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PTTI APPLICATIONS TO DEEP SPACE NAVIGATION

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ABSTRACT

Radio metric deep space navigation relies nearly exclusively upon coherent, i.e. 2-way, doppler and ranging for all precise applications. These data types and the navigational accuracies they can produce are reviewed. The deployment of Hydrogen maser frequency standards and the development of Very Long Baseline Interferometry (VLBI) systems within the Deep Space Network is making possible the development of non-coherent, 1-way data forms that promise much greater inherent navigational accuracy. These data, closely paralleling the observables taken with VLBI are much more sensitive to clock synchronizations, both time and frequency, and to instability during the measurement period itself than are the coherent data. The underlying structure between each data class and clock performance is charted. VLBI observations of the natural radio sources are the planned instrument for the synchronization task. This method and a navigational scheme using differential measurements between the spacecraft and nearby quasars are described.

I. INTRODUCTION AND BACKGROUND

Navigation for deep space probes has always required the acquisition of long arcs of precise doppler data to determine accurately and reliably the spacecraft orbit prior to planetary encounter. The Doppler data used is taken in the so-called coherent, or 2-way, mode. In this 2-way mode, a ground station within the Deep Space Network transmits a stable frequency reference to the spacecraft which transponds a coherent version of the received signal back to the ground station for Doppler detection (Renzetti, et al[1] and Melbourne[2]). At or near planetary encounter, the gravitational effects of the target body on the motion of the spacecraft are sufficient to produce a unique velocity profile in the Doppler that can be the dominant effect in determining where the spacecraft is. These effects usually arise too late, however, to be of benefit in the navigation process of determining and adjusting the planet-relative encounter conditions. The tracking which determines the planet-relative navigation accuracy is performed long before the actual encounter and when the gravity forces on the probe are of little benefit in supplying the desired location information. Here the dominant information is supplied by the diurnal motion of the tracking stations as illustrated in Figure 1. A single pass of doppler tracking data will yield information about three quantities:

1) The probe's geocentric velocity by calculating the doppler shift with the diurnal effects removed.

2) The right ascension, by determining the time of the meridian crossing via observing the null in the diurnal harmonic modulation.

3) The cosine of the declination, by determining the amplitude of the harmonic modulation.

The complete spacecraft state is determined by taking additional passes of doppler data on succeeding days and combining the information from each into a full solution. When available, ranging data greatly aids this process of determining the 6-dimensional spacecraft state to the requisite accuracy. In summary:

1) range and doppler data give direct measurements of the line of sight probe distance and its time derivative,

2) the diurnal modulation of the doppler yields an indirect measurement of the right ascension, α, and declination, δ,

3) successive passes detect changes in α and δ permitting indirect measurements of $\dot{\alpha}$, $\dot{\delta}$ and provide the data volume base for noise averaging and data consistency.

Several detailed analyses of this situation have been performed - among them are: Hamilton and Melbourne[3], Curkendall and McReynolds[4] - giving the accuracy of the observable parameters as a function of doppler measurement accuracy, tracking system calibrations, probe geometry, length of the tracking pass, tracking station location errors, and random non-gravitational forces affecting the motion of the spacecraft.

The accuracy to which the indirectly measured parameters can be estimated determine the overall accuracy of the complete orbit. Shown in Figure 2 is a plot of the system accuracy at several time points from the inception of the planetary exploration era to the present time - although the list is far from complete, the margin contains a tabulation of some of the technology improvements which enabled the accuracy evolution as shown. The accuracy achievable from a given doppler and associated data calibration and processing system is seen to be a strong function of the nominal probe declination. This is because the estimate of declination is determined by observing the amplitude of the diurnal harmonic signature, $\omega r_s \cos\delta$, and this amplitude becomes a stationary at $\delta = 0$.

The deterioration of the orbit performance at the lower range of declination can be a serious inconvenience for inner planet exploration. For example, the allowable Viking Mars arrival date space was truncated due to declinations lower than 5 deg for arrival dates past September 1976. The performance versus declination shown in the figure thus manifests itself as a mission planning constraint. Fortunately, the tempo of the geometry evolution within the inner planet system is rapid enough so that the periods of low declination and resulting loss of navigation ability are relatively short and can be usually tolerated.

The situation can be much worse for outer planet exploration. For example, the Voyager I and II Saturn encounters are both such that neither spacecraft is above 5 deg declination during the 4 months prior to the critical planetary encounter. To meet this and like situations, the dual-station planetary ranging system has been developed. Unlike the doppler system, two tracking stations are involved as is shown in Figure 3. In order to measure the troublesome declination variable these stations need to be widely separated in latitude as shown. The difference of the two range measurements is proportional to $B \sin\delta$, an observable with the right structural relationship with declination; i.e., the measurement sensitivity to declination maximizes near zero rather than becoming stationary. The declination accuracy achievable can thus approximately be given by

$$\sigma_\delta = \frac{\sigma_{\Delta\rho}}{B\cos\delta}$$

where $\sigma_{\Delta\rho}$ is the overall accuracy of the system's ability to measure the range difference, $\Delta\rho$, and B is the polar projection of the baseline. Present mechanizations require that the two range measurements be performed sequentially, rather than simultaneously, with each station obtaining a measure of the round-trip light-time between the probe and itself. The measurements can then be referred to a common epoch by either modeling the probe's motion or accumulating doppler data during the intervening period. The accuracy of this system is currently limited to a performance of approximately $\sigma_{\Delta\rho} = 4.5\sqrt{2}$m as discussed by Christensen and Siegel[5]. This measurement accuracy, working with the baseline of the tracking stations at Goldstone, CA, and Canberra, Australia, yields a declination accuracy of approximately 1 μrad over the full normal operating range of declination. It has been an iterative process, but this performance and the current Voyager mission requirements are commensurate as discussed by Jordan[6]. Figure 4 summarizes the current performance of the radio metric tracking systems for both the doppler and the ranging measurements. As shown, the doppler system provides a performance at the 0.25 μrad level for high declinations. This degrades slowly until 1 μrad is reached; the dual station ranging system then provides a level 1 μrad performance for the remainder of the declination space.

II. VERY LONG BASELINE INTERFEROMETRY

Since about the beginning of the current decade, Very Long Baseline Interferometry (VLBI) systems have been developed in parallel with the coherent doppler and ranging systems just described. References 7 - 18 which span this time period, describe the development and provide analyses necessary for a detailed understanding. In this paper we shall be content to describe VLBI only in tutorial terms, emphasizing the similarities and contrasts between the VLBI data and the coherent range and doppler already discussed.

In a typical VLBI system, each of two widely spaced antennas observes a single (broad band) radio source, e.g., a quasar, simultaneously recording the received signal over a specified frequency interval. The recordings are digital in which the received voltage is digitized at the one bit level; timing information is added so that the recordings may be cross correlated later when brought to a central site. The (expected) cross correlation function can easily be shown to be approximately (Thomas[11]):

$$E\left[R(t, \Delta\tau)\right] \propto \frac{\sin \pi W\Delta\tau}{\pi W\Delta\tau} \cos \phi_1(t)$$

Where

$\Delta\tau = \tau_g - \tau_m$

$\tau_g(t)$ = geometric delay as shown in Figure 5

τ_m = a priori delay estimate inserted to bring the correlation function to near its maximum during data processing

W = bandwidth of recorded signal

$\phi_1 = \omega_1 \Delta\tau$

ω_1 = frequency at the center of the bandpass

Using typical values of 2300 MHz and 2 MHz as the rf and recorded bandwidth respectively, this correlation function goes through one complete cycle for every change in $\Delta\tau$ equal to the period of the rf frequency (<0.5 nsec). In addition it also manifests a sin x/x characteristic envelope reaching its first null at 0.5 μsec delay. These two components are called respectively "fast fringes" and the "delay function" (see Figure 6). For these same typical values, τ (or equivalently $\Delta\rho/c$,

Figure 5) can be measured directly, by adjusting τ_m so as to maximize the delay function, to on the order of 10 nsec precision (3m in light-sec). More powerful measurements can be obtained, however, in each of two conceptually different ways:

1) Observation of the source continuously over the common visibility period of the two stations (approximately 4 hours on the baselines afforded by the DSN for sources near the ecliptic), produces a continuous record of the phase of the fast fringes versus time. The record thus obtained will contain a diurnal sinusoidal modulation term due to the Earth's rotation whose phase and amplitude are parametric in the source location and baseline parameters. This is exactly analogous to the single station coherent doppler tracking except that the equatorial baseline projection and longitude play the roles of the distance off the spin axis, r_s, and station longitude, θ, respectively. The differential frequency of the two clocks replaces the geocentric velocity term observed by the coherent data.

