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**CORRELATED K-DISTRIBUTION METHOD FOR RADIATIVE TRANSFER IN CLIMATE MODELS: APPLICATION TO EFFECT OF CIRRUS CLOUDS ON CLIMATE**Andrew A. Lacis, Wei-Chyung Wang and James E. Hansen, *Goddard Institute for Space Studies, New York, NY 10025***ABSTRACT**

A radiative transfer method appropriate for use in simple climate models and 3-D global climate models has been developed. It is fully interactive with climate changes, such as in the temperature-pressure profile, cloud distribution and atmospheric composition, and it is accurate throughout the troposphere and stratosphere. The vertical inhomogeneity of the atmosphere is accounted for by assuming a correlation of gaseous k-distributions of different pressures and temperatures. Line-by-line calculations are made to demonstrate that the method is remarkably accurate. The method is then used in a 1-D radiative-convective climate model to study the effect of cirrus clouds on surface temperature. It is shown that an increase in cirrus cloud cover can cause a significant warming of the troposphere and the earth's surface, by the mechanism of an enhanced greenhouse effect. The dependence of this phenomenon on cloud optical thickness, altitude and latitude is investigated.

*Introduction.* Radiation calculations in climate models should be designed to be properly responsive to climate changes, such as in the atmospheric temperature and in the cloud, aerosol and gaseous atmospheric composition. The major difficulties are (1) accurately integrating over complex and sometimes overlapping absorption bands, which greatly change their characteristics with height, and (2) accurately including the effects of multiple scattering.

The basis of the approach we take for both solar and terrestrial radiation, which we call the correlated k-distribution method, is a generalization of the k-distribution method used by Lacis and Hansen (1974). In this generalization the vertical inhomogeneity of the atmosphere is approximately accounted for by assuming a simple correlation of k-distributions at different temperatures and pressures. By means of 'line-by-line' calculations we demonstrate that the method is remarkably accurate even for the notoriously difficult  $9.6\mu\text{m}$  ozone band.

There are several advantages to this radiative treatment, in addition to the fact that it includes all significant atmospheric constituents. Realistic spectral properties of clouds and aerosols are employed, based on Mie scattering computations with the best available composition and refractive index information; thus the treatments in the solar and thermal regions are self-consistent and we avoid crude assumptions such as black

or 'half-black' clouds. The accurate treatment employed for solar zenith angle effects is necessary for assuring proper latitude variations of radiative heating.

*Correlated k-distribution method.* The k-distribution  $f(k)$  for a given gas and frequency interval is the probability density function such that  $f(k) dk$  is the fraction of the frequency interval for which the absorption coefficient is between  $k$  and  $k + dk$ . The basic idea of grouping frequency intervals of gaseous spectra according to absorption coefficient strengths goes back at least to Ambartsumian (1936) who used it in estimating the influence of absorption lines in stellar atmospheres. Laciš and Hansen (1974) used k-distributions to include the effects of multiple scattering; they numerically derived a mean atmospheric k-distribution for water vapor absorption and scaled the gas amount as a function of pressure and temperature to crudely approximate the effect of atmospheric inhomogeneity.

Our generalization of the k-distribution method provides a more accurate treatment of the vertical inhomogeneity of the atmosphere; it is based on the assumption that the k-distributions at all altitudes are simply correlated in frequency space, i. e., the strongest absorption occurs at the same frequencies at all altitudes and similarly for the weakest absorption. This maintenance of the same relative rank of absorption coefficients over the entire pressure range of the atmosphere is rigorously correct for a single spectral line with a fixed center and for a uniform Elsasser (1942) band model. Fig. 1 is a schematic illustration of the k-distribution method.

For a real gaseous spectrum, we expect some blurring of the assumed correlation as a result of partial overlapping of lines with different line-widths and strengths and the fact that the temperature dependence of the line-strengths is not the same for all lines. To demonstrate that this has little impact on the overall accuracy of our method we show in Fig. 2 the thermal cooling rates computed with the correlated k-distribution method and with line-by-line calculations for the 9.6  $\mu$ m ozone band.

