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Dielectrophoretic levitation of droplets and bubbles*

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Abstract

Uncharged droplets and bubbles can be levitated dielectrophoretically in liquids using strong, nonuniform electric fields. The general equations of motion for a droplet or bubble in an axisymmetric, divergence-free electrostatic field allow determination of the conditions necessary and sufficient for stable levitation. Two cases of a bubble or droplet with dielectric constant (κ_2) less than or greater than the liquid medium dielectric constant (κ_1) emerge. For $\kappa_2 < \kappa_1$, passive levitation is possible, while for $\kappa_2 > \kappa_1$, active feedback-controlled levitation is required. The design of dielectrophoretic (DEP) levitation electrode structures is simplified by a Taylor-series expansion of cusped axisymmetric electrostatic fields. Extensive experimental measurements on bubbles in insulating liquids verify the simple dielectrophoretic model. Others have extended dielectrophoretic levitation to very small particles (<100 μm) in aqueous media. Zero-gravity DEP levitation remains to be demonstrated. This paper concludes with a discussion of applications of DEP levitation to the study of gas bubbles, liquid droplets, and solid particles. Some of these applications are of special interest in the reduced gravitational field of a spacecraft.

Introduction

Strong nonuniform electric fields exert effective forces on uncharged particles, liquid droplets, and gas bubbles. Pohl named this effect *dielectrophoresis* in 1951.[1] The dielectrophoretic phenomenon is most pronounced for small particles (<1000 μm). A cusped electrostatic field can stably levitate bubbles, droplets, and small particles suspended in dielectric liquids. Further demonstrations of the manipulation and control of droplets, bubbles, and solid particles exist that are relevant to the reduced gravitational environment of space. Zero-gravity dielectrophoresis offers some interesting prospects in a number of materials processing applications that are considered in this paper.

H. A. Pohl exploited dielectrophoresis in the separation of solid dielectric particles from a liquid insulant. One proposed application has been in the separation of living from dead blood cells.[2] Lin and Benguigui have used granular solid dielectric media stressed with strong electric fields to achieve high gradient DEP separations.[3] Veas and Schaeffer used a cusped electrostatic field to levitate small droplets of one dielectric liquid suspended in another.[4] Jones and Bliss studied the dielectrophoresis of small gas bubbles in insulating liquids and levitated these bubbles using an improved electrode geometry.[5] Kallio and Jones proposed the use of DEP levitation for the measurement of solid particle and liquid dielectric constants.[6]

In this paper, I discuss theoretical and experimental dielectrophoresis. Potential applications of DEP in a number of diverse fields (including zero-gravity experimentation) are reviewed in light of this recent work.

Theory

The theory relevant to dielectrophoresis is conveniently divided into sections dealing with the basic force calculation, properties of divergence-free electrostatic fields, frequency dispersion effects (charge relaxation), and the equations of motion.

Dielectrophoretic Force

The DEP force is nonzero if either the magnitude or direction of the Lorentz force ($q\bar{E}$) exerted on the positive and negative ends of a dipole differ. The general expression is

$$\bar{F}^e = (\bar{p}_{eff} \cdot \nabla)\bar{E}, \tag{1}$$

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where $\bar{E}$ is the imposed electric field and $\bar{p}_{eff}$ is the effective dipole moment of the particle. If the particle is an insulating sphere of radius R, dielectric constant κ_2, and mass density ρ_2 in a dielectric insulator of dielectric constant κ_1 and mass density ρ_1, and if the particle is sufficiently small, then

$$\bar{p}_{eff} \approx 4\pi R^3 \varepsilon_o \kappa_1 \left(\frac{\kappa_2 - \kappa_1}{\kappa_2 + 2\kappa_1}\right)\bar{E}. \qquad (2)$$

Combining Eqs (1) and (2), the familiar result is

$$\bar{F}^e \approx 2\pi R^3 \varepsilon_o \kappa_1 \left(\frac{\kappa_2 - \kappa_1}{\kappa_2 + 2\kappa_1}\right)\nabla E^2 , \qquad (3)$$

which successfully predicts the observed behavior. Particles with $\kappa_2 > \kappa_1$ are attracted to relative electric field intensity maxima, while for $\kappa_2 < \kappa_1$, the particles are attracted to minima.

