WING SHAPE OPTIMIZATION FOR MAXIMUM CROSS-COUNTRY SPEED, WITH MATHEMATICAL PROGRAMMING

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SUMMARY

A computer program was developed to calculate numerically the speed and circling polars of an aircraft when the lift and drag characteristics of the wing airfoils are known. The planform of the wing is described by variables which are optimized so that the cross-country speed of the glider is maximum for the particular type of thermal model. Two thermal models will be compared and it can be shown that with a greater wing area than now normally used the performance can be increased.

SYMBOLS

b	half span
c_d	section drag coefficient
c_l	section lift coefficient
C_D	total drag coefficient of the aircraft
C_L	total lift coefficient of the aircraft
C_{D_i}	induced drag coefficient
C_{Do}	parasite drag coefficient
g	acceleration of gravity
H	height
r	radius of thermal
S	area
t	time
v_c	net climb rate of the glider
v_s	sink rate for the speed polar

v_{sc}	sink rate for the circling polar
$v_{thermal}$	vertical velocity of thermal
V	airspeed
V_R	average cross-country speed
V_{R_i}	cross-country speed for the i-th thermal
W	total weight
Z	object function
x_i	variables
α_i	weighting factors
ρ	air density
ϕ	sweep angle
λ	aspect ratio
ψ	bank angle

INTRODUCTION

The most important part of an airplane is a well designed wing. Only a few variables are necessary to describe the planform (fig. 1). The halfspan b and the sweep-angle ϕ are fixed. The airfoils with their lift and drag characteristics must be prescribed. The total lift of the airplane is

$$C_L = C_{L_{wing}} + C_{L_{tail}} \frac{S_{tail}}{S} \tag{1}$$

The C_L is calculated either by Multhopp's method (ref. 1) or, for swept wings, by Truckenbrodt's method (ref. 2). The tail lift is in general so small that it can be neglected. From the lift calculation the induced drag C_{D_i} (ref. 1) is obtained, too. The total drag is

$$C_D = C_{D_{profile}} + C_{D_i} + C_{Do} \tag{2}$$

$$C_{Do} = \frac{1}{S} \sum_{p} C_{Do_p} S_p \quad (p = 1,2..P) \tag{3}$$

C_{Do_p} is the drag coefficient of the p-th part, with surface area S_p. The profile drag coefficient $C_{D_{profile}}$ can be calculated from the measured Reynolds-number-dependent c_d-versus-c_l plots (fig. 2).

$$C_{D_{profile}} = \frac{1}{S} \sum_{n} c_{d_n} S_n \quad (n = 1,2..N) \tag{4}$$

From fig. 2 (measured data), fig. 3 can be determined, and by linear interpolation, the local c_{d_n} at the strip n with surface S_n is obtained. The aircraft equivalent parasite area $\sum_{p} C_{Do_p} S_p$ is assumed to be constant for a given aircraft.

SPEED AND CIRCLING POLARS

With the weight as the fifth variable (x_5 = weight), the principal performance characteristic of the glider, which is its cross-country speed, can be calculated. The speed polar is given by the equations

$$V = \sqrt{\frac{2}{\rho} \cdot \frac{W}{S} \cdot \frac{1}{C_L}} ; \quad v_s = \frac{C_D}{C_L^{3/2}} \sqrt{\frac{2}{\rho} \cdot \frac{W}{S}} \tag{5,6}$$

The circling polar is obtained from

$$v_{sc} = \frac{C_D}{C_L^{3/2} \cdot \cos^{3/2}\psi} \sqrt{\frac{2}{\rho} \cdot \frac{W}{S}} \tag{7}$$

$$\cos^{3/2}\psi = \left[1 - \left(\frac{2\ W/S}{\rho C_L\ r\ g}\right)^2\right]^{3/4} \tag{8}$$