The computations for Fig. 2 were made for standard atmospheric temperature and ozone distributions with a 1 km vertical resolution (Laciš and Wang, 1979). The solid line was obtained by means of line-by-line computations using the line coefficients tabulated by McClatchey et al. (1973) with approximately  $2 \times 10^5$  frequency intervals. The dotted curve was obtained by means of the correlated k-distribution approach with 5 probability intervals, i. e., 5 discrete values of  $k$ , each with an appropriate pressure-temperature dependence. Since the computing time is proportional to the number of frequency intervals or  $k$  values, the relative speed of the correlated k-distribution method is apparent.

The k-distribution is formally related to the transmission function by

$$T(u) = \frac{1}{\Delta v} \int_{\Delta v} e^{-k v} u dv = \int_0^{\infty} f(k) e^{-k u} dk, \quad (1)$$

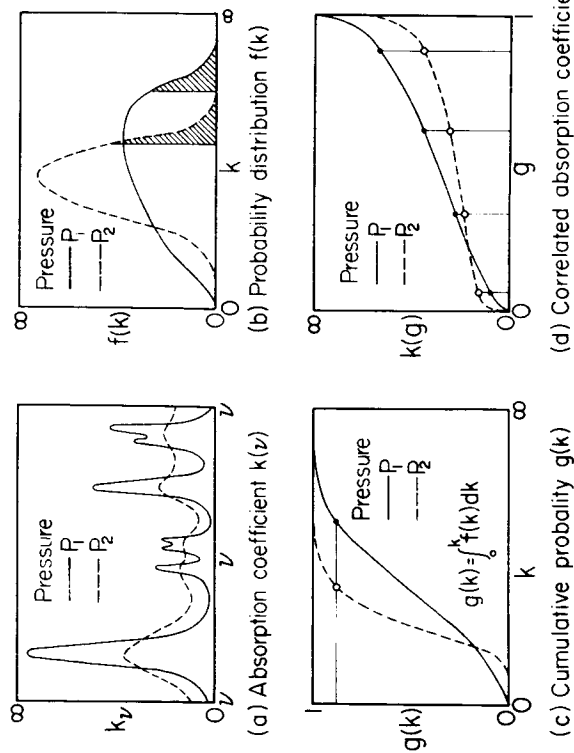


Fig. 1. Schematic illustration of the correlated k-distribution method. (a) is a schematic absorption line spectrum at two pressures. The greater pressure  $P_2$  broadens the spectral features, but the line center positions remain fixed. (b) is the probability density of absorption coefficients. The broadened spectrum has a narrower k-distribution. The shaded areas depict the strongest absorption for both pressures and are assumed to refer to the same spectral intervals for both distributions. (c) is the cumulative probability density. This integrated k-distribution makes it possible to rank probability intervals according to relative absorption coefficient strength. (d) is the correlated k-distribution. Integration over  $g$  replaces integration over  $\nu$ .

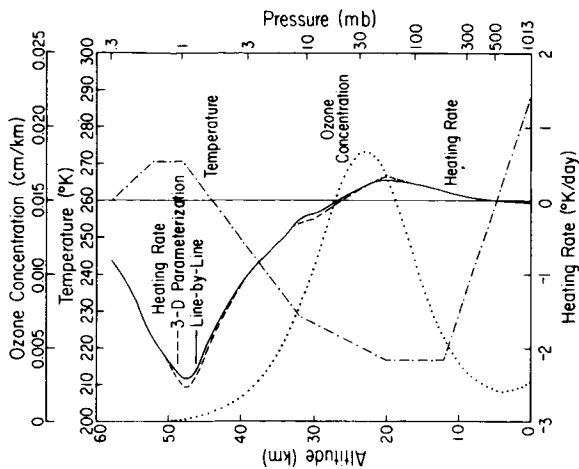


Fig. 2. Atmospheric cooling by the  $9.6 \mu\text{m}$  ozone band. Computations are for a standard atmosphere temperature distribution and a standard mean ozone concentration (Lacis and Wang, 1978) using a 1 km vertical resolution. The solid line was obtained with explicit line-by-line integration using approximately  $2 \times 10^5$  spectral intervals. The dashed line is the result obtained with the correlated k-distribution method using 5 probability intervals for the entire band.