Garton and Krasucki observed that liquid droplets and bubbles deform into prolate spheroids in the direction of the applied electric field.[7] For prolate spheroids, the DEP force becomes

$$\bar{F}^e = \frac{2\pi a b^2 \varepsilon_o (\kappa_2 - \kappa_1)}{3[1 + (\kappa_2 - \kappa_1)L_z/\kappa_1]} \nabla E^2 , \qquad (4)$$

where a and b are the semi-major and semi-minor axes of the particle, and L_z is the depolarization factor:

$$L_z = [1 + \frac{3}{5} (1 - \gamma^{-2}) + \frac{3}{7} (1 - \gamma^{-2})^2 + ...]/3\gamma^2, \text{ and } \gamma = a/b . \qquad (5)$$

The ratio γ is uniquely related to E. For small deformations, i.e., $\gamma < 1.1$, an approximation may be used:[8]

$$\gamma \approx 1 + \frac{\sqrt{3} \varepsilon_o \kappa_1 r_o}{16\delta} \left(\frac{\kappa_1}{\kappa_1 - \kappa_2} - \frac{1}{3}\right)^{-2} (\cos \theta_o)^{-1} E^2, \qquad (6)$$

where

$$\theta_o = \cosh^{-1}(3\sqrt{3}Pr_o/4\delta)/3 \text{ and } r_o \approx (R^3P/2\delta + R^2)^{1/2}. \qquad (7)$$

P is the absolute ambient pressure and δ is the interfacial tension.

Properties of Electrostatic Fields

Theorems concerning electric field intensity maxima and minima are very important in DEP levitation. Consider a static, divergence-free electric field $\bar{E}$ such that

$$\nabla \cdot \bar{E} = 0, \text{ and } \nabla \times \bar{E} = 0. \qquad (8)$$

It is easily shown[5] that

$$\nabla^2 E^2 \geq 0 . \qquad (9)$$

This inequality specifically excludes the possibility of *isolated* three-dimensional electric field intensity maxima. The existence of isolated intensity minima (including zeroes) is demonstrated by reference to examples such as the quadrupole. Because of the above properties of electrostatic fields, stable, *static* levitation of particles is possible only for $\kappa_2 < \kappa_1$. This restriction places a serious limitation on the application of DEP levitation.

Certain properties of cusped axisymmetric electrostatic fields, useful in DEP levitation cells, may be examined using a Taylor-series expansion due to Holmes.[9] This field expansion, correct to second order in excursions (ρ,z') about some equilibrium point $(0,z)$, is:[10]

$$E_z = V\left[\alpha_o + \alpha_1 z' + \alpha_2 \left(z'^2 - \frac{\rho^2}{2}\right)\right] , \qquad (10)$$

$$E_\rho = V\left[-\frac{\alpha_1}{2} \rho - \alpha_2 \rho z'\right] , \qquad (11)$$

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where

$$\alpha_n(z) = \frac{1}{\sqrt{n!}} \left. \frac{\partial^n E_z}{\partial z^n} \right|_{\rho=0,z} \tag{12}$$

and V is the applied voltage. The stability of levitation for $\kappa_2 < \kappa_1$ is determined by the dual conditions

$$\left. \frac{\partial^2 E^2}{\partial z^2} \right|_{0,z} > 0 \text{ and } \left. \frac{\partial^2 E^2}{\partial \rho^2} \right|_{0,z} > 0 \ , \tag{13}$$

where $E^2 = E_z^2 + E_\rho^2$. Using Eqs (10) and (11), Eq. (13) can be reduced to conditions on the coefficients α_o, α_1, α_2.

