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A SLAB MODEL FOR COMPUTING GROUND TEMPERATURE IN CLIMATE MODELS

Sergej Lebedeff, Gary Crane and Gary Russell, *Goddard Institute for Space Studies, New York, NY 10025*

ABSTRACT

A method is developed for computing the ground temperature accurately over both the diurnal and annual cycles. The ground is divided vertically into only two or three slabs, resulting in very efficient computation. Seasonal storage and release of heat is incorporated, and thus the method is well suited for use in climate models.

Introduction. The ground temperature should be computed accurately over both the diurnal cycle and annual cycle in a climate model. The diurnal and seasonal cycles are important because of the highly non-linear way in which latent and sensible heat fluxes from the surface depend upon temperature. Seasonal storage and release of heat is also important because it can amount to several watts per square meter and can significantly influence the ground temperature (Taylor, 1976).

The basic problem is to solve the one-dimensional heat conduction equation

$$\frac{\partial T}{\partial t} = \frac{\lambda}{c_g} \frac{\partial^2 T}{\partial z^2} = K_g \frac{\partial^2 T}{\partial z^2} , \qquad (1)$$

where c_g is the heat capacity of the ground per unit volume, λ is the thermal conductivity and K_g is the thermal diffusivity, subject to numerically specified heat flux at the upper boundary:

$$F(0) = F_{sw} - F_{lw} - F_h - F_q . \qquad (2)$$

F_{sw} is the absorbed solar radiation, F_{lw} the net thermal radiation, and F_h and F_q are the sensible and latent heat fluxes from the ground to the atmosphere.

A straightforward finite difference approach with computation of temperatures at several subsurface depths can yield accurate results. However, that method has the disadvantage of requiring (1) computation and storage of temperatures at several depths, and (2) a short time step, of the order of a few minutes or less.

Slab model. An approach requiring less resources, is to divide the ground into a small number of slabs and solve for the mean temperature of each slab. In this method we use the average temperature of the upper slab to simulate the surface temperature. That layer must be sufficiently thin that its diurnal temperature response is representative of that for the upper skin of the ground; on the other hand, it should be thick enough to allow long time steps. The minimum number of ground layers that are needed for a climate model is two, since a deep lower layer is needed for seasonal heat storage.

The equations for our slab model are derived under the assumption that the heat capacity and conductivity are uniform in each slab, and with the temperature profile within each slab approximated as a quadratic function of depth:

$$T_i(z) = \alpha_i z^2 + \beta_i z + \gamma_i. \tag{3}$$

For two layers we obtain the six parameters from the six equations:

$$\lambda_1 \left.\frac{\partial T_1}{\partial z}\right|_0 = F(0), \quad \frac{1}{z_1}\int_{-z_1}^{0} T_1 dz = \bar{T}_1, \quad T_1(-z_1) = T_2(-z_1)$$

$$\lambda_1 \left.\frac{\partial T_1}{\partial z}\right|_{-z_1} = \lambda_2 \left.\frac{\partial T_2}{\partial z}\right|_{-z_1}, \quad \frac{1}{z_2}\int_{-z_1-z_2}^{-z_1} T_2 dz = \bar{T}_2, \quad \lambda_2 \left.\frac{\partial T_2}{\partial z}\right|_{-z_1-z_2} = 0. \tag{4}$$

Thus we require that no heat cross the $z = -z_1 - z_2$ interface. The heat crossing the $z = -z_1$ interface is:

$$F(-z_1) = \frac{3\bar{T}_1 - 3\bar{T}_2 - \frac{1}{2}F(0)z_1/\lambda_1}{z_1/\lambda_1 + z_2/\lambda_2}. \tag{5}$$