where  $u$  is the gas amount. Because of its convenient mathematical properties, we choose the Malkmus (1967) model to represent  $T(u)$ ,

$$T(u) = \exp \left\{ -\frac{\pi}{2} B \left[ (1 + 4Su/\pi B)^{\frac{1}{2}} - 1 \right] \right\} \quad (2)$$

where the two parameters,  $S$  and  $B$ , are the effective line strength and line width. Since  $f(k)$  is the Laplace transform of  $T(u)$  we obtain the  $k$ -distribution directly in terms of the Malkmus model parameters,

$$f(k) = \frac{1}{2} k^{-3/2} (SB)^{\frac{1}{2}} \exp \left[ \frac{\pi}{4} B(2 - k/S - S/k) \right]. \quad (3)$$

The Malkmus model  $k$ -distribution is analytically integratable to yield the cumulative probability

$$g(k) = \int_0^k f(k') dk'. \quad (4)$$

This makes it convenient to rank intervals of  $\Delta g$  according to their relative absorption coefficient value. This procedure for ranking  $\Delta g$  retains an implicit frequency correlation with altitude and provides the basis for accurate computation of multiple scattering effects in an inhomogeneous atmosphere.

For a given gas and frequency interval, the  $k$ -distribution can be obtained in several different ways (Lacis and Wang, 1979). We use the tabulated line coefficients for  $H_2O$ ,  $CO_2$ ,  $O_3$ ,  $CH_4$  and  $N_2O$  compiled by McClatchey et al. (1973) to compute line-by-line absorption over homogeneous paths for pressures and temperatures over the range of values encountered in the stratosphere and troposphere. For each gas we obtain a table of  $S$  and  $B$  parameters by least square fitting of the Malkmus model to the line-by-line computations. This yields band model parameters which fit the line-by-line results within 1 or 2 per cent over a pressure-temperature range from the ground to a height of  $\sim 70$  km, where the assumption of local thermodynamic equilibrium begins to deteriorate. Note that the numerical approach for extracting band model parameters is not limited by the changing line shape with altitude; we can use the appropriate Voigt profile for each altitude and obtain a corresponding  $k$ -distribution in terms of its effective line strength and line width parameters.

*Climatic effect of cirrus clouds.* It is well known that clouds have two major competing effects on global climate: (1) they reflect solar radiation, which tends to decrease the surface temperature, and (2) they blanket thermal radiation from the earth's surface, reradiating at a lower temperature and thus tending to warm the surface temperature. The opinion of climatologists seems to be that a general increase in cloudiness would decrease the surface temperature, for example, according to Schneider (1972): "...the effect of a sustained increase in the average amount of cloud cover of the earth would be a decrease in the global-average surface temperature, provided that the cloud top height and cloud albedo remain unchanged."

It is recognized that cirrus clouds can present an exception to the expected cooling effect of clouds. For example, Fig. 3 (from Manabe and Strickler, 1964) shows that cirrus clouds at high altitudes can have a

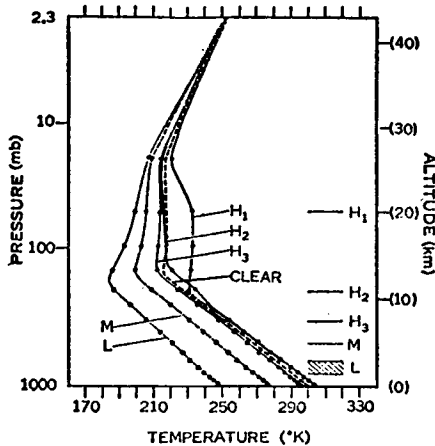


Fig. 3. Thermal equilibrium of various atmospheres with clouds. The height of overcast clouds used for each computation is shown by  $H_1$ ,  $H_2$  and  $H_3$  for high clouds, M and L for middle and low clouds. The equilibrium curve of the clear atmosphere is shown by a thick dashed line. (After Manabe and Strickler, 1964).

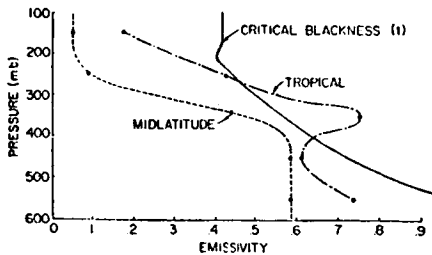


Fig. 4. Measured cirrus cloud infrared emissivities and the "critical blackness" curve from Manabe and Strickler (1964). (After Cox, 1971).