Frequency Dispersion

If the bubble, droplet, or particle and/or the fluid medium are slightly conductive, charge relaxation makes the effective dipole ρ_{eff} time dependent and dispersive. Benguigui and Lin arrive at a concise result for the time-average DEP force due to an ac dielectric field on a spherical particle with conductivity σ_2 in a fluid of conductivity σ_1[11]

$$\overline{F}^e = 2\pi R^3 \epsilon_o \kappa_1 \left[\frac{\kappa_2 - \kappa_1}{\kappa_2 + 2\kappa_1} + \frac{3\epsilon_o(\kappa_1\sigma_2 - \kappa_2\sigma_1)}{\tau(\sigma_2 + 2\sigma_1)^2(1 + \omega^2\tau^2)} \right] \nabla E_{rms}^2 \tag{14}$$

where ω is the radian frequency, E_{rms} is the rms electric field magnitude, and

$$\tau = \epsilon_o(\kappa_2 + 2\kappa_1)/(\sigma_2 + 2\sigma_1) \tag{15}$$

is a fundamental charge relaxation time. Equation (14) predicts that the DEP force is frequency-dependent, and a map of the stable regimes of levitation has been provided by Jones and Kallio.[12] (See Table 1.)

Table 1. AC Levitation Conditions

	$\kappa_2 < \kappa_1$	$\kappa_2 > \kappa_1$
$\sigma_2 > \sigma_1$	Stable levitation for $f > f_c$ only	No stable levitation at any frequency (f_c is not defined).
$\sigma_2 < \sigma_1$	Stable levitation for all frequencies (f_c is not defined).	Stable levitation for $f < f_c$ only

$$f_c = \sqrt{(\sigma_1 - \sigma_2)/\epsilon_o(\kappa_2 - \kappa_1)\tau} \ /2\pi$$

For a suddenly applied dc electric field, the DEP force is

$$\overline{F}^e(t) = 2\pi R^3 \epsilon_o \kappa_1 \left(\frac{\sigma_2 - \sigma_1}{\sigma_2 + 2\sigma_1} \right) (1 - \exp(-t/\tau)) \ \nabla E^2 + 2\pi R^3 \epsilon_o \kappa_1 \left(\frac{\kappa_2 - \kappa_1}{\kappa_2 + 2\kappa_1} \right) \exp(-t/\tau) \ \nabla E^2 \tag{16}$$

Charge relaxation, which governs the build-up and/or decay of free charge at the particle interface, causes these time- and frequency-dependent effects.

Equations of Motion

The translational equation of motion for a bubble, droplet, or particle in a nonuniform electric field has a fairly general form:

$$m_{eff} \frac{d\overline{v}}{dt} = m_g \overline{g} - b\overline{v} + \overline{F}^e \tag{17}$$

where:[13]

$$m_{eff} = \frac{4\pi}{3} (\rho_2 + \frac{1}{2} \rho_1)R^3, \ m_g = \frac{4\pi}{3} (\rho_2 - \rho_1)R^3 \tag{18}$$

ρ_1 and ρ_2 are mass densities, $\bar{g}$ is the gravitational acceleration vector, $\bar{v}$ is the velocity, and b is a damping coefficient. In viscous fluids, the Stokes drag model for viscous drag can be used:

$$b = \begin{cases} 4\pi\mu_1 R & \text{for spherical bubbles,} \\ 6\pi\mu_1 R & \text{for solid spheres,} \end{cases} \tag{19}$$

where μ_1 is the dynamic viscosity of the fluid. For a fluid droplet of dynamic viscosity μ_2:[14]

$$b = 2\pi\mu_1 R \ (2\mu_1 + 3\mu_2)/(\mu_1 + \mu_2). \tag{20}$$

Equation (17) may be reduced to a set of linear dynamic equations if the Taylor-series expansion for axisymmetric electrostatic fields is used. Consider the geometry shown in Figure 1. Gravitational acceleration is directed in the $-\hat{z}$ direction, so Eq. (17) becomes:

$$m_{eff} \ddot{z} = -m_g g - b \dot{z} + K \frac{\partial E^2}{\partial z} , \tag{21}$$

$$m_{eff} \ddot{\rho} = -b \dot{\rho} + K \frac{\partial E^2}{\partial \rho} \tag{22}$$

where:

$$K = 2\pi R^3 \epsilon_o \kappa_1 \ (\kappa_2 - \kappa_1)/(\kappa_2 + 2\kappa_1). \tag{23}$$