2) Observation of the source at a second center frequency, ω_2, produces a second measurement of the fast fringe phase, ϕ_2, at a single instant of time. Then because

$$\frac{\partial\phi}{\partial\omega} = \tau_g = \frac{\phi_2 - \phi_1}{\omega_2 - \omega_1}$$

a direct measurement of τ_g can be obtained in the short time required to achieve a high S/N for the ϕ measurements (typically 10 min.). This is the "bandwidth synthesis" technique, so called because large effective bandwidths can be obtained without the need for commensurate high recording rates, and is widely used throughout the VLBI community (Rogers 9). The geometry is exactly as for the differenced range measurement already discussed (cf. Figures 3 and 4); the τ_g measurement obtained can be used directly to estimate either the baseline projection or the source location. With spanned bandwidths, $\omega_2 - \omega_1$, on the order of 40 MHz, the precision of the measurement can easily be brought to the cm level; its accuracy is dominated by other effects such as clock performance and systematic calibration errors.

When estimating source locations with method two, a second baseline is usually employed for the second component of position. An effective combination for the two baselines is to have a large polar component associated with the first and a large equatorial projection associated with the second so that they can produce largely uncorrelated estimates of α and δ.

Thus these two methods, often referred to as narrow-and wide- band VLBI respectively, have a one-to-one correspondence with the two coherent-modes, doppler and differenced ranging, normally used for spacecraft tracking. Their normal applications are duals of each other in that VLBI is usually employed to estimate the station baselines; precise source coordinates are needed to enable this. Coherent tracking is normally used to estimate the spacecraft coordinates; precise station locations are needed for this task.

Although the VLBI and the coherent tracking modes each produce observables with identical information content as just discussed, natural-source VLBI enjoys several inherent accuracy advantages over its coherent counterpart. These were discussed in some detail in a previous paper (Curkendall[19]), but briefly they include: 1) wider bandwidth, 2) more complete calibration of charged particles, 3) a ready means for calibrating electrical path delay variations in the station electronics, 4) lack of significant proper motion in the natural sources themselves, and 5) freedom from needing to model the line-of-sight motion as is required in single station doppler tracking.

There is a single major exception to the general advantages of the non-coherent data types - they suffer from a greater sensitivity to instability of the station master oscillator. This sensitivity and the comparison of it with that of the coherent data forms is treated in detail in the following section.

III. MEASUREMENT ACCURACY VS. FREQUENCY STANDARD PERFORMANCE

The two measurement classes just discussed, coherent measurements and non-coherent VLBI measurements differ markedly by the manner in which the station's master frequency standard departures from ideal enter and corrupt the measurements.

In this section the four data types:

Data Class / Data Type	Two-Way Coherent Measurements	One-Way Non-Coherent Measurements
Narrowband	Doppler (1)	Narrowband VLBI (2)
Wideband	Differenced Ranging (3)	Wideband VLBI (4)

will each be analyzed and their sensitivity to clock performance charted.

Case 1 - Coherent Narrowband (Doppler) Data

Consider the (highly) schematic diagram of a typical coherent doppler and ranging system implementation as shown in Figure 7. Counted, or integrated, doppler is obtained by broadcasting a stable reference to the spacecraft which coherently transponds the received carrier back to the same station for comparison with the original transmitted frequency; the difference or doppler frequency is integrated by means of a counter as shown. Assume for the purposes of illustration, that a unit step in frequency error occurs for a short period of time as shown in Figure 8. This will enter the doppler extractor and be integrated to yield immediately a buildup of range error, $\Delta\rho_\varepsilon$. It will also be transmitted to the spacecraft and return a round-trip light time, τ, later and re-enter the doppler extractor, this time in the opposite sense - $\Delta\rho_\varepsilon$ will return to zero. If the doppler system has been tracking the spacecraft for T seconds, the accumulated effect of the time history of $\Delta f(t)$ is readily seen to be

$$\Delta\rho_\varepsilon \begin{cases} = \dfrac{c}{2f}\displaystyle\int_0^T \Delta f(t) - \Delta f(t-\tau)\, dt = \\ = \dfrac{c}{2f}\displaystyle\int_{T-\tau}^T \Delta f(t)dt - \int_{-\tau}^0 \Delta f(t)dt \end{cases} \qquad (T \gtrsim \tau) \qquad (1)$$

where c is the speed of light.

That is, in a data point measured at T, frequency standard performance during the first and last τ seconds counts, everything else cancels out. The factor of 2 appears so that $\Delta\rho_\varepsilon$ is the error in the one-way range as measured by the two-way instrument. It is useful to design expressions which permit calculating $\Delta\rho_\varepsilon$ assuming Δf is

i) a white noise process

ii) linear with time

i) White Frequency Noise

With the white noise assumption, the two integrals in (1) are clearly independent, the variance of $\Delta\rho_\varepsilon$ is then

$$\sigma_{\Delta\rho_\varepsilon} = \frac{c}{2}\sqrt{2\int_0^\tau\int_0^\tau h_o\,\delta(u-v)\,du\,dv} = \frac{c}{2}\sqrt{2\,h_o\,\tau} \quad (2)$$

where h_o is the spectral density of the white frequency $\Delta f/f$ process.

ii) Linear Frequency Drift

$$\text{Assume}\quad \frac{\Delta f(t)}{f} = kt$$

$$\text{Then from (1), } \sigma_{\Delta\rho_\varepsilon} = \frac{c}{2}\,k\,T\,\tau \quad (3)$$

where k is re-interpreted to be the standard deviation of the drift. Thus for frequency variations rapid relative to τ, the error builds with $\sqrt{\tau}$ and is independent of the tracking time; for frequency variations slow relative to τ, the error builds as the product, $T\tau$.

Case 2 - Non-Coherent, Narrowband (VLBI) Data

The non-coherent case is even more straightforward. Here the error in the measurement is easily seen to be proportional to the difference in the oscillators' frequencies at the two Earth-based stations integrated over the observational interval:

$$\Delta\rho_\varepsilon = \frac{c}{f}\int_0^T f_1 - f_2\,dt \quad (4)$$

where f_i = instantaneous frequency at the ith station.

The Δf notation used in (1) is dropped here to emphasize that the measurement is sensitive to more than just the change in frequency over τ or even T, it is sensitive to the "knowability" of the frequency difference. The initial frequency offset is large enough so that in any VLBI experiment it must be considered an unknown and solved for - indeed, the determination of $f_1(0) - f_2(0)$ is often the reason for the

VLBI experiment. It becomes convenient, then, to re-define the problem and focus on a modified error term as

$$\Delta\rho'_\varepsilon = \frac{c}{f}\int_0^T \Delta f_1 - \Delta f_2 \, dt \tag{5}$$

where Δf_i is understood to be the departure in the frequency from $f_i(0)$. Then

$$\sigma_{\Delta\rho'_\varepsilon} = \sqrt{2}\; c\left(\frac{\Delta f}{f}\right)_T T \tag{6}$$

where $\left(\frac{\Delta f}{f}\right)_T$ is the familiar two-sample

Allan Variance over smoothing time, T. Equation (4) is already in what is usually the most convenient form. For comparative purposes, however, it is interesting to recompute the effects arising from the white noise and linear drift models used earlier.

