OPTIMIZED LASER TURRETS FOR MINIMUM PHASE DISTORTION
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## ABSTRACT

Phase distortion due to compressible, inviscid flow over small perturbation laser turrets in subsonic or supersonic flow was calculated. The turret shape was determined by a two-dimensional Fourier series; in a similar manner, the flow properties are given by a Fourier series.

Phase distortion was calculated for propagation at several combinations of elevation and azimuth angles. A sum was formed from the set of values, and this sum became the objective function for an optimization computer program. The shape of the turret was varied to provide minimum phase distortion.

## INTRODUCTION

For many applications of a high energy laser on board an aircraft, the beam must be propagated with minimum phase distortion. The well known Strehle

ACKNOWLEDGMENT. The work reported in this paper was supported by the Air Force Weapons Laboratory, Kirtland Air Force Base, NM, and was monitored by LtCol Keith Gilbert, Dr. Barry Hogge, and Captain Richard Cook.
relation [1] gives the decrease in far field intensity as a consequence of the rms phase distortion. As a result of the dependence of index of refraction on mass density, compressible flow over laser turrets causes phase distortion. The variable density and variable index of refraction surrounding an aircraft laser turret may be the result of viscous flow phenomena or inviscid flow. This paper focuses on the inviscid flow problem.

In regard to the solution of the phase distortion problem due to inviscid compressible flow, several options exist. Location of the turret on the aircraft is an important consideration. Adaptive optics may be used. Correct shape of the turret can reduce significantly phase distortion. The approach taken in this paper is to minimize phase distortion through turret shape.

An alternate approach would be to consider combined adaptive optics and turret shape. Higher order distortions, e.g., astigmatism and coma, are more difficult to remove by adaptive optics than lower order distortions. Using this approach to design, the turret shape should be modified so as to minimize higher order distortions.

In passing, a comment should be made about adaptive optics. Adaptive optics for compensation of atmospheric turbulence and thermal blooming requires mirror displacements of a fraction of wavelength at frequencies of 25 kHz or so. In contrast, the adaptive optics for compensation of laser turret phase distortion requires mirror displacements of a few wavelengths at frequencies of a few Hertz.

An analytical model for describing laser turret geometry and the associated compressible flow field has been described in the papers by Fuhs [2] and Fuhs and Fuhs [3,4]. A companion paper in this conference proceedings [5] discusses the analytical model. The turret shape is described by a two-dimensional Fourier series.

Using the flow over a wavy wall on a circular cylinder as the basic solution, a Fourier series can be found for the potential function. In contrast to a direct numerical integration of the equations of motion of gas dynamics, an explicit analytical solution is obtained. As a result, computer time is significantly 1ess.

At a plane normal to the beam sufficiently far from the aircraft, the phase distortion is calculated. One method to represent the phase distortion is to use a series with Zernike polynomials [6]. The advantage of using a Zernike series is that the coefficients in the series are related directly to the magnitude of the various types of distortion, i.e., tilt, focus, defocus, astigmatism, etc.

A computer program [7,8] has been developed which can find the values of design variables yielding a minimum value of an objective function subject to constraints. The program has been applied to a variety of problems [9,10]. For the case at hand, the design variables are the coefficients describing the turret geometry. Also gas density inside the turret and the location of the laser turret primary mirror were treated as design variables; $X_{M}$ and $\varepsilon_{M}$ in Figure 1 define location of the mirror. The objective function was the weighted sum of phase distortion for several sets of elevation and azimuth angles. Constraints included the maximum slope of the laser turret as well as maximum turret height.

## PHASE DISTORTION

Optical path length, $L_{i}$, is defined as

$$
L_{i}=\int_{a}^{b} n(s) d s
$$

where $n$ is the index of refraction and $s$ is distance along a particular ray. Points $a$ and $b$ are on the ray. The subscript $i$ identifies the ray. The difference in


Figure 1. Geometry for Calculation of Phase Distortion.
optical path length for two adjacent rays $i$ and $j$ can be calculated; see Figure 1. The phase distortion, $P$, is the difference in optical path length divided by the wavelength.

$$
\begin{equation*}
P=\frac{L_{i}-L_{j}}{\lambda} \tag{2}
\end{equation*}
$$

The index of refraction is related to mass density, $\rho$, by

$$
\begin{equation*}
\mathrm{n}=1+\kappa^{\prime} \frac{\rho_{\infty}}{\rho_{S L}} \frac{\rho}{\rho_{\infty}} \tag{3}
\end{equation*}
$$

where $K^{\prime}$ is a weak function of wavelength in the infrared, $\rho_{\infty}$ is the freestream density, and $\rho_{S L}$ is the sea level density. The form of equation (3) was used to highlight the dependence of $n$ on altitude. The ratio $\rho_{\infty} / \rho_{S L}$ is a function of flight altitude.

