

PHYSICAL EXPLANATIONS OF THE DESTABILIZING EFFECT OF DAMPING IN ROTATING PARTS

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SUMMARY

The original explanation of shaft whipping due to internal friction by A. L. Kimball in 1923 employed a physical argument based on hysteresis in the flexural stress-strain relation. Ten years later an even simpler physical explanation of the destabilizing nature of rotating damping was briefly described by D. M. Smith. Subsequently, there have been many discussions of the effects of rotating damping on rotor stability. In most cases, the emphasis has been on the mathematical aspects of the stability analysis rather than on physical explanations. The present note provides an elaboration of Smith's physical argument. It gives a clear insight into the destabilizing mechanism and permits an elementary physical determination of the stability limit.

INTRODUCTION

The fundamental critical speed of an elastic rotor is a kind of natural barrier for rotational speed much like the velocity of sound is for an airplane. In both cases, there were historical periods when some engineers were of the opinion that safe operation would not be possible beyond the barrier. Today both barriers are routinely broken but not without careful engineering to ensure safety in the supercritical or supersonic regimes.

The first analytical treatments of supercritical rotor whirling were given by Föppl in 1895, for an undamped model, (ref. 1) and by Jeffcoat in 1919, for a model with damping, (ref. 2). In this case, art preceded science, for in 1895 some commercial centrifuges and steam turbines were already running supercritically (ref. 1). These early analyses indicated that, if anything, supercritical operation should be smoother than subcritical. Since then, many instability mechanisms (ref. 3) have been encountered, each of which poses a potential threat to the hoped-for smoothness of supercritical operation. The first of these to be diagnosed was the mechanism of internal friction or damping of relative motion with respect to the rotating system. The theory was initially described in 1923 (ref. 4) by A. L. Kimball, Jr. A fuller account is given in reference 5. This is a simple physical argument to show that damping forces in the fibers of the bending shaft have a resultant action which builds up a whirling motion when the rotation is supercritical. The argument is independent of the specific type of damping: viscous, hysteretic, or otherwise. Essentially, the same argument appears in references 3 and 6. In 1933, Baker included the damping action due to linear frequency-independent hysteresis in the equations of

motion and determined the growth rate of the unstable whirl (ref. 7). An extended presentation of the same analysis appears in reference 8.

In 1933, there also appeared the remarkable paper of D. M. Smith (ref. 9). Many modern investigations in rotor dynamics were anticipated in this paper. In particular, instabilities due to unsymmetrical rotating stiffness and unsymmetrical rotating inertia are described for the first time. Instability due to rotating damping is analyzed for a symmetrical system and the threshold frequency for viscous damping obtained for the first time. An analytical explanation for the ameliorating action of anisotropic support stiffness is also given for the first time. The basic idea of the physical explanation of destabilization due to rotating damping which is described in the present note is squeezed into a single sentence of Smith's paper at the conclusion of his analysis for the threshold frequency marking the transition from stability to instability. He also used the idea to give a simple physical explanation for the reason why anisotropic support stiffness raises the onset frequency for instability due to rotating damping.

Smith also made a number of shrewd predictions about rotor systems with several critical speeds, many of which have been verified. In the case of rotating damping, the situation has subsequently been shown (refs. 10,11) to be somewhat more complicated than he envisaged. Later investigators have also extended the theory of rotating damping to include damping mechanisms with arbitrary frequency dependence and nonlinear amplitude dependence (ref. 12).

At present, almost every engineer dealing with rotor dynamics is aware that rotating damping is destabilizing although to many it still is something of a mystery that a mechanism of energy dissipation can cause instability. Kimball's explanation, while physical and easy to follow step by step, is sufficiently intricate because of the awkward geometry of bending during simultaneous rotation and whirling that the argument as a whole is not as compelling as it might be. Smith's explanation is much simpler and persuasive but is not widely known in America. In 1970, the present author independently developed an explanation similar to Smith's which he presented several times in invited lectures based on reference 13. For nearly ten years, he believed his explanation was original until on reviewing A. B. Pippard's recent book (ref. 14) for the Journal of Applied Mechanics (ref. 15) he discovered that the explanation was, in fact, well known in England and was led back to Smith's work.