CROSS-COUNTRY SPEED

The net rate of climb of the glider in the thermal (fig. 4) is

$$v_c(r) = v_{thermal}(r) - v_{sc}(r) \tag{9}$$

The maximum climb rate must be calculated as a function of the diameter of the thermal. With these two polars (speed and circling) the cross-country speed can easily be determined (fig. 5),

$$V_R = \frac{D}{t} \tag{10}$$

(D = distance; t = time from point A to C). The loss in height is

$$H = t_1 v_s \tag{11}$$

The gain in height is

$$H = t_2 \cdot v_c \tag{12}$$

Therefore,

$$t_2 = t_1 \frac{v_s}{v_c} \tag{13}$$

and with the time t_1 to go from point B to C, and the total time t, respectively,

$$t_1 = \frac{D}{V}\ ; \tag{14}$$

$$t = t_1 + t_2 \tag{15}$$

the cross-country speed by using the equations (13) to (15) and (10) is finally

$$V_R = \frac{V \cdot v_c}{v_c + v_s} \tag{16}$$

THE OBJECT FUNCTION

The program can be used with different types of thermal models. Two examples are given in fig. 6 and fig. 7 (ref. 3,4). As object function for the optimization, an average cross-country speed is defined for each thermal model

$$Z = \sum_i \alpha_i V_{R_i} \qquad \begin{array}{l}(i = 1,2,3 \text{ for Carmichael}) \\ (i = 1,2,..4 \text{ for Horstmann})\end{array} \tag{17}$$

$$\sum_i \alpha_i = 1 \text{ and } \quad 1 \geq \alpha_i \geq 0 \tag{18}$$

The V_{R_i}'s are the cross-country speeds from the i-th thermal and α_i is its weighting function. The α_i's must be chosen by the designer and then the optimization is achieved for the particular distribution of thermals assumed.

THE OPTIMIZATION PROCEDURE

The maximum cross-country speed is calculated by a penalty optimization program (ref. 5,6), so that constraints like $x_i \geq a_i$ $(i = 1,2,..5)$ (a_i = lower bound) or others can be observed. Using the performance polars from equations (1) and (2), two sub-optimization problems are solved: (a) determination of max v_c by using C_L and r as variables in equations (7) to (9), and (b) determination of max V_R (eq. 16) by using C_L as a variable through equations (5) and (6), with max v_c fixed. V_{R_i} has to be calculated for each thermal (index i). Then Z (eq. 17) can be calculated and optimized.

RESULTS

Only gliders with a span of 15m = 2b and $\phi = 0°$ were optimized. In diagrams 1 and 2 an optimization is shown with the profile FX-61-184 (ref. 7) for two thermal models with different sets of α_i's. For a convenient representation (this is only one possibility), $\alpha_1 = \alpha_2$ was used for the Carmichael thermals and $\alpha_1 = \alpha_2$ and $\alpha_3 = \alpha_4$ for the Horstmann thermals. For comparison, the performance of the D-38, an <u>almost</u> optimized competition glider in the 15m class from the Akaflieg Darmstadt (ref. 8), is shown in both diagrams. The curves were calculated by changing only the weight of the glider. The difference between the two curves gives the gain in performance over the D-38. On the average, the gain in performance is more than 2% for both thermal models. The Carmichael thermals deliver a higher cross-country speed because it is narrow and strong. This leads to a higher optimal wing area for Carmichael thermals compared to the Horstmann thermals (see table 1 and 2). In general, the optimized wing area is significantly larger than the normally used 11m^2 area for rigid 15m wings.

In diagram 3 the wing loading and the aspect ratio are plotted versus the variable sets of α_i's for both thermal models, using the numbers from tables 1 and 2. The difference in the aspect ratio, $\Delta\lambda$, is about 1 and therefore the wing area difference, ΔS, is about $1m^2$. The average wing area is $14.4m^2$ for the Carmichael and $13.5m^2$ for the Horstmann thermals. Clearly the influence of the shape of the wing planform is small. The aspect ratio is almost constant from weak to strong thermals. This is true for both thermal models. The wing loading is the dominant factor influencing the cross-country speed. The dotted line (diagram 2) which was obtained by changing the weight, using optimal design for all α_i's equal, shows the importance of the weight (wing loading) but a relatively small influence of the wing shape. The proper wing loading, again, is more important for the Horstmann model (greater gradient) than for the Carmichael model for maximum cross-country speed.