To linearize Eqs (22) and (23), let:

$$z(t) = z_o + z'(t), \tag{24}$$

$$\rho(t) = \rho'(t), \text{ and} \tag{25}$$

$$V(t) = V_o + v'(t) \tag{26}$$

where z', ρ', and v' are all small time-dependent variables. Then, using Eqs (10) and (11),

$$\frac{\partial E^2}{\partial z} = [2\alpha_o\alpha_1 + 2(\alpha_1^2 + 2\alpha_o\alpha_2)z'] \ V_o^2 + 4\alpha_o\alpha_1 V_o v' , \tag{27}$$

$$\frac{\partial E^2}{\partial \rho} = [2(\frac{\alpha_1^2}{4} - \alpha_o\alpha_2)\rho'] \ V_o^2 . \tag{28}$$

The equation of equilibrium

$$0 = -m_g g + 2\alpha_o\alpha_1 K V_o^2 \tag{29}$$

determines the location z_o of the particle, bubble, or droplet. The perturbation equations are

$$\ddot{z}' + \frac{b}{m_{eff}} \dot{z}' - \frac{2KV_o^2}{m_{eff}} (\alpha_1^2 + 2\alpha_o\alpha_2) \ z' - \frac{4KV_o}{m_{eff}} (\alpha_o\alpha_1) \ v' = 0 \tag{30}$$

$$\ddot{\rho}' + \frac{b}{m_{eff}} \dot{\rho}' - \frac{KV_o^2}{m_{eff}} (\frac{\alpha_1^2}{2} - 2\alpha_o\alpha_2)\rho' = 0 . \tag{31}$$

For passive levitation $v' = 0$, and so Eqs (30) and (31) must have stable solutions for equilibrium to be stable. For $K < 0$ ($\kappa_2 < \kappa_1$), the prescribed conditions on α_o, α_1, α_2 can be met, but for $K > 0$ ($\kappa_2 > \kappa_1$), radial and axial stability cannot be achieved simultaneously. Thus, active means must be used to levitate particles or droplets when $\kappa_2 > \kappa_1$.

Consider the axisymmetric electrode geometry of Figure 2, which differs from the cusped configuration of Figure 1 in that the maximum electric field intensity is found along the center line. This configuration guarantees *radial* stability if

$$\alpha_1^2 < 4\alpha_o\alpha_2 . \tag{32}$$

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To provide axial stabilization, an error signal proportional to z' must be generated. Assume that

$$v'/V_o = - Gz'/L \tag{33}$$

where G is a dimensionless gain factor and L is a characteristic cell length. Then:

$$\ddot{z}' + \frac{b}{m_{eff}} \dot{z}' + \frac{2KV_o^2}{m_{eff}} [2\alpha_o\alpha_1 G/L - \alpha_1^2 - 2\alpha_o\alpha_2] z' = 0. \tag{34}$$

The condition for axial stability is

$$G/L > \alpha_1/2\alpha_o + \alpha_2/\alpha_1 , \tag{35}$$

which can be met with sufficient gain.

Equation (33) assumes that the error voltage v' is not coupled co ρ' and that the feedback is ideal with no phase shift. In a practical implemenation of active DEP levitation, deviations from such ideal behavior would be expected.

By setting $v' = 0$ and $V(t) = V_o \cos\omega t$, the parametric stability of Eqs (30) and (31) may be considered. But in the inertially dominated and the heavily damped limits, stability is impossible to achieve for $K > 0$.