PLANAR ROTOR MODEL

Consider the planar model of a rotor mounted on a flexible shaft shown in Fig. 1. The rigid ring is forced to turn at the angular rate Ω by an external source. A mass particle m is suspended elastically from the ring by four massless linear springs. The mass particle represents the central rotor of the classical Föppl-Jeffcoat model and the springs represent the flexible shaft. Gravity in the plane of the diagram is neglected. The elastic force system for small displacements is circularly isotropic. The equilibrium position is with the mass at the origin and when the mass has a radial displacement r , in any direction within the plane, the elastic restoring force is kr directed back

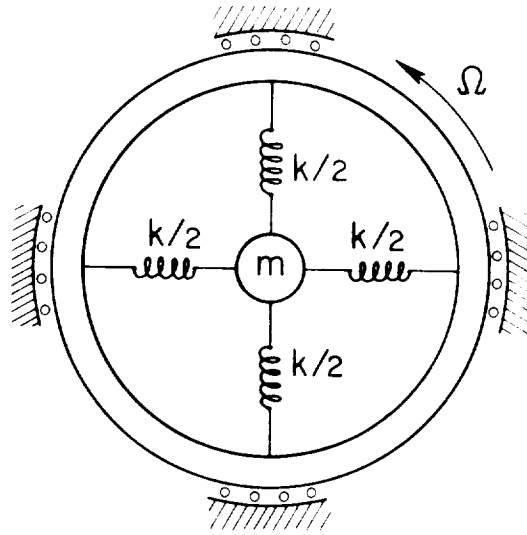


Fig. 1 Planar model of ideal balanced rotor with no damping.

toward the origin. Furthermore, this characteristic of the elastic system is independent of the rotational speed Ω . The equations of motion for the mass particle in Fig. 1 in terms of the stationary coordinates x, y of Fig. 2 are

$$m \begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} + k \begin{Bmatrix} x \\ y \end{Bmatrix} = 0 \quad (1)$$

independently of Ω . All natural motions are linear combinations of two independent modes which have the same natural frequency $\omega_n = \sqrt{k/m}$. The two independent modes may be taken as rectilinear oscillations along two diameters or as a pair of circular whirling modes, one counter-clockwise and the other clockwise. The resulting orbit, in the general case, is an elliptical path which is periodically circumnavigated with the period $2\pi/\omega_n$. Due to the isotropy of the system, it is always possible, without loss of generality, to choose the orientation of the x, y axes and the starting phase so that the general free motion has the representation

$$\begin{aligned} x &= a \cos \omega_n t \\ y &= b \sin \omega_n t \end{aligned} \quad (2)$$

where $|a|$ and $|b|$ are the major and minor semiaxes of the ellipse. It is sometimes convenient to consider the x, y plane as the locus of the complex variable

$$z = x + iy \quad (3)$$

In such a setting a counter-clockwise whirl of unit amplitude and angular rate ω is represented by

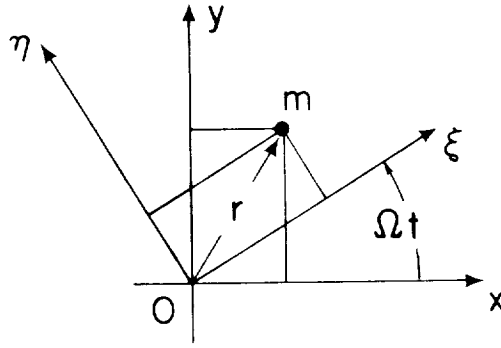


Fig. 2 Stationary coordinates x, y and rotating coordinates ξ, η for the mass particle in Fig. 1.

$$e^{i\omega t} = \cos \omega t + i \sin \omega t \quad (4)$$

The elliptical motion (2) can be represented as a superposition of counter-clockwise and clockwise whirls by inserting (2) into (3) and expressing the trigonometric functions in terms of exponentials of imaginary argument. The result is

$$\begin{aligned} z &= a \cos \omega_n t + ib \sin \omega_n t \\ &= A e^{i\omega_n t} + B e^{-i\omega_n t} \end{aligned} \quad (5)$$

where

$$\begin{aligned} A &= 1/2(a + b) & a &= A + B \\ B &= 1/2(a - b) & b &= A - B \end{aligned} \quad (6)$$