i) White frequency noise

$$\sigma_{\Delta\rho_\varepsilon} = \sqrt{2c^2\iint_0^T h_o \; \delta(u-v)\,du\,dv} = c\sqrt{2h_o T} \quad \text{(c.f.2)} \tag{7}$$

ii) Linear Frequency Drift

$$\frac{\Delta f_1 - \Delta f_2}{2} = \sqrt{2}\; kt \tag{8}$$

$$\sigma_{\Delta\rho'_\varepsilon} = \frac{\sqrt{2}\; k\; T^2}{2} \qquad \text{(c.f.3)}$$

Case 3 - Differenced Coherent (Ranging Data)

A coherent range measurement is essentially a measurement of the round-trip light time itself. The clock error introduced in such a measurement is thus the absolute frequency error integrated over the light time. A differenced ranging measurement is sensitive to the integrated frequency differences, i.e.,

$$\Delta\rho_\varepsilon = \frac{c}{2f}\int_0^T f_1 - f_2 \, dt \tag{9}$$

For precision differenced ranging measurements, this frequency difference (or more precisely, the error in the knowledge of the difference) must be held to within strict limits. For example, at the distance of Saturn, $\tau \approx 10^4$ sec., $|f_1 - f_2|$ must be $< 3 \times 10^{-13}$ for a $\Delta\rho_\varepsilon$ of less than .5 m. The implied frequency synchronization must be accomplished with traveling clocks or, as in more frequently the case, with a VLBI observation session whose object is the determinations of $f_1 - f_2$.

The error, then, in applying the synchronization to a differenced range measurement can be roughly predicted as

$$\sigma_{\Delta\rho_\varepsilon} = c \left(\frac{\Delta f}{f}\right)_{\Delta t} \tau \qquad (10)$$

where Δt is the time between synchronization and application. This expression assumes that the synchronization operation has an associated error much less than $(\Delta f/f)_{\Delta t}$. This is not always the case and the VLBI method for synchronization brings up an interesting interplay between clock performance and its measurement. In an ideal experiment where all parameters influencing the interferometer phase (geometry, media transmission effects, etc.) are known save the clock offset, $f_1 - f_2$, itself, the clock performance during the experiment of duration, T, will contribute an error

$$\sigma_{f_1 - f_2} = \frac{\sigma_{\Delta\rho_\varepsilon}}{c\ T} = \sqrt{2} \left(\frac{\Delta f}{f}\right)_T \qquad (11)$$

where $\sigma_{\Delta\rho_\varepsilon}$ is given by (6).

<u>Case 4 - Non-Coherent Wideband (VLBI) Data</u>

Here the measurement error is proportional to the clock time offset at the time of the measurement. Once again, this parameter must be measured periodically in order that the knowledge of it can be held to within reasonable limits. The expression for the error using the same nomenclature as the first three cases would be

$$\Delta\rho_\varepsilon = \frac{c}{f} \int_{"-\infty"}^{T} f_1 - f_2 \, dt \qquad (12)$$

where "$-\infty$" is understood to be the time of the last clock epoch synchronization operation. As before, if the synchronization operation is assumed accurate, the predicted standard deviation of (12) is readily seen to be

$$\sigma_{\Delta\rho_\varepsilon} = c\left(\frac{\Delta f}{f}\right)_{\Delta t} \Delta t \qquad (13)$$

where $\Delta t = T - "-\infty"$.

Evaluation of (13) for even Hydrogen maser stabilities ($\Delta f/f \approx 10^{-14}$) discloses that Δt cannot exceed 1/3 day if decimeter level measurements are sought. Because of this high sensitivity, it is common practice to include provision for both clock epoch and clock frequency synchronization integral to any natural source VLBI experiment. Equation (13) has relevance strictly only when envisioning a series of natural source measurements for clock synchronization whose results are applied to subsequent (or earlier) spacecraft tracking.

Table I summarizes the four cases just discussed and repeats the four basic sensitivity equations. Table II is an attempt to tabulate the expected metric errors arising in the same four cases in terms of the two-sample Allan Variance. There are several approximations used in writing the expressions shown and this table should be viewed more as an ordered collection of the principles discussed here rather than a set of rigorous relationships.

The non-coherent VLBI measurements are thus much more demanding of clock performance than their coherent counterparts. Indeed, the relative immunity of coherent doppler and range to clock variations enabled reasonable performance at small light-time distances ($\leq$ lunar distance) even with the crystal oscillators that were employed in the early 60's. With current rubidium standards ($\Delta f/f < 10^{-12}$/day) good performance can be achieved throughout the terrestrial planet space. In contrast, VLBI errors are dominated by clock effects when rubidium and even cesium ($\Delta f/f < 10^{-13}$/day) standards are used. The introduction of the Hydrogen maser with drifts better than 10^{-14}/day has in large part prompted the current interest in VLBI systems. At this clock performance level, the VLBI accuracy estimates given in the next section can be achieved and the non-coherent measurement class can successfully compete with its coherent counterpart.

IV. VLBI AS A NAVIGATION TOOL

The maturing VLBI technology can be applied to the spacecraft navigation problem in each of two conceptual ways: 1) calibration of the DSN for use as an otherwise conventional radiometric network, and 2) direct spacecraft signal tracking with the VLBI data acquisition and processing systems.

VLBI for DSN Calibrations

In July of '79, the first operational VLBI system (as contrasted with the R&D systems which have produced the results discussed above) will begin taking routine measurements on the California-Spain and California-Australia baselines. Initially this system, operating in conjunction with a developed precision source catalog, will have the capability of operating with an overall accuracy at the 0.05 μrad level. This capability will be exploited to calibrate the UT and PM variations, refine the knowledge of baseline vectors, and determine the relative station clock frequency offsets. The UT/PM calibration improves the knowledge of the effective station location coordinates for both range and doppler; the baseline solutions directly improve the differenced range data. Both of these error sources will be controlled to the 50 cm level as compared to their current 1-2 m effective levels. The interstation clock frequency calibration will be accurate to better than 1×10^{-13}, bringing the clock's contribution to the differenced range error (Eq. (10)) to well under 50 cm even at 10 AU.

Direct VLBI Navigation

The spacecraft can be treated as another VLBI radio source, albeit one with proper motion. Present spacecraft do not emit signals with bandwidths wide enough to really be considered wideband sources, however. Instead, the power is centered within a few MHz of the carrier, effectively enabling tracking at only the carrier center frequency so that the fast fringes may be observed but bandwidth synthesis at beneficial accuracies cannot be performed. Thus, only narrowband VLBI can be made available for already launched spacecraft. Save the one feature of the wide bandwidth, all of the inherent advantages of the non-coherent data class can be capitalized on, however.

The favored operational procedure in narrowband spacecraft VLBI is to track the spacecraft in close conjunction with the tracking of a nearby quasar ($\sim$ <5 deg away in angular measure) whose source location has already carefully been determined. In this procedure, known as ΔVLBI, the two antennas track the spacecraft for a few minutes and then quickly align on the quasar. This is repeated for the entire duration of the 4 hour tracking pass and the fringe phase history differential between the spacecraft and the source is constructed. The differences in α and δ between the two sources are then estimated. This procedure has the advantage that most of the systematic error sources, being nearly common to both the spacecraft and quasar phase histories - baseline coordinates, instrumentation calibration, neutral and charged particle media effects, and clock imperfections - are diminished by the differencing operation. The precise natural source location estimate is quickly transferred to the spacecraft by this technique.

It is relatively simple to alter the broadcast spacecraft waveform to make it suitable for wideband VLBI tracking. Modulating a high frequency subcarrier with a signal generated from a noise diode could produce noise channels suitable for bandwidth synthesis with characteristics nearly identical to that being received from the quasar. A far better technique is to transmit a specific ranging code generated onboard the spacecraft and correlate the received spacecraft signal at each station against a locally generated model of that same code in a manner similar to that used in a conventional 2-way ranging machine.