The inviscid flow over the laser
turret is assumed to be isentropic; thus the usual isentropic relation between pressure, $P$, and density, $\rho$, can be used. Further, the pressure coefficient, $C_{p}$, can be introduced with the result

$$
\begin{equation*}
\frac{\rho}{\rho_{\infty}}=\left[1+\frac{\gamma M_{\infty}^{2} C_{p}}{2}\right]^{1 / \gamma} \tag{4}
\end{equation*}
$$

The pressure coefficient for small perturbation axisymmetric flow is given by Liepmann and Roshko [11] as

$$
\begin{equation*}
c_{p}=-2 u-v^{2} \tag{5}
\end{equation*}
$$

where $u$ is the perturbation velocity in the freestream direction and $v$ is the radial velocity which is normal to the fuselage axis. Equations (1) to (5) can be combined to give an integral for the phase distortion in terms of pressure coefficient; see equation (7) of Reference [5]. When the potential function for the flow is known, both $u$ and $v$ can be calculated.

METHODS TO COMPENSATE FOR PHASE DISTORTION
The options available for compensating for phase distortion due to inviscid compressible flow over the laser turret were mentioned in the Introduction. Adaptive optics is one technique. Wolters and Laffay [12] demonstrate the effectiveness of adaptive optics.

Another method to compensate for phase distortion is to use a laser turret of proper shape. This is the approach of this paper. Turret geometry constitutes a passive technique. Turret geometry as a means to lessen phase distortion is discussed in the following sections.

A method to represent phase distortion is to use Zernike polynomials [1,6]. The polynomials, which are given in the paper by Hogge and Butts [6], are an orthonormal set of functions. The phase distortion is

$$
\begin{equation*}
P=\sum_{j=1}^{n} A_{j} F_{j} \tag{6}
\end{equation*}
$$

where $A_{j}$ is a coefficient and $F_{j}$ is the $j$ th Zernike polynomial. The summation extends from 1 to $n$, where $F_{n}$ is the highest order polynomial considered. Typically $n=10$ is adequate. The coefficient $A_{q}$ is obtained by multiplying both sides of equation (6) by $F_{q}$ and integrating over the aperture or beam cross section. All
terms in the summation vanish except for the term $j=q$. Equation (19) of Reference [5] gives the results and a formula for $A_{j}$.

An alternate method to compensate for phase distortion is to combine adaptive optics and turret geometry. Higher order phase distortions are more difficult to compensate by adaptive optics. Form an objective function which is

$$
\begin{equation*}
B=\sum W_{i} A_{i} \tag{7}
\end{equation*}
$$

where $W_{i}$ is a weighting factor for the ith coefficient in Zernike series for phase distortion. The larger $i$, the larger is the value of $W_{i}$. As the turret shape is varied, the value for $B$ changes. Using COPES/CONMIN computer program [7], the value of $B$ can be minimized through variations of turret geometry. The consequence is that the effectiveness of adaptive optics is enhanced since higher order distortions are minimized.

The technique of adaptive optics employs segmented or deformable mirrors. Phase is controlled by mirror displacement. Further, the frequency of the mirror motion is determined by the frequency of the adverse phenomenon being overcome through use of adaptive optics. Hence, different types of adaptive optics can be thought of as occupying different locations in the mirror amplitude/frequency plane. Compensation for atmospheric turbulence occurs in the low amplitude, high frequency region of the adaptive optics map. Compensation for the adverse influence of flow over the turret occurs in the high amplitude, low frequency region of the adaptive optics map. The frequency response is dictated by turret slew rates or aircraft maneuver rates.