While not necessary for the physical explanation which follows, the equations of free motion with respect to the rotating axes ξ, η of Fig. 2 are given here for completeness

$$m \begin{Bmatrix} \ddot{\xi} \\ \ddot{\eta} \end{Bmatrix} - m\Omega^2 \begin{Bmatrix} \xi \\ \eta \end{Bmatrix} + 2m\Omega \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{Bmatrix} \dot{\xi} \\ \dot{\eta} \end{Bmatrix} + k \begin{Bmatrix} \xi \\ \eta \end{Bmatrix} = 0 \quad (7)$$

In the elliptical orbit (2), the instantaneous kinetic energy is

$$T = m\omega_n^2 \left(\frac{a^2 + b^2}{4} - \frac{a^2 - b^2}{4} \cos 2\omega_n t \right) \quad (8)$$

and the instantaneous potential energy is

$$V = k \left(\frac{a^2 + b^2}{4} + \frac{a^2 - b^2}{4} \cos 2\omega_n t \right) \quad (9)$$

These energies oscillate about the same mean value with opposite phase so that the total energy E in the rotor orbit remains constant

$$E = T + V = 1/2k(a^2 + b^2) = k(A^2 + B^2) \quad (10)$$

QUALITATIVE EFFECTS OF DAMPING

Consider now the introduction of isotropic linear damping to the rotor model of Fig. 1. In the following section, arbitrary frequency dependent linear damping mechanisms will be treated. For the present, it is sufficient to consider classical viscous damping. The damping of the motion of m may be with respect to the stationary axes x, y or the rotating axes ξ, η in Fig. 2. Damping with respect to the stationary axes is represented in Fig. 3 by the four dashpots whose outer extremities are fixed in the stationary reference frame. For small motions of m , the system of dashpots develops a circularly isotropic damping force. If the absolute velocity of m is v in any direction within the plane of motion, the resultant dashpot force is $c_s v$ in the opposite direction. In particular, if m is forced to travel in a circular orbit of radius A with counter-clockwise angular rate ω_n , the damping force $c_s A \omega_n$ acts tangent to the circle in a clockwise sense.

Damping of relative motion with respect to the rotating system is represented in Fig. 4 by the four dashpots whose outer extremities are fixed in the rotating ring. For small motions of m , this set of dashpots develops a circularly isotropic damping force which acts in opposition to the velocity of m relative to the rotating system. In particular, if m is forced to travel in a circular orbit of radius A with counter-clockwise angular rate ω_n as viewed from the

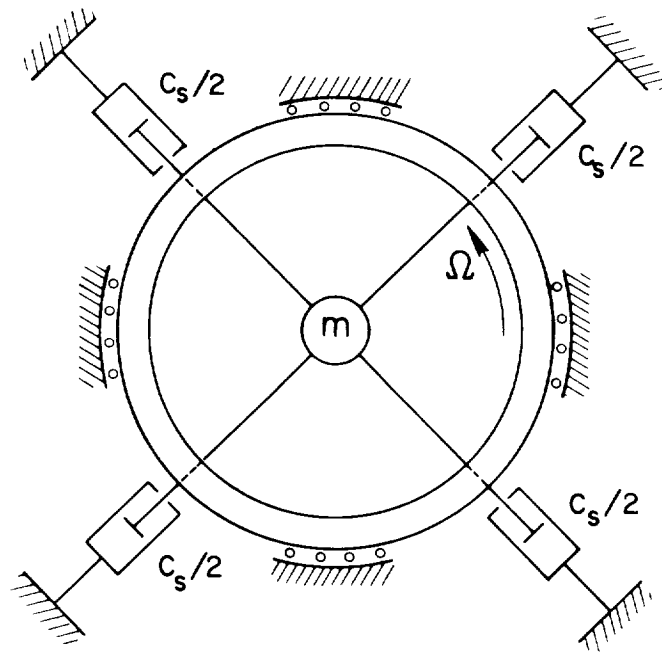


Fig. 3 Damping of motion with respect to stationary axes is represented by four symmetrically oriented dashpots.

stationary axes, it will appear to have a counter-clockwise angular rate $\omega_n - \Omega$ in the rotating frame. The damping force generated by the rotating dashpots is $c_r A (\omega_n - \Omega)$ acting tangentially to the circle in a clockwise sense.