In contrast to the current ranging technique, the signal would not be demodulated and tracked via the closed loop receiver, however. Rather, the spacecraft signal would follow the same rf chain as the quasar signal, phase calibration signals would be introduced, and detection would take place after digitization so as to preserve the near perfect commonality with the natural radio source tracking essential for careful system calibration. Each station executes this procedure, a one-way range is calculated, and the difference taken. This difference contains the differential station clock epoch error which must be carefully calibrated with conventional VLBI. This calibration, along with UT/PM calibration, can be performed via either an earlier multiple-source VLBI run and applied to the Differential One-Way Ranging (DOR) data or the DOR data can be obtained in conjunction with the track of a single very nearby quasar in a manner similar to the narrowband ΔVLBI method already described. Which method is employed operationally would be determined by the availability and strength of a nearby quasar, the number of spacecraft that need to be tracked, the availability of transmission lines for the VLBI data, the time criticality of the spacecraft results, and the ability of the clocks to hold an epoch synchronization.

An extensive error analysis and discussion of the wide-band VLBI, or DOR, technique was presented by Melbourne and Curkendall in an earlier paper[20]. As a result of that and subsequent analysis, it is felt that a confident prediction of the performance of such a system can be made at the 0.05 μrad level. The narrowband ΔVLBI can perform at similar levels for spacecraft at high declination, but would degrade with the $1/\tan\delta$ characteristic inherent for all narrowband tracking.

These accuracies, along with the accuracy of the coherent tracking earlier discussed, is summarized in Figure 4. In contrasting the wide and narrowband approaches, the constant performance of the wide-band method with declination is of the utmost importance. In addition, the tracking time required for a complete observation is much smaller for the wide-band system since no diurnal signature need be observed. Observations from two baselines are required however. The narrowband approach suffers from a greater sensitivity to systematic error sources so only the ΔVLBI mode should be used to meet a precision requirement. The wideband is more immune and offers greater tracking strategy flexibility. It is thus less dependent on the existence of a nearby quasar. This could be important for some applications since at the present time, few suitable sources have been found in the portion of the celestial sphere where the galaxy partially obscures extragalactic observations (Preston et al[21]). This results in two bands along the ecliptic plane, each measuring about 30 deg in extent, nearly devoid of suitable sources. Finally, the post correlation data processing is more difficult for the narrowband ΔVLBI data. The key to the accuracy is the accurate construction of the accumulated phase delay change over the 4 hour period for both the spacecraft and the quasar. Care must be taken to ensure that the phase is extrapolated properly to within one rf cycle (13 cm at S; 3.5 cm at X-band) during the time periods when the antennas are moving or are on the opposite source.

On a more positive note, narrowband ΔVLBI can be very useful for tracking existing spacecraft and offers a greater sensitivity for the detection of proper motion over short time periods (< 4 h).

Sensitivity of Differential VLBI to Clock Performance

Differential-, or Δ-VLBI, is intermediate in clock sensitivity between the coherent or non-coherent data forms discussed in Section III and for that reason deserves special treatment in this concluding sub-section.

This sensitivity is generally proportional to the angular separation between the natural source and the spacecraft. For example, in narrowband ΔVLBI, when the sources are both within a single beamwidth of the antennas, the normal switching between the two as described above is not necessary, both sources are tracked all the time and the effect of clock variations cancels out. All that is needed in this case are clocks with coherence times long enough to detect the signals (∿ a few minutes). Beyond a single beamwidth separation, antenna switching is necessary and the individual spacecraft and quasar records will have complimentary data outages as shown in Figure 9. For separations just greater than the single beamwidth limitation the data differencing operation would difference the two data streams as close together in time as possible or one full cycle time, denoted as Δt in the diagram. Consider for the moment, that the clock instability is the sole error source, the measured Δρ for the quasar would be:

$$\Delta\rho_{mq}(T) = \Delta\rho_q(T) + \frac{c}{f}\int_0^T f_1 - f_2 \, dt \qquad (14)$$

An identical expression for the spacecraft measurement, $\Delta\rho_{m\,s/c}(T)$ can also be written. Their difference with the time shift, Δt is

$$\begin{aligned} \Delta\Delta\rho_m(T, \Delta t) &= \Delta\rho_{mq}(T + \Delta t) - \Delta\rho_{m\ s/c}(T) \\ &= \Delta\rho_q - \Delta\rho_{s/c} + \frac{c}{f}\int_T^{t+\Delta t} f_1 - f_2 \, dt \end{aligned} \qquad (15)$$

It is useful to calculate the expected rms value of the clock induced error for the white frequency noise and linear drift examples used earlier.

i) White Frequency Noise

$$\sigma_{\Delta\Delta\rho_\varepsilon} = c\sqrt{2h_o\ \Delta t} \qquad \text{(c.f.2\&7)} \tag{16}$$

ii) Linear Frequency Drift

$$\sigma_{\Delta\Delta\rho_\varepsilon} = \frac{ck}{2}(\Delta t^2 + 2T\Delta t) \qquad \text{(c.f.3\&6)} \tag{17}$$

In comparing these expressions with those derived earlier, note that if the switching time is shorter than the spacecraft round trip light time (Δt can be on the order of 10 minutes, - is often several hours), the ΔVLBI data is actually less sensitive to the white frequency noise than is two-way counted doppler. The expression for the linear drift model is a good illustration that the sensitivity to more systematic clock errors is intermediate to that of the coherent and non-coherent forms discussed in Section III.

For larger source separations, the situation is somewhat more complex. As the angle increases, it soon becomes apparent that if both sources are tracked over the same time periods (e.g. t = 0 to T) except for the alternating outages, the cancellation of other error sources will not be nearly so complete as if better strategies had been employed. For example, suppose that the sources have equal declinations but differ in their right ascensions and that the dominant error source is the tropospheric scale height. It should be clear that in this circumstance, the best strategy would be to track each source over the same range of hour angles and stagger the time intervals as shown in Figure 10.

In this circumstance, the effective Δt would grow to a much larger value than is strictly needed for the switching time operation as is illustrated in the figure. More generally, the global tracking scenario and the effective Δt must be chosen to minimize the sum of all the error sources, a problem beyond the scope of this short discussion. Once chosen however, the clock's contribution to the accumulated error can be written directly in terms of the Allan Variance as

$$\sigma_{\Delta\Delta\rho_\varepsilon} = \sqrt{2}\, c \left(\frac{\Delta f}{f}\right)_{\Delta t} \Delta t \tag{18}$$

In wideband ΔVLBI, the tracking strategy is simplified to a single observation of each source. The clock's contribution to the error in the difference of the two resulting time delay measurements are 1) the error suffered internal to the measurement itself, and 2) the clock offset drift in between the two measurements, spaced Δt apart. The determination of the first of these is beyond the scope of this discussion (but should be small), the second is given by (18) just as in the narrowband case. The real difference between the narrow and wideband cases is that the narrowband transformation to right ascension and declination estimates is itself more sensitive to a time delay error change (by about a factor of 5 at high declination, growing gradually worse as declination is reduced) than is the wideband transformation sensitivity to time delay error.

A single numerical example should serve to put these relationships in perspective. During the 1979 Voyager encounters with Jupiter, a narrowband ΔVLBI demonstration is planned whose accuracy goal is set at the .05 μrad level. A single quasar, OJ287, is being used as the reference natural source throughout the several month experimentation period; the appropriate Δt is as large as 45 min. The error control needed to achieve this accuracy is approximately 7 cm of range change error during the 4 hour integration period. If 1 cm is the clock's allocation, the two-sample Allan Variance clock performance required is (from (18):

If $\sigma_{\Delta\Delta\rho_\varepsilon} = .01\text{m}$

$\Delta t = 45$ min.

$$\frac{\Delta f}{f} = \frac{\sigma_{\Delta\Delta\rho_\varepsilon}}{\sqrt{2}\, c\, \Delta t} = 8.8 \times 10^{-15}$$

If wideband ΔVLBI were possible, the same accuracy level could be achieved with a total error budget of about 40cm. If 1/7 of this were allocated to the clock, the $\Delta f/f$ specification could be relaxed to 5×10^{-14}. Alternately, and probably of more practical importance, a clock operating at 8.8 x 10^{-15} could be used to lengthen the time between epoch calibration and spacecraft use; i.e., the 45 min. could be stretched to several hours.