ANALYTICAL MODEL
The linearized potential equation for axisymmetric flow is

$$
\begin{equation*}
\pm \beta \phi_{\mathrm{xx}}+\phi_{\mathrm{rr}}+\frac{1}{\mathrm{r}} \phi_{r}+\frac{1}{\mathrm{r}^{2}} \phi_{\theta \theta}=0 \tag{8}
\end{equation*}
$$

The ( + ) sign is for subsonic flow, and the ( - ) sign is for supersonic flow. The quantity $\beta$ is

$$
\begin{equation*}
\beta^{2}=\left|1-M_{\infty}^{2}\right| \tag{9}
\end{equation*}
$$

where $M_{\infty}$ is the freestream Mach number. The potential $\phi$ is the perturbation potential and yields the perturbation velocities

$$
\begin{equation*}
u=\frac{\partial \phi}{\partial x} \quad ; \quad v=\frac{\partial \phi}{\partial r} \quad ; \quad w=\frac{1}{r} \frac{\partial \phi}{\partial \theta} \tag{10}
\end{equation*}
$$

The velocities appearing in equation (5) can be obtained from equation (10). The boundary conditions for equation (8) and a wavy wall on a circular cylinder are discussed in Reference [4]. A solution is obtained for one spatial frequency for the wavy wall. The solution is the basic function from which a Fourier series for the flow is constructed. The turret is represented by two polynomials

$$
\begin{equation*}
f(x)=1+\sum_{k=1}^{K} \tilde{a}_{k} x^{k} \tag{11}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{f}(\theta)=1+\sum^{\mathrm{p}} \tilde{\mathrm{~b}}_{j} \theta^{j} \tag{j}
\end{equation*}
$$

To obtain a symmetric turret in the $\theta$-direction, only even values of $j$ are used in equation (12). In terms of $f(x)$ and $f(\theta)$, the turret geometry is

$$
\begin{equation*}
R(x, \theta)=R_{0}+\varepsilon f(x) f(\theta) \tag{13}
\end{equation*}
$$

where $R$ is the radial distance to the surface of the turret or fuselage. Equation (13) is represented by a Fourier series which leads to the coefficients in the Fourier series for the potential flow.

Figure 2 shows the geometry. The maximum turret height is $\varepsilon$, and the length of the turret is 2l. The meaning of $\theta_{\max }$ is that $f(\theta)$ is zero for $|\theta|>\theta_{\max }$. Figure 3 is an artist's concept of the laser turret. Two comments are applicable to


Figure 2. Geometry of a Small Perturbation Laser Turret on a Circular Fuselage.


Figure 3. Artist's Concept of Laser Turret.

Figure 3. First, the turret will become operational at some future date when laser canopies can be manufactured: Second, the model does not include perturbation effects of wings, blade antennas, and similar items.

Figure 4 illustrates the coordinate system used to describe the direction of the beam relative to the aircraft. A Cartesian coordinate system $X, Y$, $Z$ is oriented as shown. The Z-axis forms the polar axis for a spherical coordinate system. The beam is at azimuth angle $\phi$ and elevation $\gamma$.


Figure 4. Coordinate System for Direction of Laser Beam Propagation.

The design variables become $\tilde{a}_{k}$ and $\tilde{b}_{j}$. Conditions at $|\theta|=\theta_{\max }$ and $|x|=\ell$ reduce the number of independent design variables. Typical conditions are

$$
\begin{equation*}
R\left(\theta_{\max }, x\right)=R(\theta, \ell)=R_{0} \tag{14}
\end{equation*}
$$

and

$$
\begin{equation*}
\left.\left.\frac{\partial R}{\partial x}\right)_{x= \pm \ell}^{=0} \quad ; \quad \frac{\partial R}{\partial \theta}\right)_{Y=\theta_{\max }}=0 \tag{15}
\end{equation*}
$$

The intercept between a ray and the turret surface is a particularly difficult problem in analytical geometry. An iteration scheme was used to find the intercept as follows:
a. At a given point $s=s^{\prime}$ on the ray, calculate $x, \theta, R$.
b. For these values of $x$ and $\theta$, calculate the radial component of the turret surface $R_{T}$.
c. Iterate to find a value of $s^{\prime}$ such that $R=R_{T}$.

The iteration procedure is described in detail by Vanderplaats [13].
In equation (1), the integration starts at and ends at $b$. The start of integration, $a$, is at the primary mirror surface. The end of integration, $b$, is at a point sufficiently far from the laser turret so that additional integration by an amount $\delta s$ yields a negligible change in the value of the integral. Since integration starts at the mirror surface, a portion of the ray between $a$ and $b$ is within the laser turret. Hence the density within the turret is a factor in the phase distortion. Even if the air external to the turret were uniform, a phase distortion could be generated by the air within the turret. In this study, the turret window is assumed to be of zero thickness so as to be distortionless.