The effects of these damping mechanisms on the free motion of the rotor can be argued qualitatively as follows. Suppose that the damping forces are very small in comparison with the spring forces. This implies that a free orbit will undergo only a small change during one period $2\pi/\omega_n$ as a result of the damping. If initially the orbit is a counter-clockwise circle of radius A , the preceding discussion shows that when the rotor rotation rate Ω is subcritical; i.e., less than the natural frequency of the free motion ω_n , both the stationary and rotating dashpots act to retard the motion of m . The rotor does work on the dashpots and the total energy E of the rotor orbit is diminished. If initially the orbit is a superposition of a counter-clockwise whirl of radius A and a clockwise whirl of radius B as in (5) a similar argument shows that when $\Omega < \omega_n$ the rotor does work on the dashpots in both whirls and that both A and B are diminished after a period $2\pi/\omega_n$. Any rotor orbit generated by an accidental disturbance will thus be damped out and the system is stable.

When the system rotates supercritically, the rotating dashpots do work on the rotor and add energy to the rotor orbit. This destabilizing action can be seen by returning to Fig. 4 and considering the case of a counter-clockwise whirl of radius A at absolute angular rate ω_n when $\Omega > \omega_n$. The relative motion now is

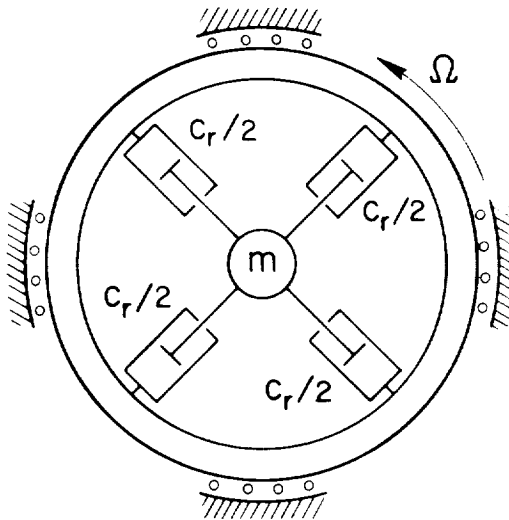


Fig. 4 Damping of relative motion with respect to rotating axes is represented by four symmetrically oriented dashpots.

a backward (i.e., clockwise) whirl with rate $\Omega - \omega_n$. The resultant dashpot force acts tangentially to the circular path in a forward (i.e., counter-clockwise) sense with magnitude $c_r A (\Omega - \omega_n)$. The supercritical rotation of the system thus acts to drag the rotor forward around its orbit through the rotating dashpots.

This action may be seen in an even clearer fashion if, instead of the four dashpots in Fig. 4, it is imagined that the interior of the ring is filled with a massless viscous fluid which rotates with the ring and acts to retard the relative motion of the mass m with respect to the ring. When the mass has a circular orbit in the same absolute sense as the ring rotation, but at a slower rate, the viscous drag pulls the mass forward and adds energy to its orbit.

The model described by Pippard (ref. 14) is essentially equivalent. The rotor is modelled by a conical pendulum with a heavy bob suspended in the gravity field by a string of length L . The natural modes are forward and backward conical whirls with the same natural frequency $\omega_n = \sqrt{g/L}$ (for small cone angles). Damping is modelled by allowing the pendulum bob to dip into a glass of water. If the glass is on a variable speed turntable, damping with respect to a rotating frame can be demonstrated. "If the fluid is caused to rotate more slowly than the pendulum bob in its circular orbit, the bob experiences a retarding force and sinks toward the vertical. But if the fluid rotates faster than the bob, it urges the bob onwards and causes its orbit radius to increase"--- (ref. 14). See Fig. 6 below.