TABLE I: Comparison of Data Type Sensitivity to Station Master Clock Errors

Data Class / Data Type	Two-Way Coherent Measurements	One-Way Non Coherent Measurements
Narrowband	Doppler Clock Frequency Drift During Pass Integrated Over Round Trip Light Time to S/C $\Delta\rho_\varepsilon = \frac{c}{2f}\left[\int_{T}^{T+\tau} f(t)\,dt - \int_{o}^{\tau} f(t)dt\right]$	Narrowband VLBI Interstation Frequency Difference Integrated Over Tracking Pass Duration $\Delta\rho_\varepsilon = \frac{c}{f}\int_{o}^{T} f_1 - f_2\,dt$
Wideband	Differenced Ranging Interstation Clock Frequency Difference Integration Over Light Time $\Delta\rho_\varepsilon = \frac{c}{2f}\left[\int_{o}^{\tau} f_1 - f_2\,dt\right]$	Wideband VLBI Interstation Clock Epoch Error at Measurement Time $\Delta\rho_\varepsilon = \frac{c}{f}\int_{``-\infty"}^{T} f_1 - f_2\,dt$

f_i = instantaneous frequency of i^{th} station clock

T = Tracking Pass Duration

τ = Round Trip Light Time to S/C

NOTE: All errors are given in range difference or integrated range-rate (m).

Table II.

Approximate Evaluation of Expected Metric Error, $\sigma_{\Delta\rho_\varepsilon}$, In Terms of the Two Sample Allan Variance $\left(\frac{\Delta f}{f}\right)^2_t$.

Data Class / Data Type	$\sigma_{\Delta\rho_\varepsilon}$ Two-Way Coherent Measurements	$\sigma_{\Delta\rho_\varepsilon}$ One-Way Non-Coherent Measurements
Narrowband	$\frac{\sqrt{2c}}{2}\left(\frac{\Delta f}{f}\right)^{*}_{\tau}\tau$ *only strictly true for white phase and frequency noise processes	$c\left[\frac{\Delta f_{\varepsilon_s}}{f} \oplus \sqrt{2}\left(\frac{\Delta f}{f}\right)^{+}_{\Delta t} \oplus \sqrt{2}\left(\frac{\Delta f}{f}\right)_{T}\right]T$
Wideband	$\frac{c}{2}\left[\frac{\Delta f_{\varepsilon_s}}{f} \oplus \left(\frac{\Delta f}{f}\right)^{\dagger}_{\Delta t}\right]\tau$ † an approximate interpretation of Allan Variance	$c\left[\Delta\tau_{\varepsilon_s} \oplus \frac{\Delta f_{\varepsilon_s}}{f}\Delta t \oplus \left(\frac{\Delta f}{f}\right)_{\Delta t}\Delta t\right]$

where

- c = speed of light
- $\left(\frac{\Delta f}{f}\right)_t$ = two sample Allen Variance evaluated at t smoothing time
- τ = round trip light-time to coherent transponder
- T = time from beginning of $\Delta\rho$ integration
- Δf_{ε_s} = error from frequency synchronization operation
- $\Delta\tau_{\varepsilon_s}$ = error from clock epoch synchronization operation
- Δt = time from clock synchronization to T
- $\oplus$, an operator impling addition of errors in the rms sense

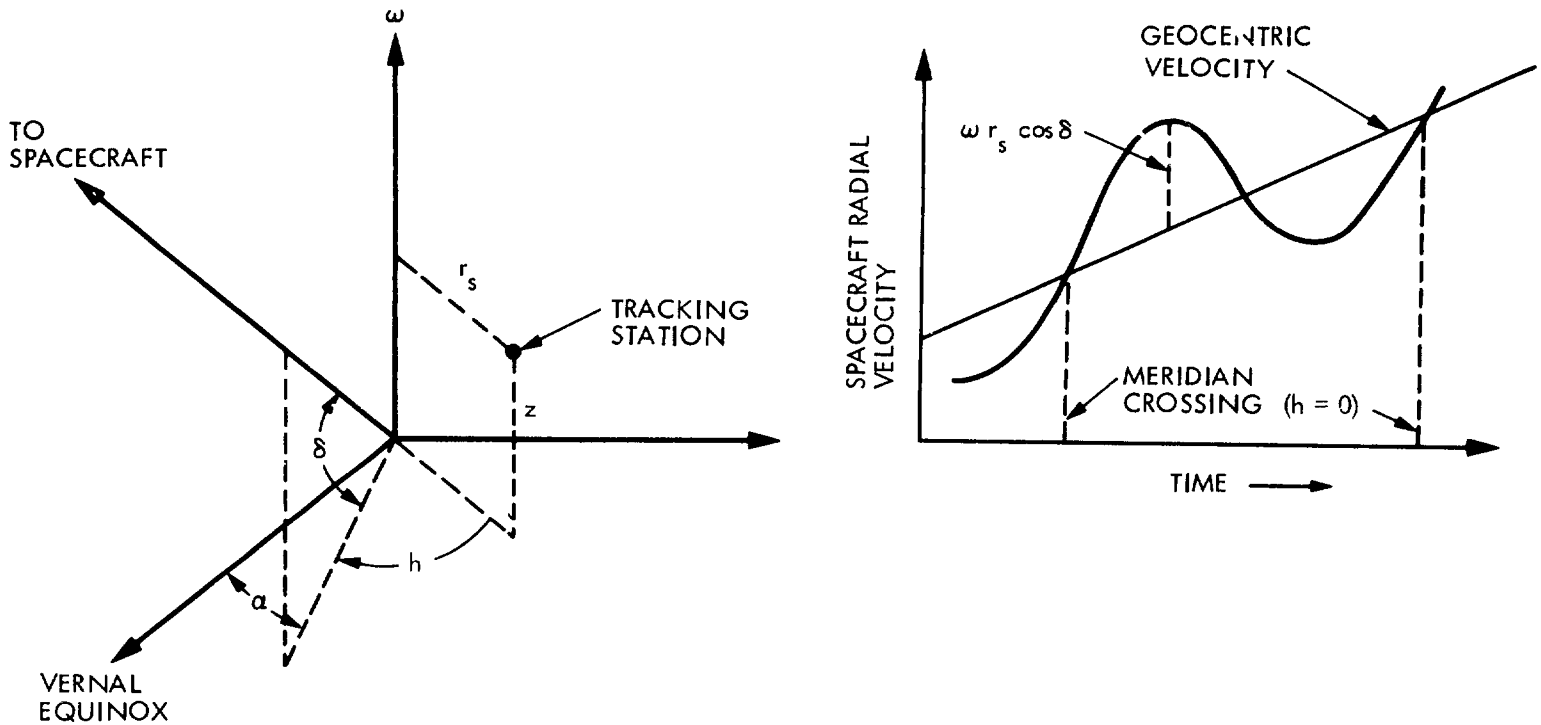

Figure 1. The Primary Information Source for Pre-Encounter Navigation Arises from the Diurnal Motion of the Tracking Station.