OPTIMIZATION OF LASER TURRET SHAPE
For any given azimuth, $\phi$, and elevation angle, $\gamma$, the phase distortion of any ray in the beam can be calculated using the center ray of the beam as a reference. At a specified beam orientation, the phase distortion typically will be calculated using equation (7) of Reference [5] at two radial locations for each of eight angular locations, i.e., $\eta$ of Figure 1 occurs every $45^{\circ}$. Furthermore, to provide optimum overall system performance, several beam orientations are considered. By squaring the values of phase distortion and summing over all rays and orientations, a measure of total performance, $S$, is obtained as

$$
\begin{equation*}
s=\sum_{\phi} \sum_{\gamma} W_{\phi \gamma} \sum_{z} \sum_{\eta} P^{2} \tag{16}
\end{equation*}
$$

The variable $z$ is defined in Figure $1 ; z$ gives the radial location within the beam. $W_{\phi \gamma}$ is a weighting function. Values of $W_{\phi \gamma}$ are determined from mission studies. For a particular mission, the laser beam may be pointed most of the time at a particular direction, i.e., particular values of $\phi$ and $\gamma$. For that direction, $W_{\phi \gamma}$ is larger. For more extensive mission studies, sufficient information can be obtained so that a meaningful function can be defined for $W_{\phi \gamma}$; the function is a two-dimensional probability density function giving the probability the beam points in the direction specified by $\phi$ and $\gamma$.

The objective of the optimization was to minimize $S$ by determining the proper combination of design variables. The design variables have been mentioned earlier. To summarize, the design variables are as follows:

| independent variables from | $\tilde{a}_{k}$ |
| :--- | :--- |
| independent variables from | $\widetilde{b}_{j}$ |
| mirror location | $\varepsilon_{M}, X_{M}$ |
| density within turret | $\rho_{t}$ |

The analysis capability presented in this paper and Reference [5] has been coded in FORTRAN to produce maps of phase distortion for given azimuth and elevation angles. Figures 5 to 7 are phase distortion maps. For all three maps the windward side of the beam is at the top of the map. This fact can be determined from the equation for phase distortion
$P=\frac{K^{\prime}}{\lambda} \frac{R_{0}}{\rho_{S L}}\left\{\left(s_{j}-s_{i}\right) \dot{\rho}_{t}+\rho_{\infty} \int_{s_{j}}^{\infty}\left[1+\frac{\gamma M_{\infty}^{2} C_{p} p^{\frac{1}{\gamma}}}{2}\right]_{j} \frac{d s}{R_{0}}-\rho_{\infty} \int_{s_{i}}^{\infty}\left[1+\frac{\gamma M_{\infty}^{2} C^{\frac{1}{\gamma}} p_{1}}{2} \frac{d s}{R_{0}}\right\}\right.$


Figure 5. Phase Distortion Map.
Azimuth, $\phi=0^{\circ}$; Elevation, $\gamma=45^{\circ} . \quad M_{\infty}=0.5$.
$\rho_{t} / \rho_{\infty}=1.0$. Cosine Turret.


Figure 6. Phase Distortion Map. Azimuth, $\phi=0^{\circ}$; Elevation, $\gamma=45^{\circ} . M_{\infty}=2.0$. $\rho_{t} / \rho_{\infty}=1.0$. Cosine Turret.


Figure 7. Phase Distortion Map. Azimuth, $\phi=0^{\circ}$; Elevation, $\gamma=90^{\circ}$. $M_{\infty}=2.0 . \quad \rho_{t} / \rho_{\infty}=0.309$. Optimized Turret.

In equation (17), $s_{j}$ is the distance from the surface of the primary mirror to the intercept of ray $j$ with the laser turret surface. The laser radiation has wavelength, $\lambda$. When the pressure coefficient $C_{p}$ along ray $j$ exceeds that along ray $i$, a positive contribution to the value of phase distortion occurs. Refer to Figure 2 or Figure 4. For an elevation angle of $\gamma=45^{\circ}$ and the laser beam in the plane of symmetry of the fuselage, $s_{j}>s_{i}$. Consequently, the $\operatorname{term}\left(s_{j}-s_{i}\right) \rho_{t}$ in equation (17) is also positive on the windward side. A positive phase distortion, $P$, means a lag of wavefront for ray $j$ compared to wavefront of ray $i$.