Returning to the planar rotor model with both stationary damping, Fig. 3, and rotating damping, Fig. 4, a forward whirl at absolute angular rate ω_n will be retarded by the stationary damping force and will be urged on by the rotating damping force when the rotation is supercritical, $\Omega > \omega_n$. Whether the net effect is to remove or to add energy to the whirl orbit, depends on which force is larger. Neutral stability occurs when the forces are equal. There is then no change in the orbit energy and the free motion persists indefinitely. The quantitative determination of the stability borderline is discussed in the following section, taking account of the frequency dependence of the damping mechanisms.

In the case of a backward whirl, both the rotating dashpots and the stationary dashpots act to retard the whirl. The orbit energy of a backward whirl is always decreased by either rotating or stationary damping. For completeness, the equations of motion for the planar rotor model of Fig. 2 with the damping elements of Figs. 3 and 4 are given here. In the stationary coordinates, the equations for the damped system are

$$m \begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} + (c_s + c_r) \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} + k \begin{Bmatrix} x \\ y \end{Bmatrix} + c_r \Omega \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = 0 \quad (11)$$

in place of (1) for the undamped system. In the rotating coordinates used in (7), the equations for the same damped system are

$$m \begin{Bmatrix} \ddot{\xi} \\ \ddot{\eta} \end{Bmatrix} + (c_s + c_r) \begin{Bmatrix} \dot{\xi} \\ \dot{\eta} \end{Bmatrix} + (k - m\Omega^2) \begin{Bmatrix} \xi \\ \eta \end{Bmatrix} + 2m\Omega \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{Bmatrix} \dot{\xi} \\ \dot{\eta} \end{Bmatrix} + c_s \Omega \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{Bmatrix} \xi \\ \eta \end{Bmatrix} = 0 \quad (12)$$

Note that in the stationary coordinates of (11), the rotating damping is represented by both a normal damping term and by a quasi-gyroscopic term. In the rotating coordinates of (12), the stationary damping is represented in like fashion.

QUANTITATIVE EFFECTS OF DAMPING

In the preceding section, it was shown that at the stability borderline for a forward whirl the resultant forward drag force of the supercritically rotating damping balances the backward drag of the stationary damping. To give a quantitative expression of this we introduce the concept of loss factor. When a material or mechanical element undergoing harmonic deformation at frequency ω has both elastic and damping behavior, the ratio of the amplitude of the damping force to the amplitude of the elastic force is called the loss tangent, or loss factor, η . In general, η depends on both the amplitude and frequency of the deformation. For linear damping mechanisms, η is independent of amplitude. There are many real damping mechanisms that can be satisfactorily modeled as linear frequency-dependent processes; e.g., polymers, and metals when the dominant damping is due to transverse thermal currents (refs. 13,16). For the planar rotor model of Fig. 1, we can define a stationary loss factor $\eta_s(\omega)$ by taking the ratio of the tangential drag force, due to stationary damping, to the radial spring force when the mass particle executes a circular orbit with radius A at frequency ω . When $\omega = \omega_n$, the backward drag force D_s due to the stationary damping is

$$D_s = \eta_s(\omega_n) kA \quad (13)$$

Similarly, a rotating loss factor $\eta_r(\omega)$ is defined as the ratio of the tangential drag force, due to rotating damping, to the radial spring force when the mass particle executes a circular orbit with radius A at a circular frequency ω , with respect to the rotating system. In a forward whirl at absolute rate ω_n , the circular frequency with respect to the system rotating at rate Ω is $\omega_n - \Omega$. Thus, when the system rotates supercritically, the drag force D_r due to the rotating damping is forward and has the magnitude

$$D_r = \eta_r(\Omega - \omega_n) kA \quad (14)$$

for an orbit of radius A . If $D_r = D_s$, we have a steady orbit of fixed radius A . If $D_r < D_s$, there is net retardation and the orbit energy decreases. If $D_r > D_s$, there is net acceleration and the orbit energy increases. Alternatively, the stability is decided by the relative magnitudes of the rotating and stationary loss factors as indicated in Fig. 5. The stability borderline condition occurs when the loss factors are equal

$$\eta_r(\Omega - \omega_n) = \eta_s(\omega_n) \quad (15)$$

The running speeds Ω which satisfy (15) depend crucially on the frequency dependence of the rotating damping mechanism and on the magnitude of the stationary damping.