<div align="center">Experiment</div>

Force measurement

Most attempts to measure the dielectrophoretic force and thus check the validity of Eq. (3) involve the use of electrode structures with well-known electrostatic field solutions. Jones and Bliss[5] used wedge-shaped electrodes to measure forces on small gas bubbles in insulating liquid dielectrics. Pohl and Pethig[15] describe experiments with an isomotive electrode structure which features a DEP force that is independent of position within the cell. (See Figure 3.) Within the accuracy of measurement, the experimental results are consistent with DEP theory.

Levitation

DEP levitation makes possible some interesting applications of dielectrophoresis in terrestrial and extra-terrestrial environments. Veas and Schaeffer levitated small droplets of one liquid suspended in another[4] using a cusped field. Jones and Bliss[5] levitated both air bubbles and liquid droplets in dielectric liquids. Jones and Kallio[12] extended the measurements to various liquids, solid particles, and glass microballoons using the simple electrode geometry shown in Figure 4.

The matter of electrode shape has been pursued further by Jones and McCarthy[16], who show that the effectiveness of a DEP levitation cell is very sensitive to electrode shape, as shown in normalized plots of the DEP force versus axial particle position in the cell. (Refer to Figure 5.) This observed sensitivity is consistent with the analytical work of Pohl and Pollock.[17]

Frequency-Dependent Effects

Jones and Kallio[12] attempted to verify the theory concerning variable-frequency levitation of semi-insulating particles using ac and dc electric fields. The results are shown in Table 2.

Some variable frequency DEP levitation experiments using very small cells to levitate 50 μm solid particles[18] and biological materials[19] in aqueous media have also been reported.

Dielectric Constant Measurement

The use of DEP levitation for measurement of dielectric constant was reported in 1978[6]. All measurements are made with respect to a well-characterized standard dielectric liquid. A small gas bubble is levitated at a fixed position in the cell and the required voltage is noted. Then, the known liquid properties (mass density, dielectric constant) are used to calculate a geometric constant for the cell. The unknown is then introduced to the cell. In the case of an unknown liquid, a gas bubble is again levitated at the same position, or in the case of an unknown solid particle, the particle itself is levitated at the same position. The new voltage is noted and used in an analytic expression to compute the unknown dielectric constant.

Table 2. Results of some ac and dc levitation experiments.

Sphere	Liquid	f_c (Hz)	τ_1 (s)	Successful Levitation? ac[a]	dc
Polystyrene	Transformer oil	0.02	18	No	No
Polystyrene	Silicone fluid		240	Yes	
Polystyrene	Corn oil		0.55	Yes	
Polystyrene	Castor oil		0.40	Yes	Yes
Polyvinyl acetate	Transformer oil		18	No	No
Polyvinyl acetate	Silicone fluid		240	No	
Polyvinyl acetate	Corn oil	0.7	0.55	No	No
Polyvinyl acetate	Castor oil		0.40	Yes	
Glass microballoon	Transformer oil	7	0.18	Yes	No
Glass microballoon	Corn oil	4	0.55	Yes	No
Solid glass	Corn oil		0.55	No	
Solid glass	Castor oil		0.40	No	

[a] $f \gtrsim 45$ Hz for all ac experiments.

Loomans and Jones[20] have investigated certain size-dependent phenomena (principally elongation) that have an effect on the accuracy of this DEP dielectric constant measurement scheme. They found up to 0.5% discrepancy in the volume normalized DEP force between small ($\lesssim 200$ μm) and large (~ 1200 μm) diameter bubbles levitated in a cusped electrostatic field. (See Figure 6.)

