
Exciter output versus length

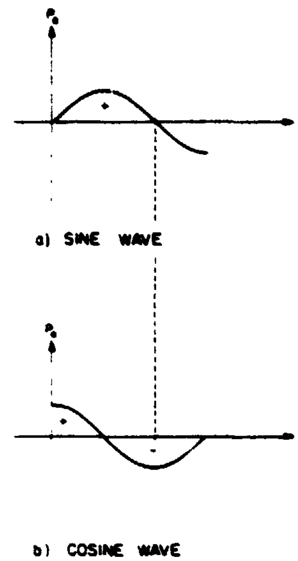

Figure 4
Force variation within the exciter

which is shown in Figure 3 as a function of exciter length.

The nulls and peaks which occur in the response are caused by the alternation of electric pressure while the jet is inside the exciter. If the pressure is sinusoidal, as shown in Figure 4a, the net impulse given to the jet over one period of excitation will be zero, since the jet is influenced by equal and opposite half cycles. Thus if the jet is within the exciter for a whole period of excitation, (i.e., the exciter is one wavelength long), the response will show a null. On the other hand, there will be a peak for lengths which are one half wavelength long. While this effect is easy to see from Figure 4a, some care must be used in interpreting such diagrams, since they may suggest erroneous nulls, depending on the choice of phase at the inlet. For example, if the sinusoidal force were chosen to be a cosine, as in Figure 4b, it would be easy to infer that the null would occur when the exciter is only one half wavelength long. This is not true, because the cosine is one special choice from all of the phases which occur during a complete cycle.

Because of the peaks and nulls in the frequency response, the output of the exciter can not be increased indefinitely simply by increasing the length of the exciting electrodes. Instead, an optimum length, which may be fairly small by macroscopic standards, must be selected to achieve maximum output. This explains why the electrodes used in earlier work on EHD excitation were usually not very efficient.

Downstream Response

The approximate surface displacement far downstream is given by[1]

$$\delta = \frac{\delta'_o}{2\mu\, U_o}\, e^{\mu x} \tag{25}$$

where

$$(\mu a)^2 = \alpha^2 \left(\frac{\omega a}{U_o}\right)^2 \left[1 - \left(\frac{\omega a}{U_o}\right)^2\right] \tag{26}$$

The breakup length can be determined by extrapolating the downstream response to the point of breakup, ℓ_B to give the breakup length equation,

$$\ell_B = \frac{1}{\mu} \ln\left(\frac{2a\mu U_o}{\delta_o'}\right) \tag{27}$$

Thus the breakup length can be calculated as soon as the exciter output is known.

Experiments

Apparatus

Several experiments were performed in order to test the predictions of the EHD exciter theory. The nozzle and associated apparatus were mounted on a specially designed frame which allowed adjustment and positioning suitable for the experiments. The fluid used in most of these experiments had a density of 1030 kg/m^3 and a surface tension of 39 mN/m. The jet formed by the nozzle had a radius of 16.5 um. Its velocity was determined by measuring the wavelength of the disturbance at a known frequency.

The exciter electrodes used in most of these experiments consisted of steel plates of various nominal thicknesses (3-7 mil, or 75-175 um) through which holes were drilled. The holes ranged in nominal diameter from 3-7 mils (75-175 um). Microscopic examination of the electrodes revealed that the holes were essentially straight sided, although occasional burrs could be seen.

Procedure

Since the electrodes consisted of a single piece of metal, they had to be in position before the jet was turned on to avoid splashing and electrical shorting between exciter and ground. As a compromise between speed and complexity, we first located the approximate axis of the jet by focussing a microscope on the nozzle orifice along the nozzle axis. The electrode, mounted on a three axis micromanipulator, was then moved into position so that the desired exciter hole appeared to be on the nozzle axis, as seen through the microscope. The microscope was then removed to the side, where the jet and hole would both be visible at an oblique angle, and the jet turned on. The original electrode placement was usually within a few mils of the desired placement, could be quickly adjusted to center the electrode and allow the jet to stream through unhindered.

The jet was then turned off to allow the electode to be cleaned by wiping with absorbant paper towels. Upon restarting, the jet always went through the hole cleanly, without splashing or blocking, even for the 3 mil hole, once the initial centering procedure had been carried out. In most cases, the jet would restart cleanly even if the electrode had not been wiped clean.