Because of the dominating rule of the weight for the gliders, the curves in diagram 4 were calculated for various masses by using the optimal design with all α_i's equal. The optimal variables are given in table 3. The comparison between the 7 profiles shows that the best rigid profile is the FX-61-184, and the best profile with flaps is the FX-K-170

CONCLUSION

An increase of performance of more than 2% is possible with an increase in wing area. It is not necessary to design a glider for extreme (strong and weak) thermal conditions. The use of average weather (all α_i's equal) for the optimization of a glider results in an almost optimal design for all weather conditions. But the glider must be built as light as possible and should be able to carry up to 150 kg water ballast. The thermal model (Carmichael, Horstmann, or perhaps others) plays a minor part in the design but is extremely important for the proper choice of the water ballast to maintain maximum performance. Here it matters whether to believe in the Carmichael, Horstmann, or other thermal models. The program can then be used for a single variable optimization (the weight) to calculate the optimal water ballast curves for any type of weather condition.

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Table 1: Optimal Variables for 15m Gliders with the Profile FX-61-184 (Carmichael thermals, $C_{Do}S = 0.04m^2$)

OPTIMAL VARIABLES	OPTIMAL 15m GLIDERS					D-38
	$\alpha_1=\alpha_2=0$ $\alpha_3=1$	$\alpha_1=\alpha_2=0.2$ $\alpha_3=0.6$	$\alpha_1=\alpha_2=\alpha_3$	$\alpha_1=\alpha_2=0.45$ $\alpha_3=0.1$	$\alpha_1=\alpha_2=0.5$ $\alpha_3=0$	
x_1 (m)	1.31	1.19	1.23	1.14	1.19	0.94
x_2 (m)	1.11	1.07	1.09	1.04	1.02	0.753
x_3 (m)	0.49	0.47	0.51	0.45	0.45	0.376
x_4 (m)	3.78	4.17	3.87	3.62	4.25	4.5
x_5 (kg)	249	289	327	360	381	300
DERIVED VALUES						
S (m^2)	14.89	14.6	14.76	13.65	14.2	11
λ	15.11	15.4	15.22	16.48	15.85	20.45
max C_L/C_D	34.3	34.8	34.8	35.7	35.5	37

Table 2: Optimal Variables for 15m Gliders with the Profile FX-61-184 (Horstmann thermals, $C_{Do}S = 0.04m^2$)

OPTIMAL VARIABLES	OPTIMAL 15m GLIDERS				D-38
	$\alpha_1=\alpha_2=0$ $\alpha_3=\alpha_4=0.5$	$\alpha_1=\alpha_2=0.25$ $\alpha_3=\alpha_4=0.25$	$\alpha_1=\alpha_2=0.4$ $\alpha_3=\alpha_4=0.1$	$\alpha_1=\alpha_2=0.5$ $\alpha_3=\alpha_4=0$	
x_1 (m)	1.32	1.19	1.149	0.999	0.94
x_2 (m)	0.99	0.884	1.012	0.995	0.753
x_3 (m)	0.507	0.384	0.394	0.389	0.376
x_4 (m)	4.05	4.48	4.28	3.679	4.5
X_5 (kg)	254	328	390	432	
DERIVED VALUES					
S (m^2)	14.53	13.2	13.5	12.64	11
λ	15.48	17.04	16.67	17.8	20.45
max C_L/C_D	34.6	35.9	36.0	36.7	37

Table 3: Optimal 15m Gliders with Different Profiles
(Horstmann thermals, $C_{Do}S = 0.04m^2$)

	$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4$ Profile						
OPTIMAL VARIABLES	FX-60-126	FX-61-163	FX-66-S-196V1	FX-61-184	FX-K-131	FX-K-150	FX-K-170
x_1 (m)	0.94	1.07	0.98	1.19	1.18	1.03	1.077
x_2 (m)	0.76	0.927	0.919	0.884	0.99	0.866	0.911
x_3 (m)	0.379	0.395	0.37	0.384	0.38	0.39	0.37
x_4 (m)	4.37	4.08	3.37	4.48	4.9	4.26	4.14
x_5 (kg)	290	311	316	328	348	344	367
DERIVED VALUES							
S (m^2)	10.76	12.8	11.52	13.2	14.08	12.1	12.74
λ	20.9	17.58	19.53	17.04	15.98	18.6	17.66
max C_L/C_D	36.4	35.7	34.3	35.9	38.5	37.6	37.1

x_1 = chord at the root
x_2 = chord at the break
x_3 = chord at the tip
x_4 = span to the break