Bubble and Particle Control

Appropriate design of electrodes permits the manipulation of bubbles and particles suspended in dielectric liquids. Figure 7a illustrates one periodic electrode structure that can be used for the conveyance of bubbles or particles. Depending on how the "bubble ladder" is connected, bubbles, etc., are collected and held at various locations between the rings. If the rings are connected to multiple-phase ac voltage, a synchronous pumping and delivery of the bubbles or particles should be possible. Another simple means of conveyance for bubbles and particles is shown in Figure 7b. Four electrode rods of alternate polarity are arranged parallel, forming a two-dimensional null along the axis. Bubbles or any particle with $\kappa_2 < \kappa_1$ are attracted to this null and, therefore, can be conveyed along the axis of the bubble conduit using buoyancy or other means.

Work to be Done

Active feedback levitation of particles with $\kappa_2 > \kappa_1$ has not been demonstrated yet. Such a demonstration would greatly extend the capabilities of DEP levitation. No zero-gravity demonstrations of DEP phenomena have been attempted. Experimental verification of zero-gravity levitation of bubbles, fluid droplets, or particles at a null in an electrostatic field is needed to evaluate the dielectrophoretic effect in extra-terrestrial applications.

Conclusion

Dielectrophoresis offers some unique capabilities for the control and manipulation of small uncharged gas bubbles, liquid droplets, and solid particles suspended in liquid media. Investigators have successfully levitated and controlled various particles (up to 1 mm in diameter) in dielectric fluid media using intense electric fields. Using more modest electric fields, levitation experiments have been extended down to small particles (<50 μm) in aqueous media. The limit on dielectrophoresis imposed by Joule heating is most harmful in large electrode geometries or in highly conducting liquids, where electric fields of sufficient strength for DEP cannot be imposed. Charge relaxation produces significant frequency-dependent effects on the dielectrophoretic force, which can be used to characterize the electrical and dielectric properties of unknown solid and liquid media.

In applications not requiring stable levitation, dielectrophoretic forces can be used for separation, agglomeration, collection of bubbles, droplets, and particles with no real restrictions on relative dielectric constants. But a fundamental electrostatic theorem, which permits local three-dimensional field intensity minima, while excluding such maxima, imposes the condition $\kappa_2 < \kappa_1$ for stable static levitation. This condition must be generalized for the dispersive case (see Table 1). However, active feedback may be used to levitate particles when $\kappa_2 > \kappa_1$.

Applications

The study of bubbles, droplets, and particles suspended in liquid media is important in many fields of science and technology. Dielectrophoretic phenomena, since they represent an important case of interactions of electric fields with these small multi-phase systems, merit attention. Some general categories of DEP experiments of at least potential interest in the weak gravitational environment of space are discussed below.

● Zero-gravity hydrodynamics — Levitation of bubbles and droplets in space provic_s the opportunity to study reduced and zero gravity hydrodynamics. DEP levitation might afford unique opportunities to observe the dynamics of uncharged dielectric liquid droplets ana bubbles.

● Multi-phase separations — Some preliminary work on using nonuniform electric fields to separate constituents in gas/liquid and solid/liquid systems in zero gravity has been reported. New space manufacturing processes which require separation could benefit from DEP techniques.

● Biology in space — New opportunities for biological experimentation on living cells exist in reduced gravity. DEP levitation can be used to position cellular materials in vitro and to perform certain dielectric measurement diagnostics.

● Electrochemical experiments — Limited experience exists in the study of solid particles in aqueous media using DEP levitation. Using variable-frequency techniques, the dispersive nature of double layers in electrclytes may be investigated using certain types of small particles.

● High-voltage engineering — DEP levitation affords a means to study solid/liquid/gas interactions in the presence of strong electric fields. Bubbles and particulate impurities are known to be important in the breakdown of dielectric liquids. Experience with liquid insulants in zero gravity is very limited. Thus, dielectrophoretic levitation may be a useful investigative tool in extra-terrestrial, high-voltage engineering applications.

● Dielectric measurements — The dielectric constant measurement scheme described elsewhere in this paper offers a unique cpportunity for the nondestructive evaluation of solid dielectric particles from power to audio frequencies and rf. Simple 1% measurements on liquids have already been demonstrated.