Several arrangements were used to provide different voltage levels. An operational amplifier could be connected directly to the electrodes, giving voltage up to 150V. In order to increase the voltage, the output of the operational amplifier was often connected to either a wide band or a narrow band transformer. The secondary winding of the transformer furnished higher AC voltages than the amplifier, and additional increases in the effective voltage could be achieved by connecting the secondary in series with a DC power supply. With this arrangement, peak voltages up to 800V were obtained. This voltage was not limited by breakdown in the exciter, but by the available power supplies and amplifiers.

In carrying out the experiments, the jet was turned on and the voltages adjusted to the desired value. The position of the breakup was then measured by a micrometer which moved a long focal length microscope to the breakup point while the phase of the viewing strobe was slowly rolled. The breakup position can be measured very precisely (+10 um) with this technique if the jet is quiet. At very long breakup lengths (>4 mm), the natural breakup of the jet caused the breakup point to vary erratically. Lateral movement of the electrode varied the breakup point, probably by increasing the electric field strength at the surface of the jet. This effect was used to center the jet before each measurement, since the point of minimum excitation should correspond to a centered jet.

Frequency Response

The simple theory presented above predicts maxima and minima in the frequency response of the exciters. In order to test these predictions, frequency sweeps were performed on a number of exciters. The easiest feature of the response to check is the null which occurs when the frequency corresponds to an entire wavelength of the disturbance inside the jet at any time. Figure 5 depicts the frequency response for an electode which is 7 mils (178 μm) thick.

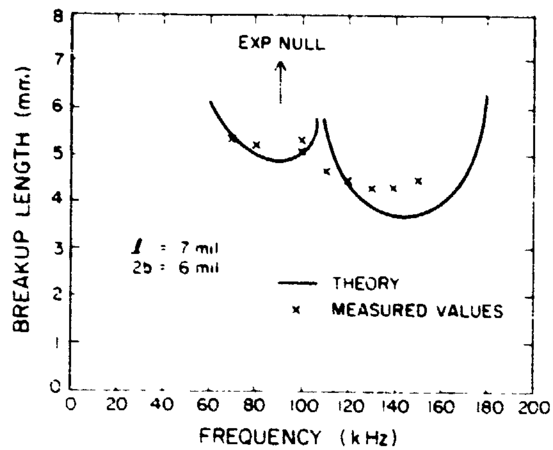

Figure 5
Frequency response for a 7 mil exciter

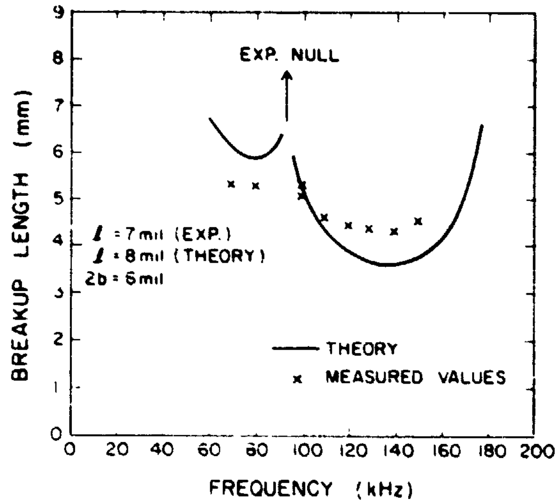

Figure 6
Same thickness, but 8 mil theory

For this length, the null is expected at 108 kHz. Since this null occurs near the maximum growth rate of the jet, it could be observed experimentally by varying the frequency to produce the longest apparent breakup length. This null frequency (shown in Figure 5.3 by an arrow) is lower than expected from theory by approximately 15%. Reasonable variations in the fringing length and jet velocity could not account for the discrepancy, but increasing the effective length of the exciter from 7 to 8 mils gave better agreement, as shown in Figure 6. This extension in length might be justified by the presence of burrs at either end of the hole, since such burrs were often observed in microscopic examination of the holes. Another possibility is the effect of fringing in extending the effective length of the electrode beyond the physical limits of the conductor.