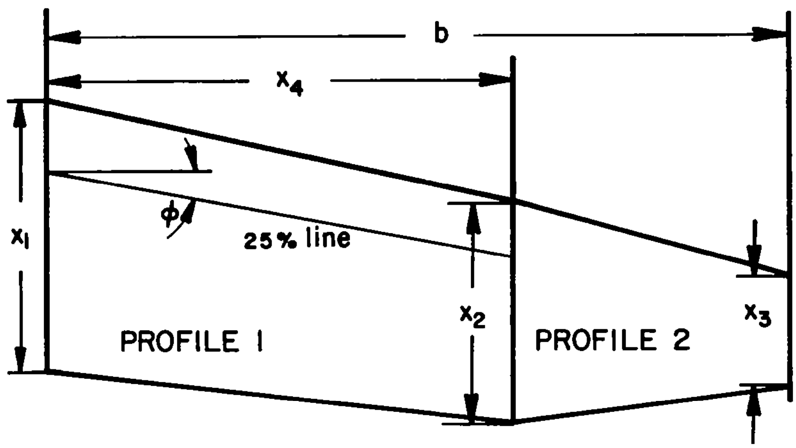

Fig. 1: Wing planform

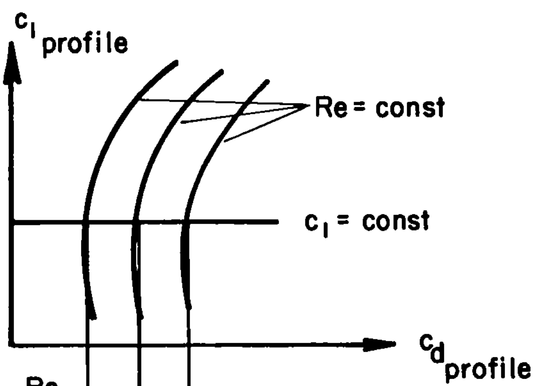

Fig. 2: Reynolds-number-dependent lift and drag characteristics

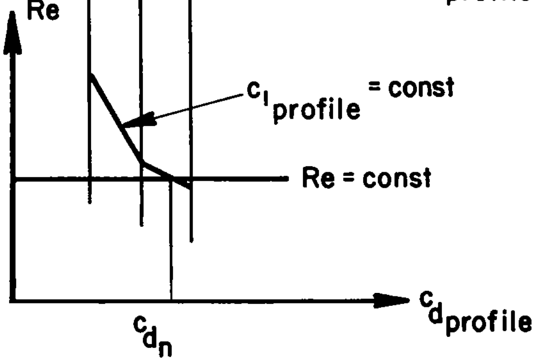

Fig. 3: Reynolds-number-dependent $c_{d_{profile}}$ for c_l=constant

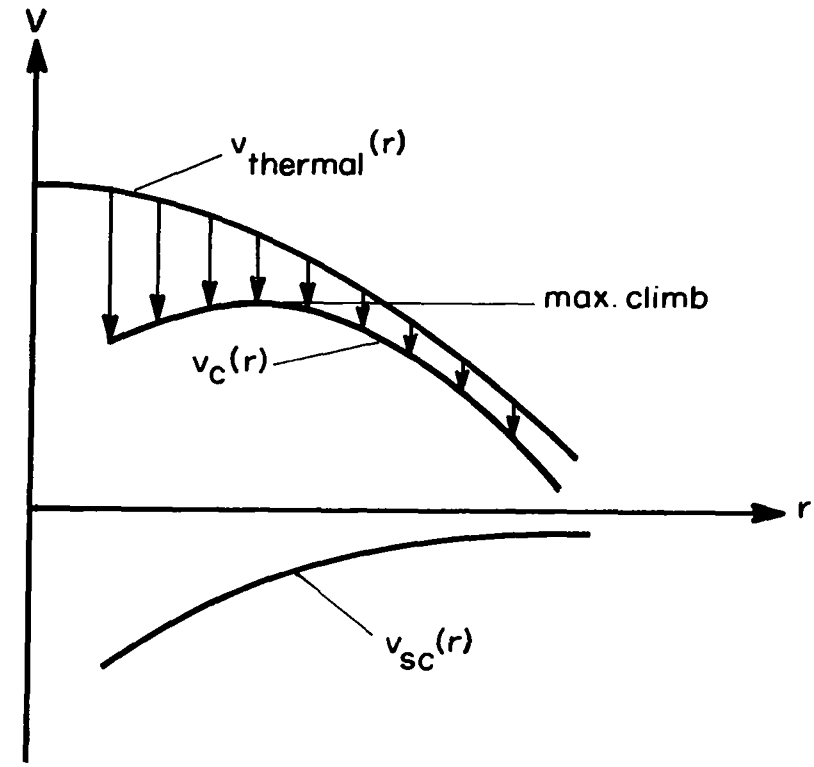