The planar rotor model also provides a clear insight into the energy relationships during whirling of a rotor with stationary and rotating damping mechanisms. Consider a forward whirl with radius A and angular rate ω_n . If the system is rotating supercritically at rate Ω , the external agent responsible for maintaining the rotation rate Ω delivers energy to the system. This power input is mostly dissipated in the stationary and rotating damping mechanisms. If there is excess

input power, energy will be added to the whirl orbit, if the input power is inadequate, energy will be removed from the whirl orbit. Quantitatively, we have the following powers, when $\Omega > \omega_n$

$$\begin{aligned}
 P_{\text{input}} &= D_r A \Omega &= kA^2 \Omega \eta_r \\
 P_{\text{diss,rot}} &= D_r A (\Omega - \omega_n) &= kA^2 (\Omega - \omega_n) \eta_r \\
 P_{\text{diss,sta}} &= D_s A \omega_n &= kA^2 \omega_n \eta_s \\
 P_{\text{orbit}} &= (D_r - D_s) A \omega_n &= kA^2 \omega_n (\eta_r - \eta_s)
 \end{aligned}
 \tag{16}$$

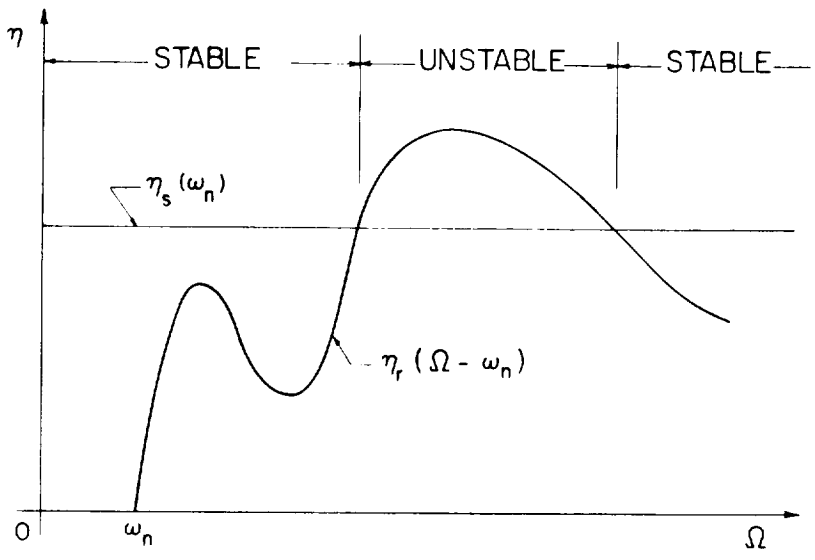


Fig. 5 Forward whirl is unstable for supercritical rotation speeds Ω at which the loss factor of rotating damping is greater than the loss factor of stationary damping.

Note that these satisfy the requirement of energy conservation

$$P_{\text{input}} = P_{\text{diss,rot}} + P_{\text{diss,sta}} + P_{\text{orbit}} \tag{17}$$

At a stability borderline, we have no power flow into, or out of, the orbit which leads back to the requirement (15). Note also that if the frequency dependence of $\eta_r(\omega)$ is such that the instability onset rotation Ω is very close to ω_n then the relative amount of energy dissipated in the rotating damping mechanism will be very small in comparison to the energy dissipated in the stationary damping mechanism even though it is the rotating damping force which is responsible for the instability.

The power flow into the orbit in (16) can be used to estimate the growth rate of the orbit in the unstable regime. Assuming exponential growth of orbit radius

$$A = A_0 e^{\alpha t} \quad (18)$$

with small growth rate α , the energy added to an orbit with initial radius A_0 and angular rate ω_n would be approximately

$$P_{\text{orbit}} \cdot \frac{2\pi}{\omega_n} = 2\pi k A_0^2 (\eta_r - \eta_s) \quad (19)$$

During this period, the radius increases from A_0 to $A_0 \exp \{2\pi\alpha/\omega_n\}$ and the increase in orbit energy as given by (10) is

$$\Delta E = k A_0^2 (e^{\frac{4\pi\alpha}{\omega_n}} - 1) \approx 4\pi k A_0^2 \frac{\alpha}{\omega_n} \quad (20)$$

Equating (19) and (20), we find

$$\alpha = 1/2\omega_n (\eta_r - \eta_s) \quad (21)$$

as the estimated growth rate. In (21), $\eta_r(\omega)$ is to be evaluated at $\omega = \Omega - \omega_s$ and $\eta_s(\omega)$ is to be evaluated at $\omega = \omega_n$.