Other applications of dielectrophoretic effects, in general, and DEP levitation, in particular, may be envisioned. For example, processing and ultrasonic cleaning of small spherical particles, such as laser targets, might be possible using DEP levitation. Levitated particles can be rotated to improve the uniformity of coating operations in wet processing.[12]

Dielectrophoretic phenomena offer unique means to control and manipulate small, uncharged gas bubbles, liquid droplets, and solid particles suspended in liquid media. In the reduced gravitational environment of space, these nonuniform electrostatic field effects can be used to provide a controllable body force which is localized and tailored to requirements by proper electrode design. A better understanding and appreciation of dielectrophoresis will enhance the basic investigations of drops and bubbles and may open some new opportunities for space processing of materials.

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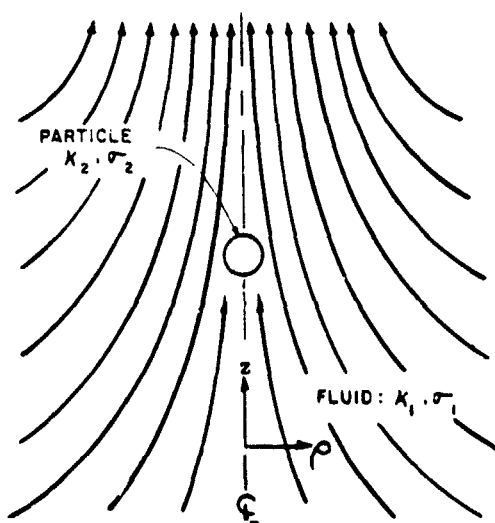20. Jones, T.B., and Loomans, L.W., "Size Effect in Dielectrophoretic Levitation," 1981, in preparation.

Figure 1. Axisymmetric cusped electrostatic field for passive dielectrophoretic levitation. ($K_2 < K_1$)
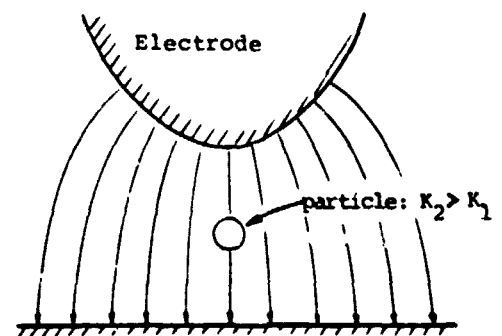

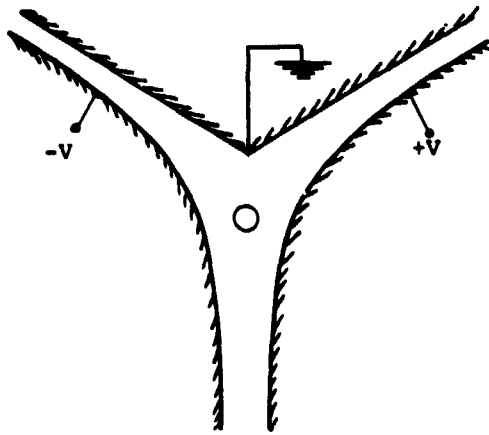Figure 2. Axisymmetric electrostatic field for active feedback dielectrophoretic levitation.