Fig. 4: Circling Climb Rate

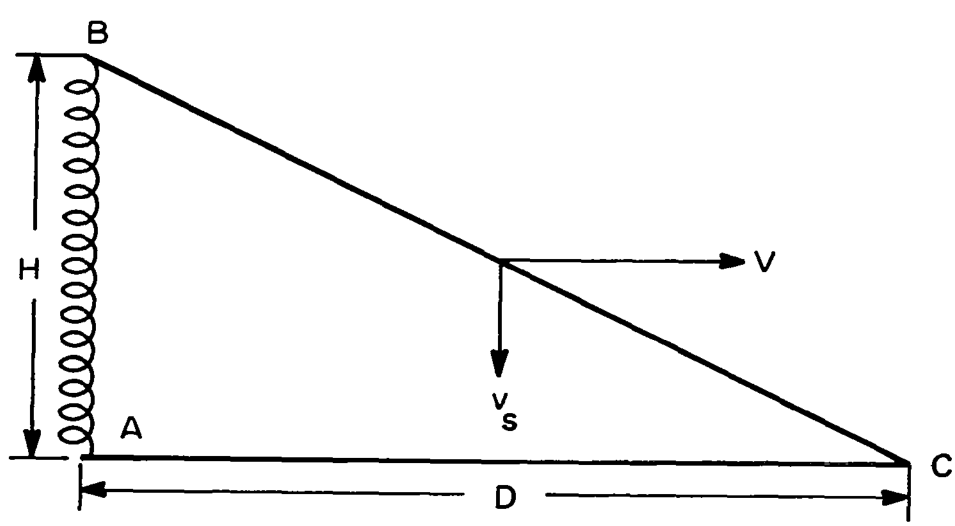

Fig. 5: Typical Flight Section

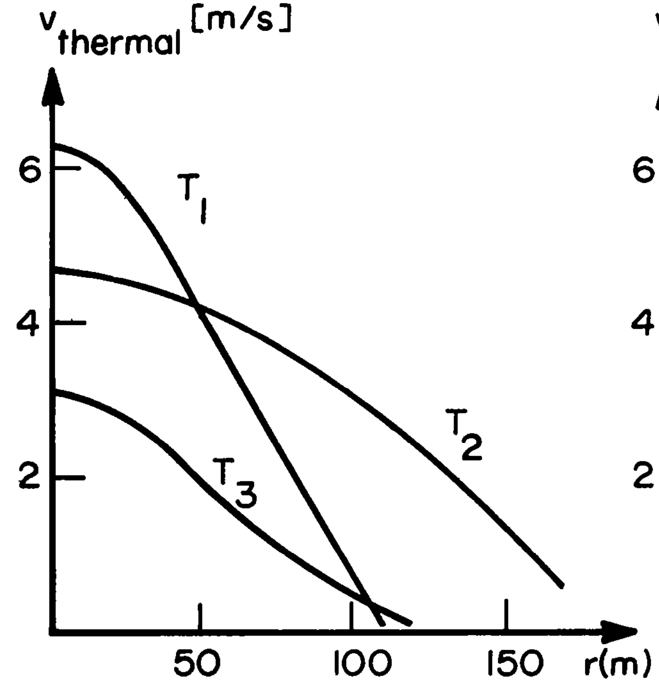

Fig. 6: Carmichael Thermals

T_1 = strong

T_2 = wide

T_3 = weak

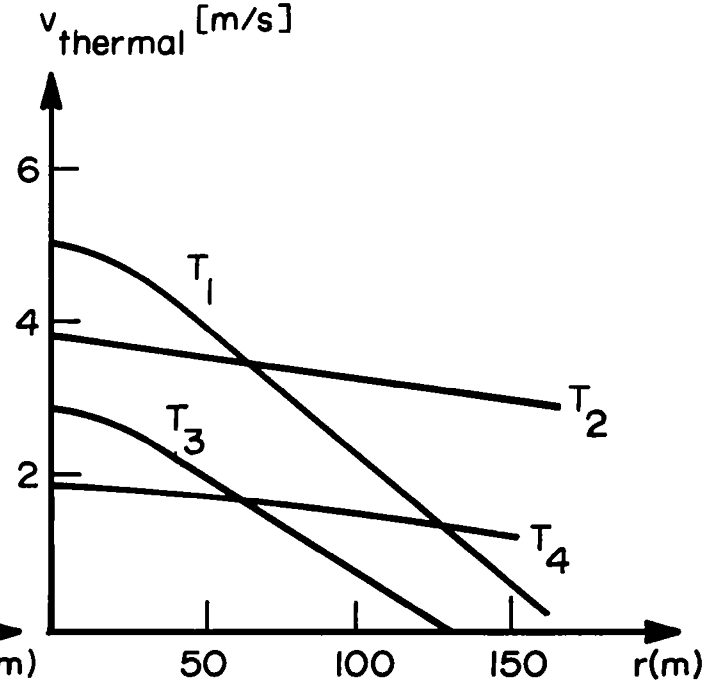

Fig. 7: Horstmann Thermals

T_1 = narrow, strong