The laser turret geometry and the associated flow were coded in subroutine form compatible with the general purpose optimization program.

## COPES/CONMIN [7]

The COPES/CONMIN program solves the design problem of the following form:

$$
\begin{equation*}
\text { Minimize } \quad F(\vec{x}) \tag{18}
\end{equation*}
$$

subject to the constraints

$$
\begin{equation*}
G_{j}(\vec{x}) \leq 0 \quad, \quad j=1 \cdot \cdot \cdot \cdot m \tag{19}
\end{equation*}
$$

where $F(\vec{x})$ is called the objective function and is defined by equation (16). The vector of design variables, $\vec{x}$, contains the design variables summarized above. $G_{j}(\vec{x})$ are the constraints. The constraints that were considered at one time or another during the study were the maximum slope of the turret in the streamwise direction, no discontinuity at turret fuselage boundary, and the conditions of equation (15). The slope was restricted, at the most, to a value of 0.3 since the linearized flow equations become inaccurate for larger values. The choice of objective and constraint functions is somewhat arbitrary; the only restriction is that both functions must be continuous functions of the design variables, $\vec{x}$, with continuous first derivatives. In general, $F(\vec{x})$ and $G_{j}(\vec{x})$ may be any linear or nonlinear functions of $\vec{x}$.

TWO EXAMPLES OF LASER TURRET DESIGN
Two design examples are presented here, the first being for subsonic flow and the second being for supersonic flow. The design conditions are listed in Table I. Calculations were conducted for six beam orientations and sixteen rays within the beam.

| Table I. Design Conditions |  |
| :---: | :---: |
| AERO-OPTICS |  |
| Mach number Case 1 | $M_{\infty}=0.5$ |
| Case 2 | $M_{\infty}=2.0$ |
| Ratio of heat capacities | $\gamma=1.4$ |
| Wavelength of laser radiation | $\lambda=3.8$ micron |
| Density ratio | $\rho_{\infty} / \rho_{\text {SL }}=0.3$ |
| Constant for index of refraction | $K^{\prime}=0.00023$ |
| GEOMETRY |  |
| Fuselage radius | $\mathrm{R}_{0}=1.0$ |
| Spacing of turrets | $L=5.0$ |
| Mirror location | $\varepsilon_{M}=1.125$ |
|  | $X_{M}=0$ |
| Turret length | $\ell=2.0$ |
| Turret height | $\varepsilon=0.2$ |
| Mirror radius | $\mathrm{R}_{\mathrm{m}}=0.05$ |
| Maximum angle extent of turret | $\theta_{\max }=60^{\circ}$ |

GEOMETRIC BOUNDARY CONDITIONS

| $x / \ell$ | $f(x)$ | $d f(x) / d x$ | $\theta / \theta_{\max }$ | $f(\theta)$ | $d f(\theta) / d \theta$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| -1.0 | 0 | 0 | $\pm 1.0$ | 0 | varies |
| 0 | 1.0 | varies | 0 | 1.0 | 0 |
| 1.0 | 0 | 0 |  |  |  |

able I Continued. Design Conditions

| Beam ortentations |  |  |
| :---: | :---: | :---: |
| Beam Number | Azimuth, $\phi$, degrees | Elevation, $\gamma$, degrees |
| 1 | 0 | 45 |
| 2 | 0 | 90 |
| 3 | 0 | 120 |
| 4 | 45 | 45 |
| 5 | 90 | 30 |
| 6 | 90 | 60 |

## PHASE DISTORTION CALCULATION POINTS

Rays defined by all combinations of:
radius within beam $\quad z / R_{0}=0.025,0.050$
angle within beam $\quad \eta=0,45,90$, . . 315
Note: All rays are shown in Figure 1.