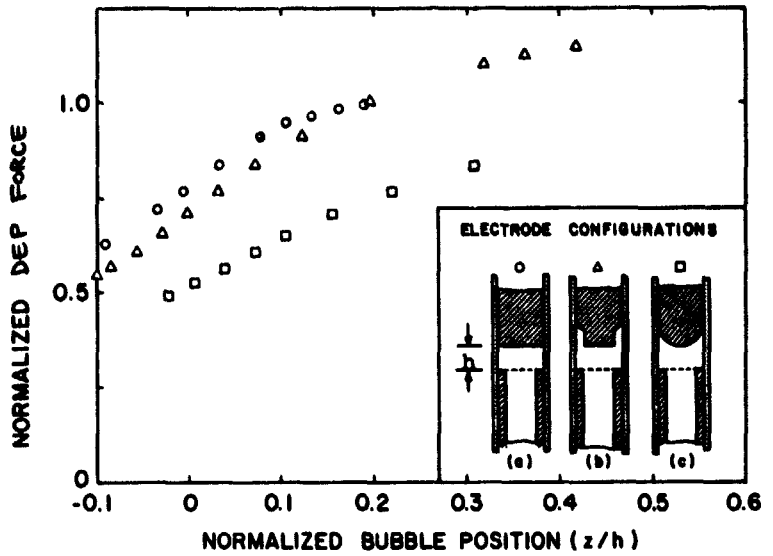Figure 3. Isomotive electrode structure of Pohl and Pethig [15].

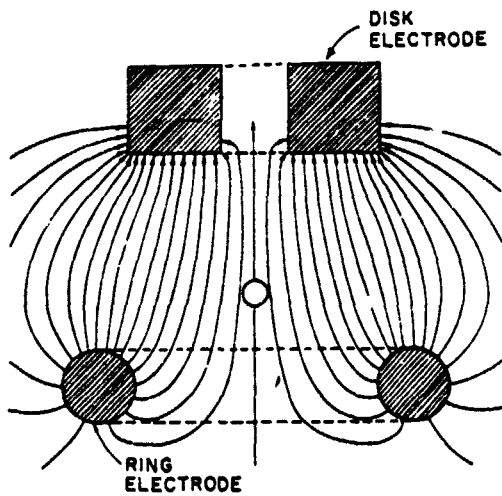Figure 5. Typical normalized DEP force data for various electrode geometries.

Figure 4. Cusped levitator geometry employing ring and disk electrodes.
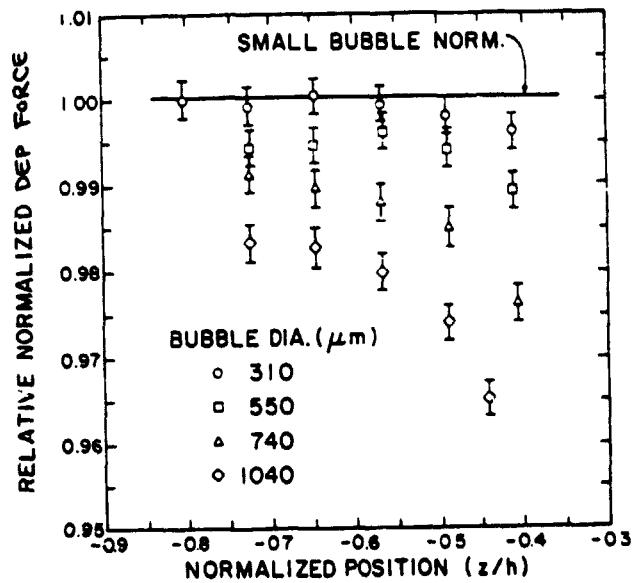

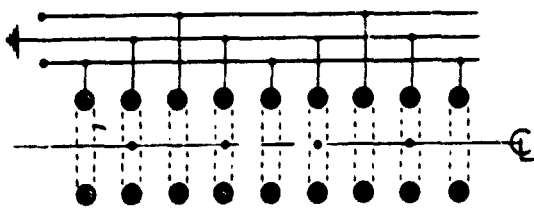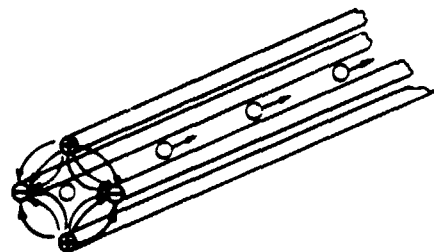Figure 6. Normalized DEP force data versus position for bubbles of various sizes.

Figure 7. a) Periodic electrode structure for synchronous DEP bubble or particle transport.

b) DEP bubble/particle conduit.