The preceding results can, of course, also be derived by solving (11) or (12). If in (11), for example, we set $z = x + iy$ and $\omega_n^2 = k/m$ the complex variable $z(t)$ must satisfy the equation

$$\ddot{z} + \frac{c_s + c_r}{m} \dot{z} + (\omega_n^2 - i\frac{c_r}{m}\Omega)z = 0 \quad (22)$$

Exponential terms of the form

$$z = A \exp \{(\alpha + i\beta)t\} \quad (23)$$

satisfy (22) if

$$\begin{aligned} \beta^2 &= \omega_n^2 - \alpha^2 - \alpha \frac{c_s + c_r}{m} \\ \alpha &= \frac{(\Omega - \beta)c_r - \beta c_s}{2\beta} \end{aligned} \quad (24)$$

These relations apply with constant c_s and c_r if the damping is viscous. For other linear frequency-dependent damping mechanisms with loss factors $\eta_s(\omega)$ and $\eta_r(\omega)$ we can use the relations (see ref. 13)

$$c_s = \frac{k\eta_s(\beta)}{|\beta|} \quad c_r = \frac{k\eta_r(\Omega - \beta)}{|\Omega - \beta|} \quad (25)$$

on the assumption that $\alpha \ll \beta$. This assumption is verified a posteriori for light damping. Then, correct to first order in the loss factors, we have from (24)

$$\beta_1 = \omega_n$$

$$\beta_2 = -\omega_n$$

$$\alpha_1 = 1/2\omega_n[\eta_r(\Omega - \omega_n) \operatorname{sgn}(\Omega - \omega_n) - \eta_s(\omega_n)] \quad \alpha_2 = -1/2\omega_n[\eta_r(\Omega + \omega_n) + \eta_s(\omega_n)] \quad (26)$$

To this order of accuracy, the general solution to (11) is

$$z(t) = A_1 \exp\{(\alpha_1 + i\omega_n t)\} + A_2 \exp\{(\alpha_2 - i\omega_n t)\} \quad (27)$$

This is a superposition of Archimedean spirals. The forward spiral has a growth rate α_1 which is positive if $\Omega > \omega_n$ and $\eta_r(\Omega - \omega_n) > \eta_s(\omega_n)$. The magnitude of α_1 in this case agrees with the estimate (21). If $\Omega < \omega_n$ then α_1 is negative and the forward spiral decays. The growth rate α_2 of the backward spiral is always negative.

CONCLUSIONS

Visualization of damping as a drag force on the orbit provides a clear explanation of the destabilizing effect of rotating damping. When the rotation is faster than the whirl, rotating damping drags the orbiting particle forward. When stationary damping is also present, the stability borderline is readily determined by balancing the backward and forward drags. A key notion here is that a forward whirl at rate ω_n with respect to stationary axes appears to be a backward whirl at rate $\Omega - \omega_n$ with respect to a system rotating supercritically at rate Ω . The growth rate of unstable whirls (or the decay rate of stable whirls) is readily estimated by a simple energy balance.

As a final note, we call attention to D. M. Smith's extension of the previous argument to rotor systems with anisotropic non-rotating support elasticity. If the bearings in Fig. 1 are not rigidly supported, but are elastically supported so that the compliance in the x - direction is greater (say) than that in the y - direction, the free motion of the undamped system becomes a superposition of a horizontal oscillation at frequency ω_1 and a vertical oscillation at frequency $\omega_2 > \omega_1$. The more anisotropic the supports, the greater the separation between ω_1 and ω_2 . The resultant free orbits no longer whirl with a fixed sense and ---"there is no tendency to set up a whirl of the type which can be dragged forward by rotary damping until the rotary damping forces have been so far increased by rising speed that they are commensurate with the difference between elastic restoring forces in the two principal directions"---(ref. 9). Thus, anisotropic bearing supports should raise the onset speed Ω for instability due to rotating damping. Smith (ref. 9) and others (ref. 17) have verified this by applying Routh's criterion to the equation of motion.