The final equations for the mean temperatures of the two slabs are:

$$z_1 c_1 \frac{d\bar{T}_1}{dt} = F(0) - F(-z_1), \tag{6}$$

$$z_2 c_2 \frac{d\bar{T}_2}{dt} = F(-z_1). \tag{7}$$

We use a similar formulation for ocean ice but the no flux condition at $z = -z_1 - z_2$ is replaced by:

$$T_2(-z_1 - z_2) = T_{fo}. \tag{8}$$

T_{fo}, the freezing point for ocean, is taken as 271.6 K. The heat crossing the $z = -z_1$ interface is

$$F(-z_1) = \frac{12\bar{T}_1 - 18\bar{T}_2 + 6T_{fo} - 2F(0)z_1/\lambda_1}{4z_1/\lambda_1 + 3z_2/\lambda_2}, \tag{9}$$

and the heat crossing the ice-ocean interface:

$$F(-z_1 - z_2) = 3\lambda_2 \frac{\overline{T}_2 - T_{fo}}{z_2} - \frac{F(-z_1)}{2} . \tag{10}$$

The final equations for the two layer model for ocean ice are:

$$z_1 c_1 \frac{d\overline{T}_1}{dt} = F(0) - F(-z_1) , \tag{11}$$

$$z_2 c_2 \frac{d\overline{T}_2}{dt} = F(-z_1) - F(-z_1 - z_2) . \tag{12}$$

The equations for our three layer model are determined in a similar way.

The thicknesses of the layers were determined empirically by comparison with an exact periodic solution and an accurate 21 layer finite difference solution. For $K_g = 4 \times 10^{-7} m^2 s^{-1}$ the optimum depths for two layers were found to be $z_1 \sim 10$ cm, $z_2 \sim 4$ m, while for three layers $z_1 \sim 5$ cm, $z_2 \sim 25$ cm, $z_3 \sim 4$m. For other values of K_g these depths should be scaled in proportion to the square root of K_g. Note that for the particular thermal diffusivity which we employ the damping depth, at which the amplitude of the temperature variation is reduced by e^{-1}(cf. Sellers, 1972), is ~ 10 cm for the diurnal temperature wave and ~ 2 m for the annual temperature wave.

To illustrate the errors in the two and three layer models, we show the results of computations for the case $K_g = 4 \times 10^{-7} m^2 s^{-1}$ (the same as case 1 of Deardorff, 1978). For these tests we considered the simple case of no atmosphere above the ground ($F_h = F_q = 0$), with

$$F_{sw} = (1 - A_g) \; S_o \cos\theta_o , \tag{13}$$

$$F_{lw} = \varepsilon \sigma T_1^4 , \tag{14}$$

ground albedo $A_g = 0.24$, solar constant 1.354×10^3 W m^{-2} and ground emissivity $\varepsilon = 0.9$; θ_o is the solar zenith angle and σ the Stefan-Boltzmann constant. This idealized case yields unrealistic temperature variations at the surface, but it simplifies the computations and their interpretation.

Fig. 1 compares the diurnal and annual surface temperature variations computed with several different models. The computations are for the simplified case of no atmosphere described above. Our two layer and three layer results are about equally accurate for the annual temperature variation, as expected since both models use one deep layer for annual heat storage. Three layers are clearly needed if it is desired to represent both the amplitude and phase of the diurnal ground temperature variations, as well as the seasonal heat storage. We emphasize that the Arakawa (1972) and Deardorff(1978) schemes were not designed for use in climate models, and we have included them only to illustrate the necess-