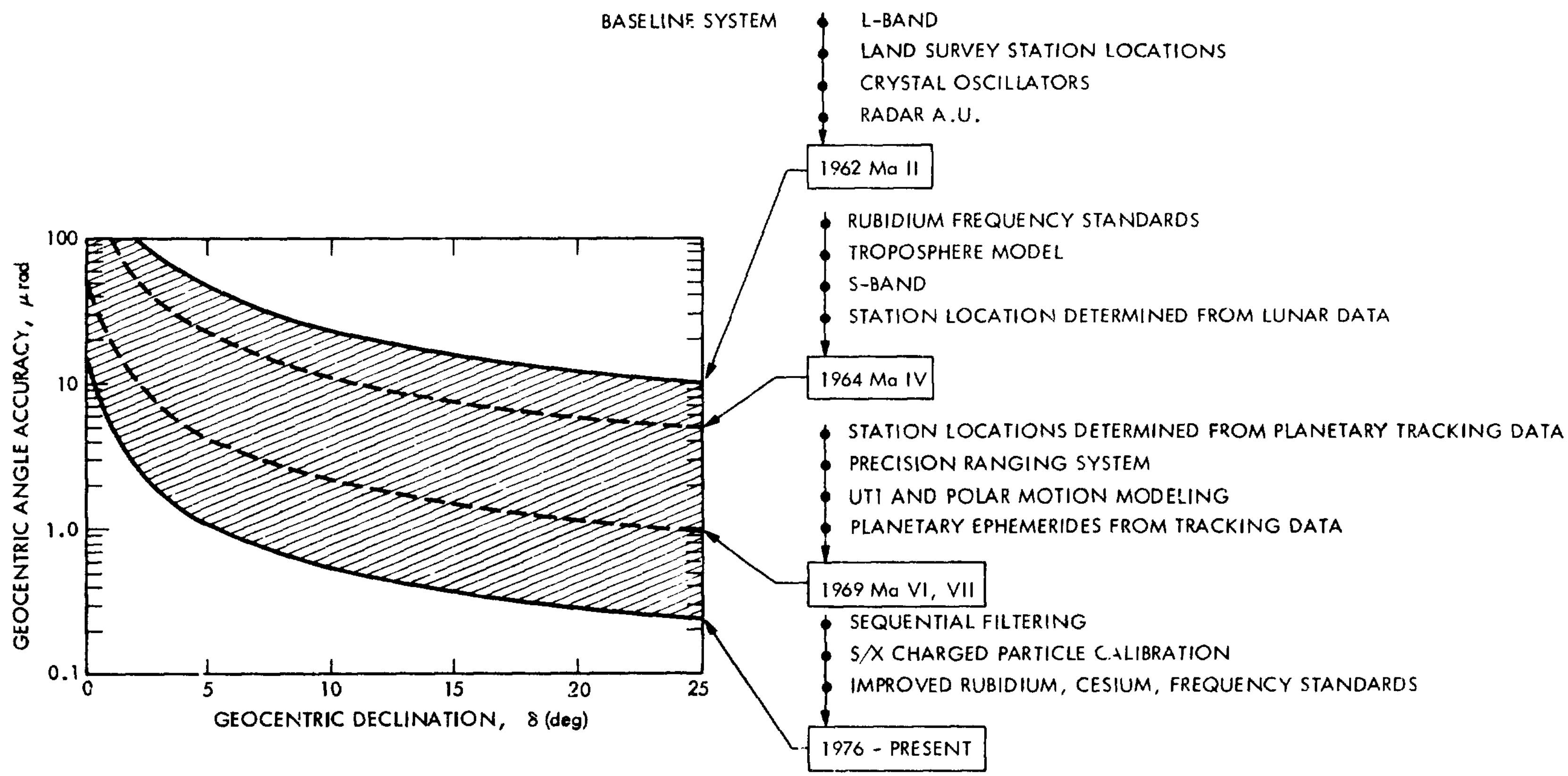

Figure 2. Doppler System Performance versus Time and Spacecraft Declination

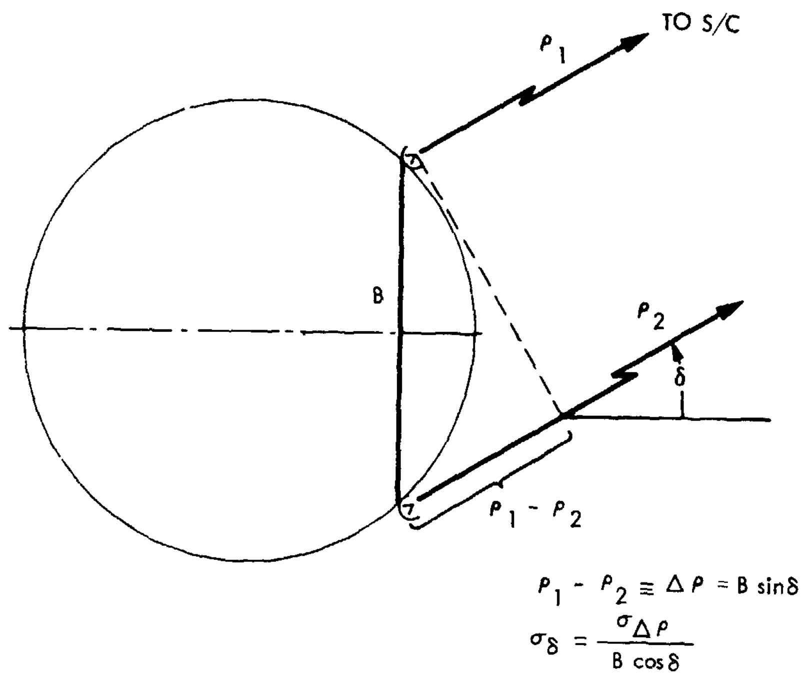

Figure 3. The Difference of Two Ranging Measurements from Stations Widely Separated in Latitude can Accurately Measure Declination of Probe

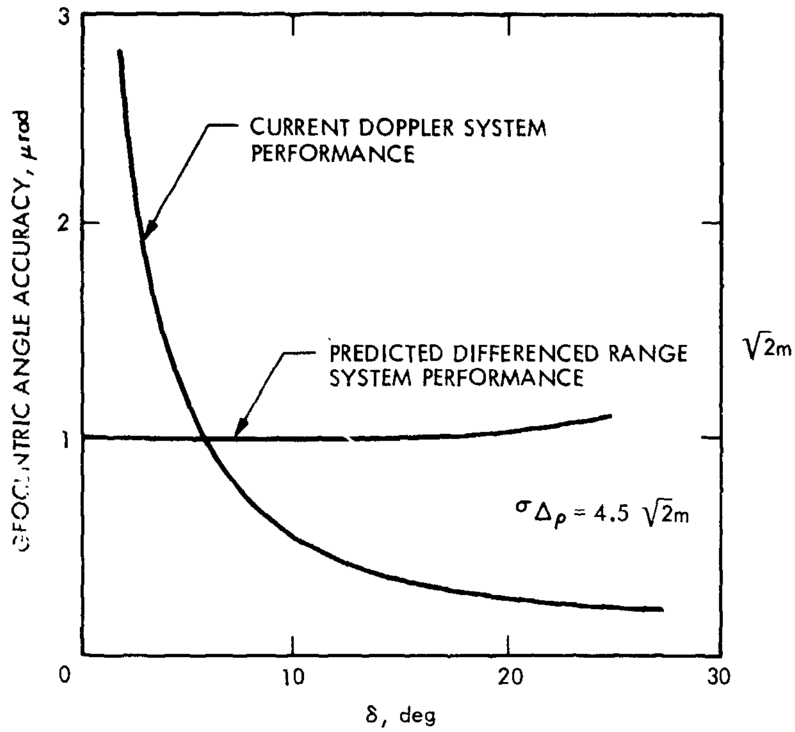

Figure 4. Accuracy Performance of Doppler and Ranging System versus Nominal Probe Declination.

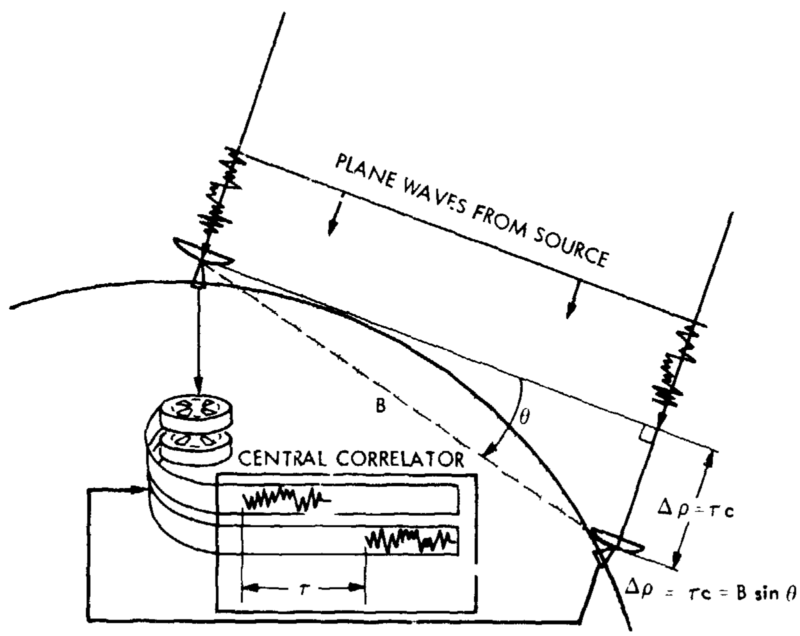

Figure 5. Wideband VLBI can Estimate Source Location by Measuring the Differential Time of Arrival.