APPENDIX

The description of the destabilizing action of rotating drag forces given above can be extended to give a heuristic explanation of the destabilizing tendency of fluid filled journal bearings when the rotational speed exceeds twice the natural frequency of the rotor. Here, instead of modelling the rotor as a particle travelling in an orbit within a fluid bath whose drag either retards or advances the particle, we model the rotor as a cylindrical journal of radius r rotating within a cylindrical bearing with a small radial clearance h ($h \ll r$). When the journal is centered and rotating at angular rate Ω , the incompressible viscous fluid in the gap is pumped circumferentially (longitudinal flow is neglected). The fluid velocity varies linearly across the channel of thickness h , from $v = r\Omega$ at the journal to $v = 0$ at the bearing. This flow can be decomposed into a mean flow with uniform velocity $v = r\Omega/2$ and no vorticity and a residual flow with no mean flow and large vorticity.

Next, consider a small circular whirl of the rotor with amplitude A ($A \ll h$) and angular rate ω_n where as before ω_n is the rotor natural frequency. As the center of the journal traverses a circle of radius A , a wave of thickness variation moves around the channel between the journal and the bearing. At the thinnest point, the thickness is $h - A$ and at the thickest point the thickness is $h + A$. If we follow the position of, say, the thinnest point, we note that it appears to whirl at the rate ω_n ; i.e., the circumferential velocity of the wave of thickness variation is $r\omega_n$.

Finally, we conclude that such a natural whirling motion will be retarded or aided by the mean flow of the pumped fluid depending on whether the mean fluid velocity $r\Omega/2$ is less or greater than the circumferential velocity of the wave of thickness variation $r\omega_n$. Neglecting other mechanisms, the stability borderline is determined by a balance between the destabilizing tendency of the mean flow and the stabilizing tendencies of the residual high-vorticity flow. A necessary condition for the onset of instability is $\Omega > 2\omega_n$.

The action of the mean fluid flow in a fluid-filled bearing, in dragging around a wave of thickness variation when $\Omega > 2\omega_n$ is thus proposed as a heuristic physical explanation for the classical oil-whip phenomenon.

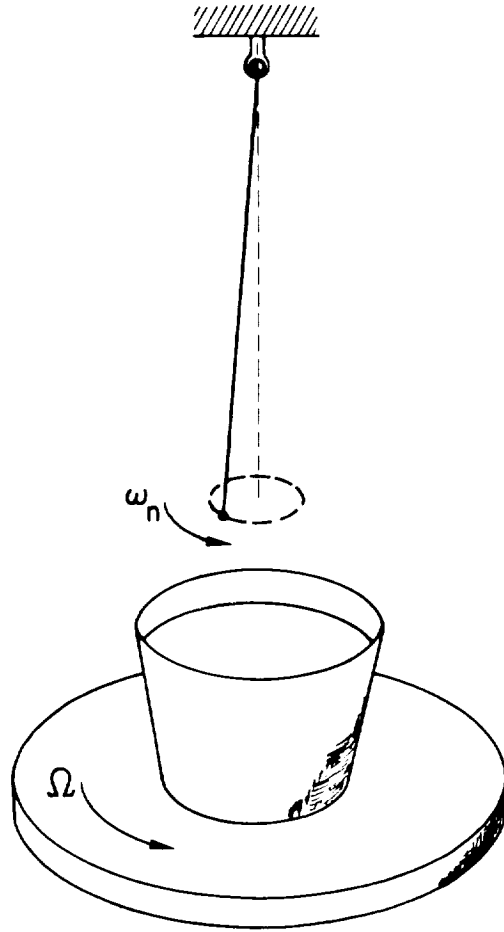


Fig. 6 When $\Omega < \omega_n$, fluid drag retards pendulum bob and radius of orbit decreases; when $\Omega > \omega_n$, fluid drag pulls bob around orbit and radius of orbit increases.

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