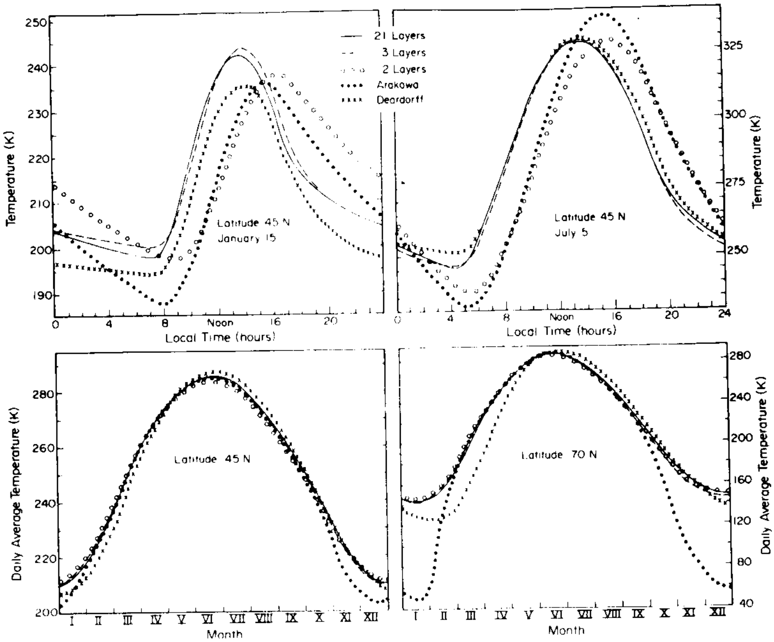

Fig. 1 Diurnal and annual surface temperature variations computed with several different models for the simple case of no atmosphere and thermal diffusivity $K_g = 4 \times 10^{-7} m^2 s^{-1}$. The Arakawa (1972) and Deardorff (1978) schemes were not designed for use in climate models, and are included here only to illustrate the necessity of having a deep layer for annual heat storage.

ity of having a deep layer for annual heat storage. Arakawa's scheme has one layer which is used to represent the diurnal temperature variation. Deardorff's method has two layers, with the lower layer damping the upper layer so as to give a realistic phase to the diurnal temperature variation.

Fig. 2 compares the diurnally averaged heat flux into the ground for the same models as in Fig. 1. The Arakawa (1972) model has a negligible diurnally averaged flux, since it consists of a single thin layer with a zero flux lower boundary condition. Although the two layers in the Deardorff (1978) model are thin, there can be a substantial diurnally averaged heat flux into (or out of) the ground because the method does not employ a zero flux lower boundary condition.

Discussion. The ground temperature model we have presented permits efficient and accurate computation of both diurnal and seasonal temperature variations with a minimum number of layers. It is therefore well suited for use in global climate models.

The diurnally averaged heat flux into the ground is usually small in comparison to typical radiative, latent and sensible heat fluxes, even

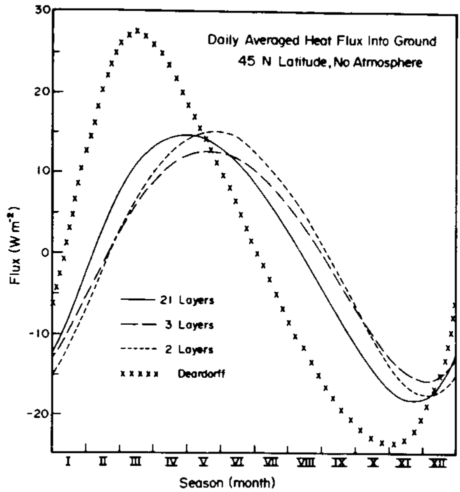

Fig. 2 Diurnally averaged heat flux into the ground for the same models as in Fig. 1. The calculations are for the simple case of no atmosphere at 45 N latitude, as described in the text. Arakawa's model, since it is a single thin layer with zero flux lower boundary condition, has negligible diurnally averaged heat flux.

for this idealized case in which the moderating effects of the atmosphere are excluded. Thus a large impact of seasonal heat storage on short time scales should not be expected, but the cumulative effect on seasonal time scales may be significant. Specific tests of the impact will be included in the three-dimensional climate model described elsewhere in these proceedings. A previous test has been made in a general circulation model by Delsol et al. (1971), who used a ground thickness of 5 m with a lower boundary condition of T=280 K; however, they ran the model only 14 days which is too short for an adequate test.

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