

## Interaction effects in thermocapillary bubble migration

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The interaction effects between two bubbles migrating along their line of centers under the influence of an imposed thermal gradient are considered in the quasi-static limit. Results are reported for representative values of the governing parameters.

A gas bubble, when placed in a liquid with a non-uniform temperature field, will move in the direction of the temperature gradient. Such motion is a consequence of the dependence of surface tension on temperature. The temperature gradient at the bubble surface gives rise to a surface tension gradient. The resulting tangential stress at the interface will cause motion of the neighboring liquid in the direction of increasing surface tension, or toward the colder pole of the bubble. By reaction, the bubble will propel itself toward the warmer regions of the liquid. The thermocapillary motion of isolated bubbles has been the subject of much study. Both theoretical descriptions<sup>1-4</sup>, and experimental observations in general agreement with theory<sup>1,3,5,6</sup> have been reported.

Under conditions of creeping motion, the buoyant rise velocity of a gas bubble is proportional to the square of its diameter whereas its thermocapillary velocity (for negligible convective energy transport) is proportional to the diameter. Thus, it may be expected that on earth, very small bubbles will be influenced significantly by temperature gradients. In contrast, in reduced gravity processing, capillarity induced motion of bubbles and droplets will be important over a wide range of bubble sizes. Thus, there is renewed interest in this subject. At Clarkson, for instance, theoretical models of thermocapillary motion are being developed<sup>7-10</sup> and a reduced gravity experiment has been flown in collaboration with Westinghouse Research and Development Center on a NASA rocket in their SPAR (Space Processing Applications Rocket) program<sup>11</sup>. More rocket experiments as well as Space Shuttle experiments are being planned<sup>12,13</sup>.

The problem of the isolated bubble moving in a thermal gradient has received considerable attention as mentioned earlier. However, in many practical applications, interactions with neighboring bubbles and boundaries will be important. We have begun addressing such problems theoretically in the quasi-static limit. Work on bubble migration normal to a plane fluid or solid surface is reported in (8). Elsewhere in this Proceedings<sup>14</sup>, a brief discussion of the problem of thermocapillary motion of bubbles inside drops is presented in the context of applications to the containerless processing of glasses and other materials in free fall. The axially symmetric interaction problems for two unequal bubbles has been solved using bispherical coordinates, and a complete treatment is being reported elsewhere<sup>15</sup>. Details will be given in (16). Here, only a brief summary of the results will be presented.

When two bubbles, possibly of unequal diameters, are placed in a liquid possessing a linear temperature field such that the line of centers of the bubbles is parallel to the temperature gradient, the resulting problem is axially symmetric. Both bubbles will move in the direction of the temperature gradient. The problem, in general, is unsteady. However, when the Reynolds and Marangoni numbers are very small, the quasi-static assumption<sup>17</sup> may be made in order to obtain first order results. This involves ignoring the unsteady accumulation terms as well as the convective transport terms in the equations of conservation of momentum and energy. In physical terms, in part, it is assumed that the vorticity and temperature distributions for a given spacing of the bubbles relax quickly to their steady representations for that configuration compared to the time scale for the bubbles to move an appreciable distance and change the configuration. Further, the convective transport terms are ignored compared to molecular transport. For the small bubbles in glass melts used in our rocket experiments, the quasi-static assumption is a good one.

General solutions of the quasi-static equations in bispherical coordinates are available<sup>18,19</sup>. They are specialized for the boundary conditions of the present problem, a straightforward, but tedious process<sup>15</sup>. The quasi-static thermocapillary velocities of the two bubbles then are calculated by setting the net hydrodynamic force on the bubbles to zero.

It is convenient to discuss the results in the context of an interaction parameter  $\Omega$

which represents the ratio of the velocity of a given bubble in the presence of the second bubble to its velocity when isolated.  $\Omega$  will depend on the ratio of bubble radii,  $\lambda$

( $\lambda = \frac{R_{II}}{R_I}$ ) and the scaled separation distance between the bubbles,  $D$ . ( $D = \frac{d}{R_I}$ ). Here "d" is the actual separation distance,  $R_I$ , the radius of bubble I, and  $R_{II}$ , the radius of bubble II. Due to the neglect of convective transport effects, the order of two bubbles in the temperature gradient has no influence on the results. A representative set of values of  $\Omega_I$  and  $\Omega_{II}$  for the two bubbles as a function of scaled separation distance  $D$  is reported in Table I for a typical value of  $\lambda$ .  $\lambda = 1.5$  represents the case of bubble II being larger than bubble I.

TABLE I

Interaction Parameters  $\Omega_I$  and  $\Omega_{II}$  as a Function of  $D$  for  $\lambda = 1.5$

D	$\Omega_I$	$\Omega_{II}$
2.6	1.1432	0.9662
3.0	1.0728	0.9832
8.0	1.0033	0.9993

From the results in Table I, it may be seen that the larger of the two bubbles (II) moves slightly slower than it would if isolated. In contrast, the smaller bubble (I) moves more rapidly than it would, when isolated. Also, as the bubbles are separated further, the interaction becomes less important, and  $\Omega$  approaches unity. We might also observe that in view of the above behavior in the case of unequal bubbles, it is not surprising that when they are of equal size, both bubbles move at the same velocity that they would possess, if isolated.

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