CONSTRAINT IN SLOPE

$$
-0.3 \leq \frac{\mathrm{df}(\mathrm{x})}{\mathrm{dx}} \leq 0.30
$$

for $\quad \theta=0$


As a reference, flow over a cosine-shaped turret was calculated. The equations for turret geometry were

$$
\begin{equation*}
f(x)=1.0-0.50\left(\frac{x}{\ell}\right)^{2}+0.0625\left(\frac{x}{l}\right)^{4} \tag{20}
\end{equation*}
$$

and

$$
\begin{equation*}
f(\theta)=1.0-1.824\left(\frac{\theta}{\theta_{\max }}\right)^{2}+0.832\left(\frac{\theta}{\theta}\right)_{\max }^{4} \tag{21}
\end{equation*}
$$

Phase distortion maps are shown in Figures 5 and 6 for the turret specified by equations (20) and (21). The perturbation velocities, $u$ and $v$, were calculated in the plane of symmetry of the fuselage and are shown in Figures 8 and 9. Figure 8 is for the subsonic flow example, and Figure 9 is for the supersonic flow example. The radial perturbation velocity, $v$, is dictated by the boundary condition at the turret surface. Hence $v$ is identical in both Figures 8 and 9. The axial perturbation velocity, $u$, is different for subsonic flow as compared to supersonic



Figure 9. Perturbation Velocity for a Cosine Turret in Supersonic Flow.
flow. In subsonic flow, the maximum value of $u$ occurs at $x=0$, while in supersonic flow, at $x=0$, the value of $u$ is zero. In supersonic flow, the flow is compressed ( $u<0$ ) on the forward or windward side of the turret; on the leeward side of the turret, the flow is expanded (u>0).

Results for the two examples are summarized in Table II. In Table II the values of the coefficients $\tilde{a}$ and $\tilde{b}$ are listed for both the initial and optimized laser turrets. Using these values of $\tilde{a}$ and $\tilde{b}$, the laser turrets have been drawn and appear in Figures 10 to 12. Figure 10 is the cosine-shaped turret used as reference.

Table II. Summary of Laser Turret Design Results

| TURRET GEOMETRY |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Polynomial | Initial | Turret | Case 1 | Optimized (Subsonic) | Turrets <br> Case 2 ( | (Supersonic) |
| Exponent | $\mathfrak{a}$ | 亿 | $\stackrel{\sim}{a}$ | $\widetilde{\mathbf{b}}$ | $\underset{\mathbf{a}}{ }$ | $\tilde{\mathrm{b}}$ |
| 0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| 1 | 0 | 0 | 0 | 0 | 0.2651 | 0 |
| 2 | -0.5 | -1.824 | -1.5596 | -1.83 | -0.6077 | -1.858 |
| 3 | 0 | 0 | -0.0006 | 0 | -0.1326 | 0 |
| 4 | 0.0625 | 0.8315 | 0.5923 | 0.8426 | 0.1163 | 0.8933 |
| 5 | 0 | 0 | 0.00015* | 0 | 0.0166* | * 0 |
| 6 | 0 | 0 | -0.0662* | -0.005* | -0.0067* | * -0.0282* |

* Design Variable

VALUE OF OBJECTIVE FUNCTION, S, AND DENSITY RATIO

| Quantity | Initial Turret <br> Subsonic Supersonic | Opti <br> Case 1 (Subsonic) | Turrets Case 2 (Supersonic) |
| :---: | :---: | :---: | :---: |
| S | 36.022 .69 | 31.22 | 1.55 |
| $\rho_{t} / \rho_{\infty}$ | 0.70 .3 | 0.7 | 0.3094 |




Figure 12. Optimized Laser Turret for Supersonic Flow. $M_{\infty}=2.0$.

Figure 11 is the optimized laser turret for subsonic flow. Figure 12 is the optimized laser turret for supersonic flow. Comparing Figures 10 and 12 , very little difference between the cosine-shaped and the optimized turret for supersonic flow can be seen. However, reference to Table II shows the optimized turret has odd powers for $f(x)$; note that $a_{1}=0.2651, a_{3}=-0.1326$, and $a_{5}=0.0166$.

For the calculations summarized in Table II, all the weighting values $W_{\phi \gamma}$ were unity. Using the COPES/CONMIN optimization computer program, the objective function, $S$, defined by equation (16), was reduced from 36.02 to 31.22 for the laser turret optimized for subsonic flow. The reduction is 13 per cent. The objective function, $S$, for the laser turret designed for supersonic flow was reduced from 2.69 to 1.55 . The reduction is 42 per cent. Also note that a density within the turret, $\rho_{t}$, less than ambient helps to reduce the phase distortion; see Table II.

EXAMPLE OF LASER TURRET GIVING LEAST AND WORST PHASE DISTORTION
To illustrate the range of values of the objective functions, $S$, that can be obtained by varying the turret geometry, both the best and worst laser turret were designed. Details are given in Table III. The cross sections of the initial turret, the best turret, and the worst turret are shown in Figure 13. For the case at hand, the initial turret yielded $S=0.0115$. The worst turret gave $S=0.0918$ which is a change of 690 per cent. The best turret has $S=0.0012$ which is an improvement in $S$ of 890 per cent. The range from the worst to the best is $0.0918 / 0.0012=78.5$.