T_2 = wide, strong

T_3 = narrow, weak

T_4 = wide, weak

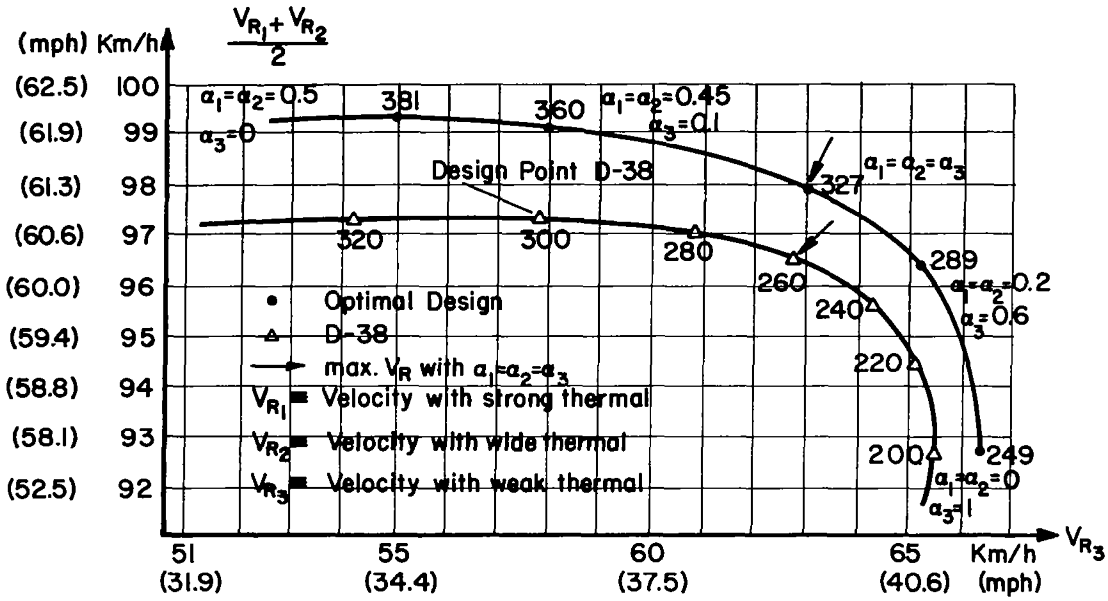

Diagram 1: Optimal performance of a 15m glider with the profile FX-61-184 for the Carmichael thermals. The numbers on the curves are total weights of the gliders in kg. ($C_{Do}S = 0.04m^2 = \text{const.}$)