This exciter exhibits the predicted null in the frequency response, but does not test the exciter at the thickness which is expected to give the shortest breakup length. Since short breakup length is desired in practical printers, the frequency response of a shorter (3 mil or 76 μm) electrode was also measured. The results of this measurement, along the the predictions of the theories, are shown in Figure 7.

Just as in the earlier measurements, the theory predicts both the magnitude and the shape of the frequency response quite well (±0.2 mm or 10%) whether the fringing fields are included or not, although the fringing approximation seems to give better results at the shortest breakup lengths. To appreciate this agreement, it should be kept in mind that the theory, which rests on the fundamental equations of electrostatics and fluid mechanics, contains no adjustable parameters. The breakup length is predicted only in terms of geometrical measurements, applied voltage, and material properties.

Voltage response

Compared to acoustic excitation, EHD excitation is often relatively weak, so that the magnitude of the drive is extremely important in practice. The theory presented above predicts that the excitation pressure is proportional to the square of the effective voltage, so that a relatively large increase in the excitation may be obtained by increasing the voltage at which the exciter operates. This prediction was tested experimentally by varing both the AC and DC voltage levels over a wide range, and measuring the breakup length.

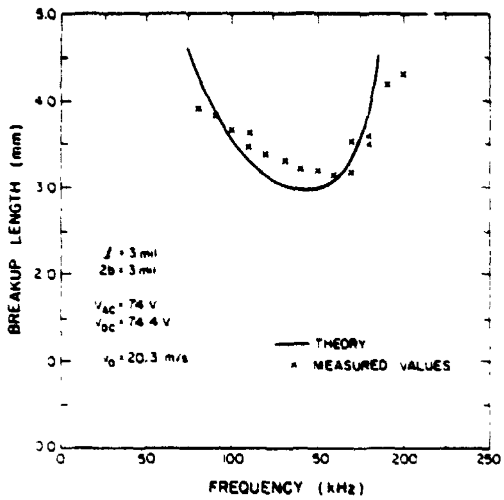

Figure 7
Frequency response for a 3 mil thickness

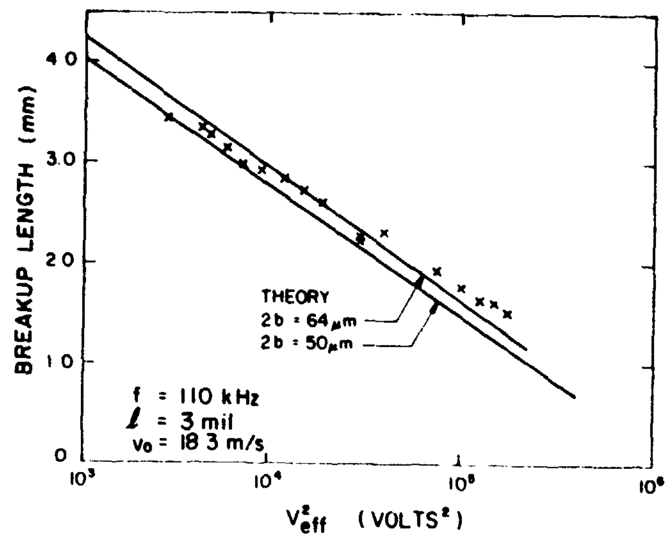

Figure 8
Breakup length versus effective voltage

Since the breakup length depends logarithmically on the excitation, a plot of breakup length against the logarithm of the effective voltage should give a straight line. Experimental values of breakup length are plotted in this way in Figure 8, along with the predictions of the simpe theoretical model. Both the magnitude and slope of the breakup length follow the predictions well, although the slope of the line appears to be somewhat flatter than expected. These results give us some confidence in extrapolating the design to even higher voltages to achieve a shorter breakup length if necessary, although these lengths are already comparable to those used in acoustic excitation.

Acknowledgments

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References

1. Crowley, J. M., "The Theory of Electrohydrodynamic Droplet Generators," to be publ'shed

2. Lee, H. C., "Drop Formation in a Liquid Jet," IBM Journal of Research and Development, July, 1974, pp.364-369

3. Melcher, J. R., Field Coupled Surface Waves, MIT Press, Cambridge, Mass., 1963