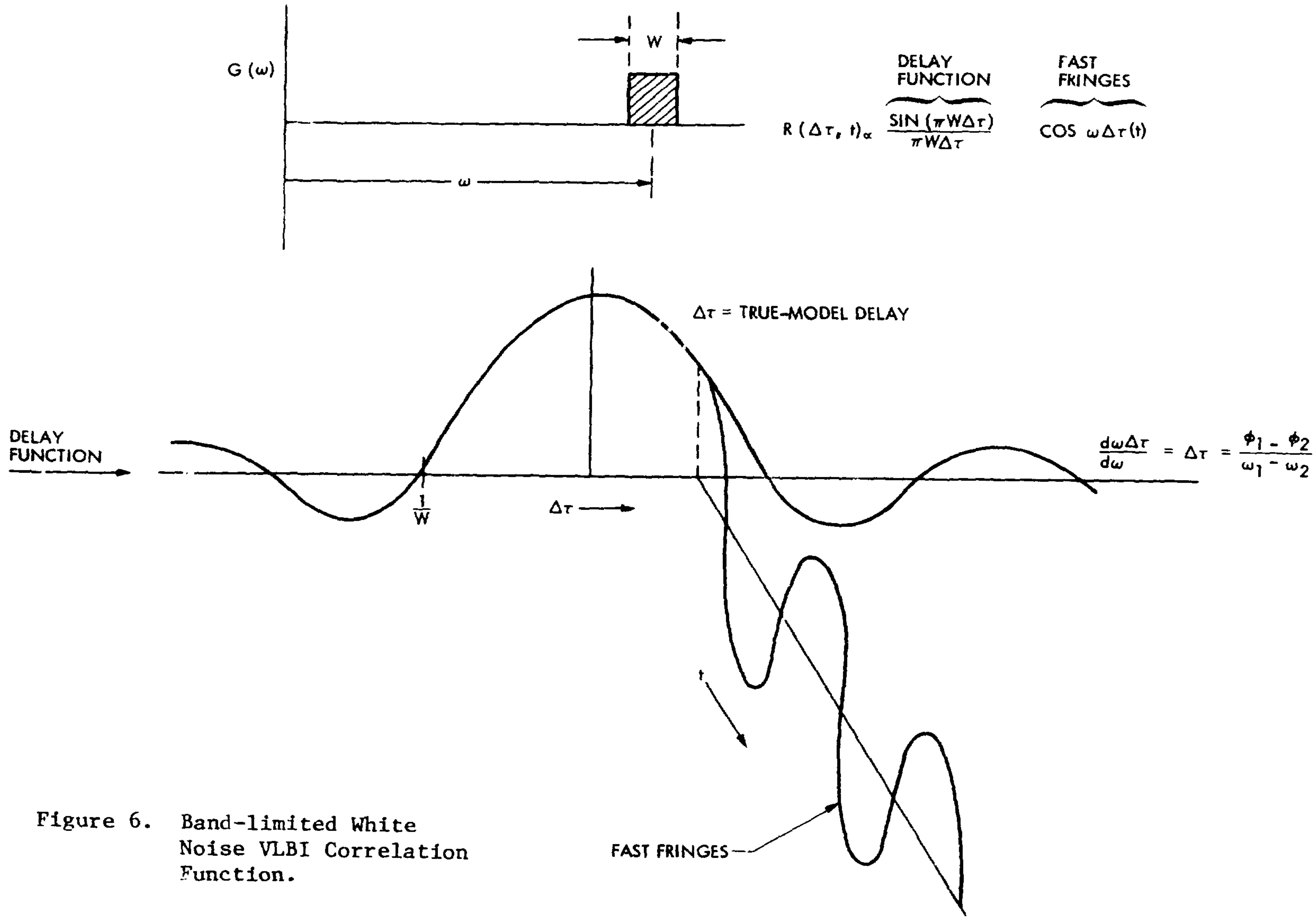

Figure 6. Band-limited White Noise VLBI Correlation Function.

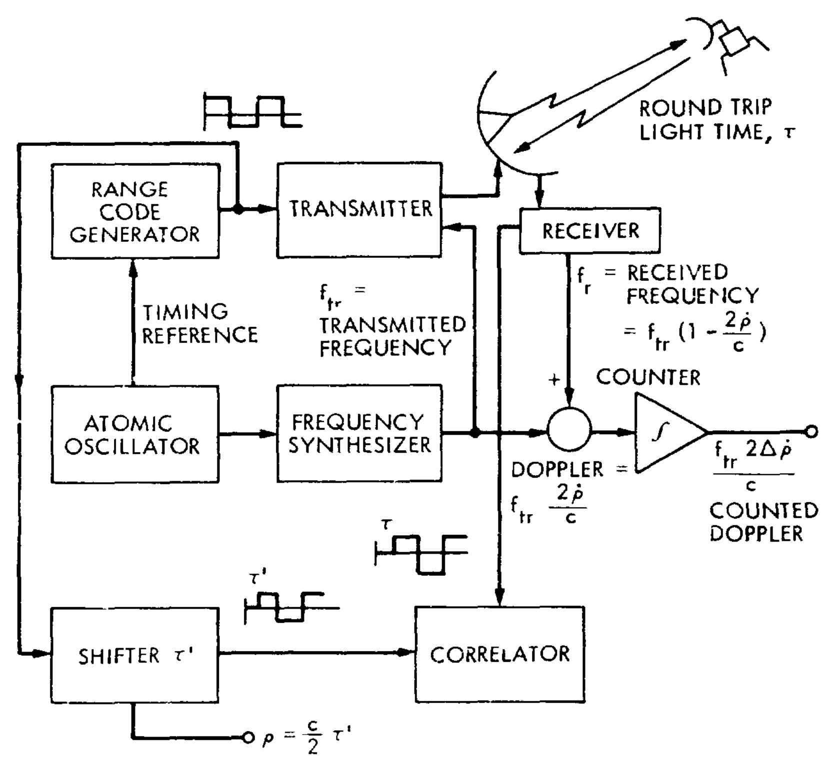

Figure 7. Coherent Doppler, Range Ground Tracking System

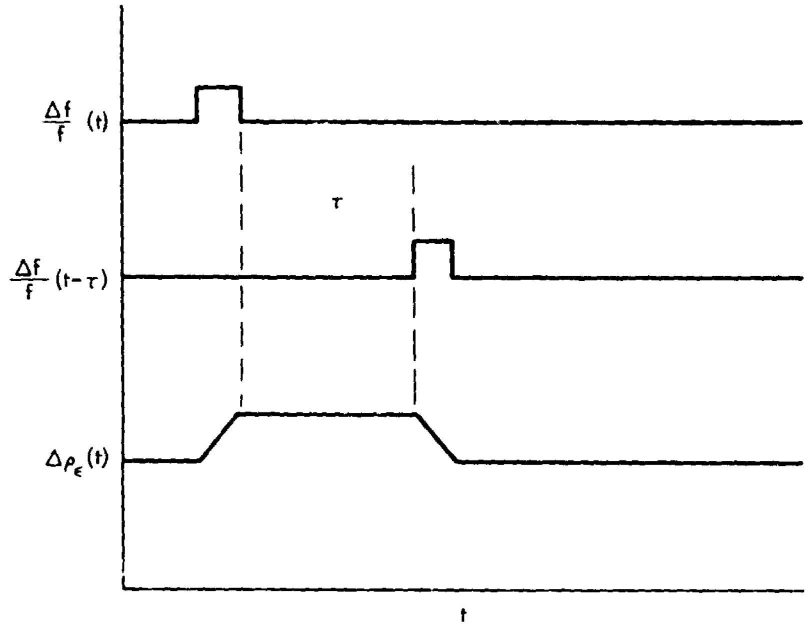

Figure 8. Counted Doppler Response, $\Delta\rho_{(-)}$ To a Unit Step of $\Delta f/f$.