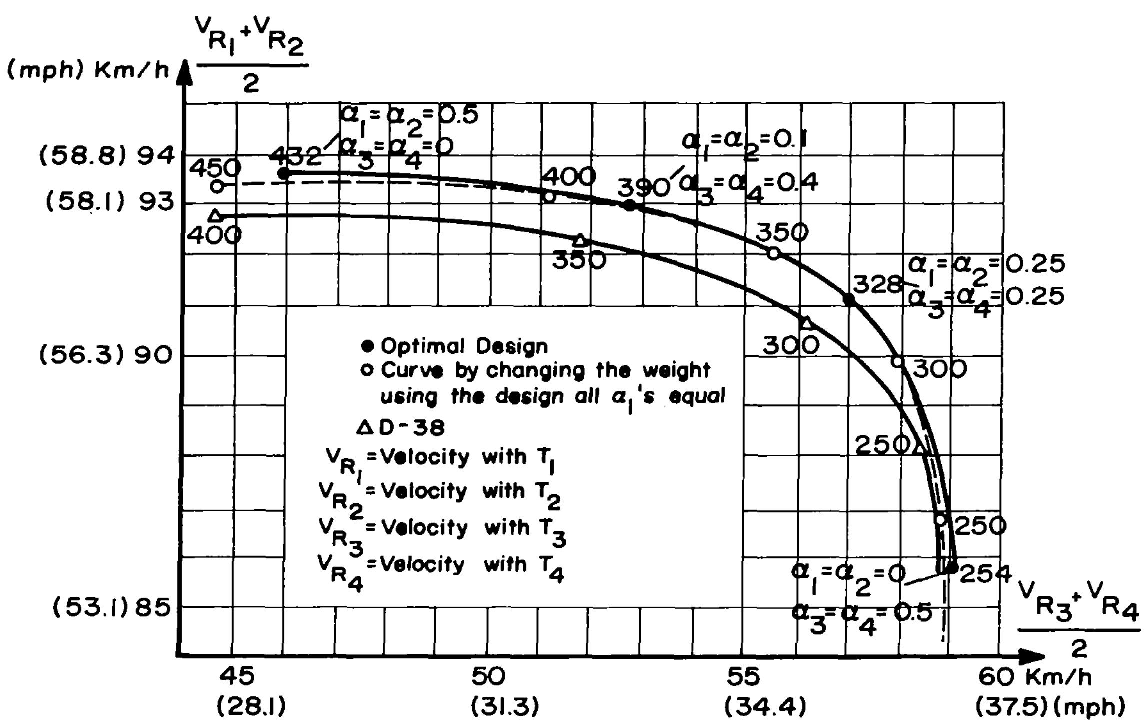

Diagram 2: Optimal performance of a 15m glider with the profile FX-61-184 for the Horstmann thermals. The numbers on the curves are total weights for the gliders in kg. ($C_{Do}S = 0.04m^2 = \text{const.}$)