Table III has the coefficients $A_{j}$ for the Zernike polynomials. The phase distortion can be represented by equation (6) using $A_{j}$ from Table III. The reader should compare $A_{j}$ for the initial turret with the other two turrets. The best turret has a slightly larger value for $A_{4}$. The value of $A_{9}$ is reduced greatly.

For the laser turret giving the worst distortion, all coefficients are increased except for $A_{4}$. The focus coefficient is slightly smaller. The average value $A_{1}$ has little significance.

Compared to the two examples of the previous section, the turrets in this section were optimized for only one beam direction. The beam direction was at an azimuth, $\phi=45^{\circ}$ and an elevation, $\gamma=45^{\circ}$.

COMPUTER CODE FOR LASER TURRET OPTIMIZATION
An extensive and versatile computer code has been written by Vanderplaats and Fuhs [14]. The computer code is based on References $[2,3,5,7,8,13]$. The program calculates the optical path length and phase distortion arising from the density field surrounding a laser turret in compressible flow. Further, the program finds the optimum turret shape yielding minimum phase distortion. The optimization and control codes are thoroughly discussed in Reference [14]. Sample data input and

Table III. Summary of Results for Phase Distortion

| Zernike <br> Coefficient | Physical <br> Significance | Initial <br> Turret | Laser Turret Shape <br> Least <br> Distortion | Worst <br> Distortion |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{A}_{1}$ | average value | $-3.331 \mathrm{E}-04$ | $3.967 \mathrm{E}-04$ | -4.193 Em 03 |
| $\mathrm{~A}_{2}$ | x-tilt | 0.02129 | $4.391 \mathrm{E}-03$ | 0.05995 |
| $\mathrm{~A}_{3}$ | y-tilt | $-7.643 \mathrm{E}-03$ | $5.422 \mathrm{E}-03$ | -0.03595 |
| $\mathrm{~A}_{4}$ | focus | $-3.947 \mathrm{E}-04$ | $4.759 \mathrm{E}-04$ | $-3.650 \mathrm{E}-03$ |
| $\mathrm{~A}_{5}$ | astigmatism | $6.576 \mathrm{E}-05$ | $3.320 \mathrm{E}-05$ | $3.748 \mathrm{E}-04$ |
| $\mathrm{~A}_{6}$ | astigmatism | $9.183 \mathrm{E}-04$ | $3.973 \mathrm{E}-05$ | $9.944 \mathrm{E}-04$ |
| $\mathrm{~A}_{7}$ | coma | $-1.638 \mathrm{E}-05$ | $-7.172 \mathrm{E}-06$ | $-5.610 \mathrm{E}-04$ |
| $\mathrm{~A}_{8}$ | coma | $-5.123 \mathrm{E}-06$ | $3.136 \mathrm{E}-05$ | $-8.846 \mathrm{E}-04$ |
| $\mathrm{~A}_{9}$ | coma | 0.01656 | $3.428 \mathrm{E}-03$ | 0.04549 |
| $\mathrm{~A}_{10}$ | coma | $-5.951 \mathrm{E}-03$ | $4.285 \mathrm{E}-03$ | -0.02840 |

Mach number $=.500$
Flight altitude $=$ sea level
Turret height/fuselage radius $=.200$
Beam radius/fuselage radius $=.05$
Elevation angle $=45^{\circ}$
Azimuth angle $=45^{\circ}$
sample output are given. The material is presented in sufficient detail so that Reference [14] constitutes a user's manual for LASTOP.
(a) Nominal turret. Objective function has value of 0.0115 .

(b) Turret yielding greatest distortion. Objective function has value of 0.0918 .

(c) Turrct yielding least distortion. Objective function has value of 0.0012 .

Figure 13. Cross Section Shape of Turrets in the Plane of Symmetry.

A versatile analysis and computer program has been developed which optimizes laser turret geometry to obtain minimum phase distortion. The turrets are located on a fuselage of circular cross section. Turret slope is limited so as not to exceed the perturbation allowed by the linearized equations for the flow. The computer code is described in Reference [14].

Examples have been given which show the decrease in objective function, $S$ in equation (16), that can be achieved.

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