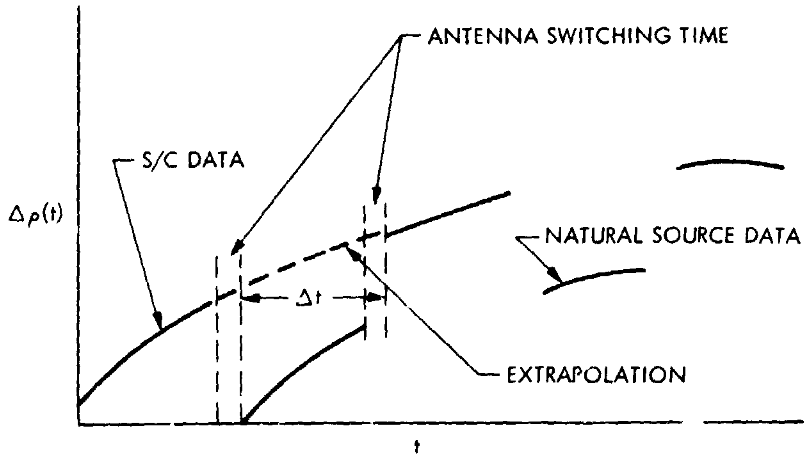

Figure 9. Narrowband ΔVLBI Phase Record Showing Alternate Data Gaps Due to Antenna Switching.

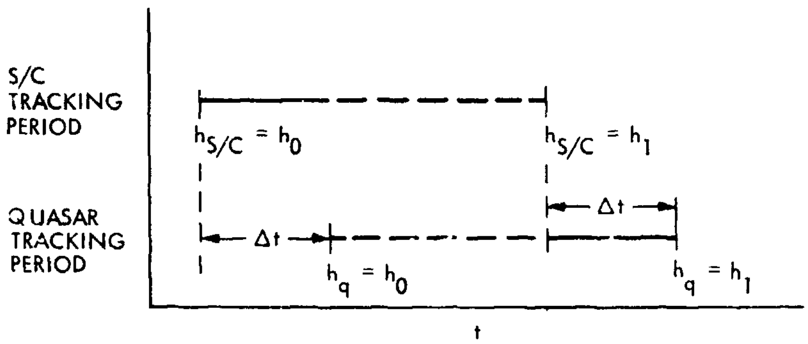

Figure 10. Narrowband ΔVLBI Tracking Scenario

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Questions and Answers

DR. TOM CLARK, NASA Goddard Space Flight Center:

I might update a couple of comments that were made in that one. I believe there has been one successful spacecraft-to-quasar delta VLBI experiment which was done by Shapiro et al. at MIT, and Newhall et al. in your shop, on the differential position of the Viking Orbiter, and the OJ-287 quasar. That was done about a year ago, and the data is still being processed, but it appears that it was successful.

I just wanted to stress one thing in terms of the angular measures that were mentioned here. The units, milliseconds of arc and so forth, were stressed a few times, and I just wanted to remind you what a millisecond of arc was.

If I took this quarter and gave it to Dave, and told him to take it back to Pasadena and tried to look at it from here, that is about a millisecond of arc. That is half a nanoradian.

I would point out that there is at least one pair of quasars which have been done to a few tens of picoradians in terms of differential positions by a delta VLBI type technique. They are only a degree apart on the sky, but it is the size, roughly, of George's eyeball on this thing, in the same analogy as before.

MR. CURKENDALL:

Yes. Not only has there been one experiment with the Viking spacecraft, there are something like 20, an unknown number of which are successful, sitting in our data hoppers.

By my remark, I meant I don't think anybody has been ever able to run an experiment where the accuracy had to be seven centimeters, and then be able to prove that it was. And I think that is accurate; it has probably never been done. And I think this is the same thing whether you can really wring that kind of performance out of the system or not.

DR. CARROLL ALLEY, University of Maryland:

Unfortunately, I arrived late for your talk. You may have mentioned this at the beginning. But it is worth pointing out, I think, that with this coherent tracking, and with a good transponder on these deep spacecraft, that there is a potential of measuring low frequency gravitational waves that may well be occurring in the universe. That is, the part in 10^{14} that you are striving for is about three orders of magnitude too insensitive, according to the current estimates of my friend Kip Thorne and other people.

Nevertheless, I would like to submit, as in an earlier discussion today, that we do not know everthing about the universe, and that there may well be gravitational radiation of higher amplitudes than is predicted, and I hope that you will be keeping a very careful watch, even at the part in 10^{14} level. The strain induced, the delta L over L in the distance between the spacecraft and the earth, is the same as the strength of the graviational radiation.

MR. CURKENDALL:

I understand.

DR. ALLEY:

And so there may just be some relic radiation of this amplitude around that would show up in such measurements.

MR. CURKENDALL:

Yes. I think your word "relic" is the key here. The thing I don't like about the Kip Thorne calculations is it goes something like this: If you assume the collapse of something like 10^8 solar masses, you will get a differential movement between the spacecraft and the earth.

And maybe this occurs something like once every 30 years. You will get a differential change in length between the earth and the spacecraft of about one and a half millimeters; and the problem of sitting around for 20 years waiting for that to happen is immense.

What seems more feasible to me is to look at what you have just said: Sit there and look for the background radiation. If you can put an X-band uplink on the spacecraft, and get masers down to a few parts in 10^{15}, the numbers already work out that you should be able to see enough gravitational energy that would close the universe. And that is much better than waiting around.

And the advantage of that experiment is if you go out and you look and you don't see anything, you can go home. And that is really important.

DR. VICTOR REINHARDT, NASA Goddard Space Flight Center:

Just one question and it is a partial comment: Is there any possibility of using telemetry information to increase the bandwidth and narrowband VLBI to give you an effective wideband?

MR. CURKENDALL:

The telemetry, right now, runs about 360 kilohertz subcarriers and at around 100 kilobits a second. You see, relative to the earth's satellites, where the signal and the noise are so much better, the telemetry rate coming back is not that high. And that is not very much bandwidth spreading.

We are doing that as part of this demonstration program. In fact, we have a mode where we leave the subcarrier on, but turn off the modulation so you get the nice pure sine waves, and you look at the harmonics of those sine waves.

It is just like the Goddard side-tone ranging system, in terms of being able to detect that.

But we detect it through the open loop RF chain, and calibrate with the quasar signals. And so, yes; you can do it as a restricted bandwidth sort of thing.

What we hope to put on Galileo is tones of plus-minus 20 megahertz on down, and that ought to do it.

MR. DAVID W. ALLAN, National Bureau of Standards:

I would think that this level of sensitivity, you would be sensitive to lunar crustal tidal movements of the mantle of the earth. This was not mentioned. I was just wondering if this is a problem. Is this not of the order of a meter or two? I am not sure.

MR. CURKENDALL:

Yes, they are; and I think our Chairman can give better words on that. I think the bottom line is that you have to model it; but it is thought to be modelable.

DR. TOM CLARK, NASA Goddard Space Flight Center:

Yes, there are a number of these little subtle, insidious effects that affect the stations on the earth. The particular one with tides is not as bad as many of the others because the tides have a semi-diurnal signature, and it is very easy to pull semi-diurnal signatures out, because all of the things that Dave was talking about were diurnal signatures. It is those diurnal ones that are much more insidious and give you a lot more trouble; things like diurnal polar motion, for example.

MR. CURKENDALL:

Do you know what you are most sensitive to? I once did a spectrum analysis, and for reasons I have never quite understood, you are really most sensitive to twice diurnal rates. With a given amount of power, you are more sensitive to something with a 12-hour period than a 24-hour period. I have never quite understood that, physically.