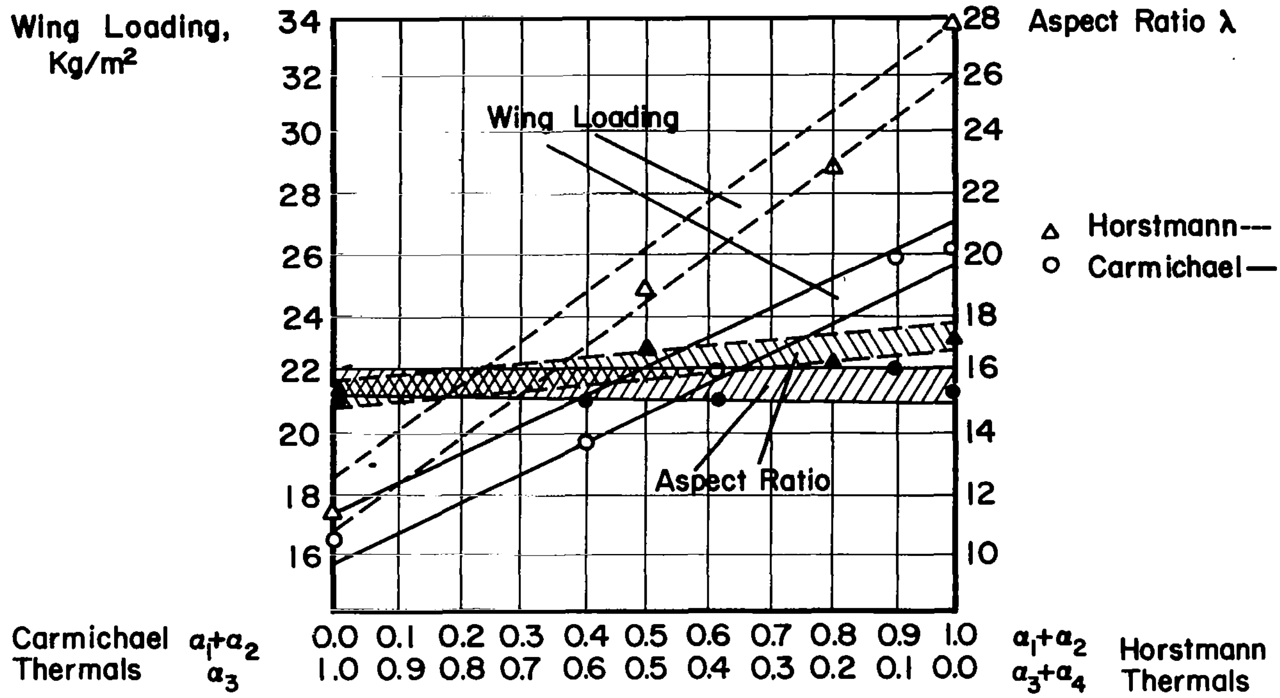

Diagram 3: Comparison Between Carmichael and Horstmann Thermals.

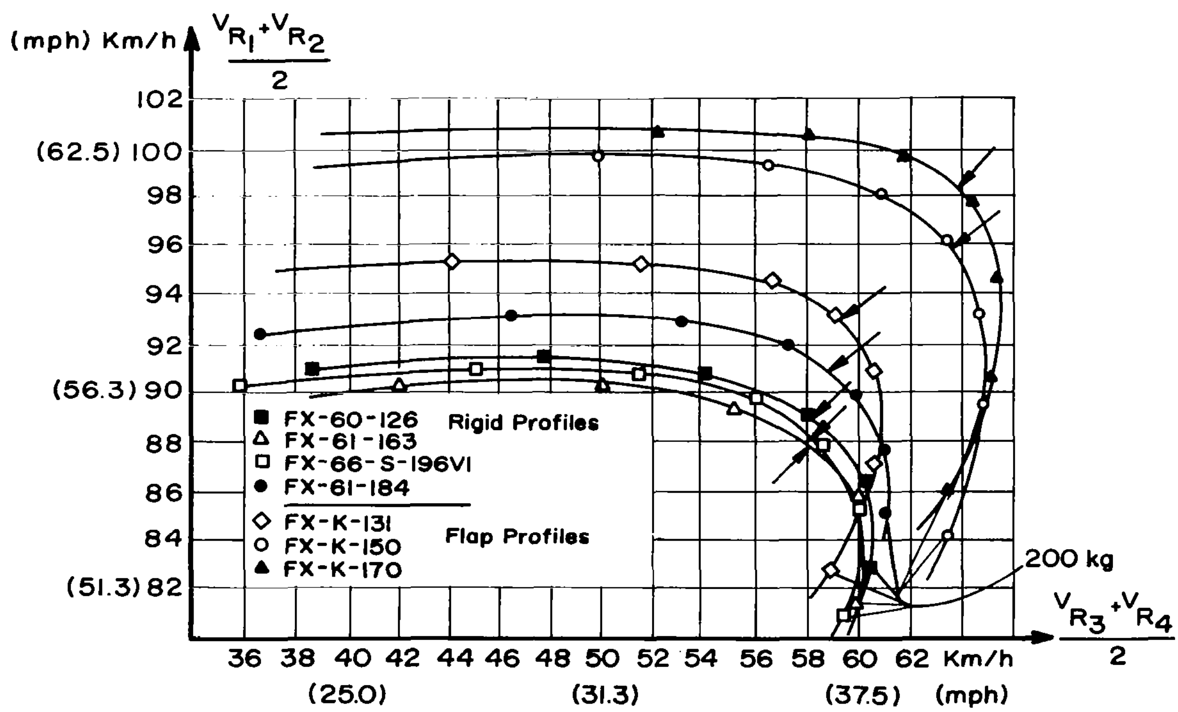

Diagram 4: Performance of 15m gliders using different profiles for the Horstmann thermals. Curves are calculated by using the optimal design (design point) with all α_i's equal. The change of the total mass from point to point is 50 Kg ($C_{Do}S = 0.04m^2$)