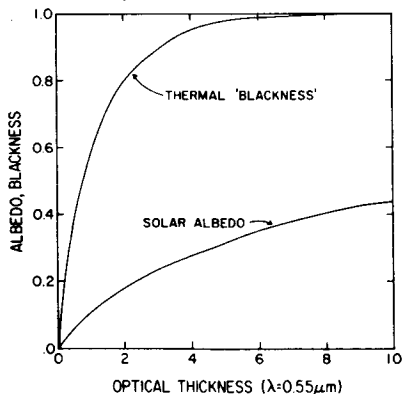


Fig. 5. Albedo and blackness of cirrus clouds as a function of optical thickness, computed for ice particles of  $25 \mu\text{m}$  effective radius. The 'blackness' is the sum of the cloud absorptivity and reflectivity for blackbody radiation of 250 K; the reflectivity contributes at most 0.05 to the blackness.

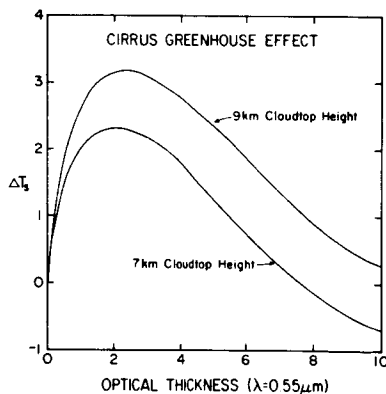


Fig. 6. Effect of cirrus clouds on surface temperature (in degrees K) as a function of the cirrus optical thickness and cloudtop height. The cirrus are assumed to cover 10% of the sky. Calculations are for a global average temperature profile, which is also similar to the mean profile at  $30-40^\circ$  latitude.

heating effect, though the magnitude is small compared to the cooling by other clouds. According to Manabe and Strickler (1964): "If the height of cirrus clouds is greater than 9 km and its blackness for infrared radiation is larger than 50 percent, cirrus has a heating effect on the temperature of the earth's surface." The cirrus infrared emissivities measured by Cox (1971) also suggest that a warming effect by cirrus is small and rather unusual, cf., Fig. 4 his measurements are interpreted as indicating that in the tropics only cirrus at altitudes 250-400 mb cause heating, and at midlatitudes all cirrus cause cooling.

However, the definition for blackness used by Cox (1971) differs from that of Manabe and Strickler (1964) which thus leaves the conclusions of Cox open to question. The nature of the calculations of Manabe and Strickler was to use a fixed cirrus albedo of 21% and to look at how the surface heating or cooling would change for different assumed infrared emissivities. In general the calculations of Manabe and Wetherald are extremely valuable, but in order to investigate the effect of cirrus on the temperature profile it is essential that the solar and thermal spectral regions be treated in a self-consistent fashion using spectrally dependent optical properties of water ice. The method described above permits this computation to be done with reliable accuracy.

Results for the solar albedo and thermal 'blackness' are shown in Fig. 5 as a function of optical depth. It is apparent that the thermal blackness (defined to be 1 minus the total transmission) is much more rapidly saturated than is the solar albedo. Typical cirrus cloud emissivities of  $\sim 0.4$  to  $\sim 0.6$  measured by Cox (1971) correspond to optical depths of  $\sim 0.5$  to  $\sim 1.0$  and should have corresponding albedos of  $\sim 5\%$  to  $\sim 10\%$  rather than the widely used value of 21% estimated by Haurwitz (1948). As shown in Fig. 6, cirrus clouds have a substantial warming effect on the surface temperature. It is only when the optical thickness becomes very large and the clouds appear at low altitudes that a cooling effect is experienced.

The most effective optical thickness for the heating (greenhouse) effect is approximately  $\tau = 2$ , for which the thermal blackness is nearly saturated while the solar albedo is still rather small. The warming effect is also sensitive to cloudtop height; thus high clouds, which reradiate the absorbed surface radiation at a lower effective temperature, provide greater warming than low clouds.

*Discussion.* These results, which were computed for mean global atmospheric conditions, show a substantial warming effect due to cirrus clouds. The same qualitative conclusion holds for tropical and polar latitudes. The magnitude of this greenhouse effect is sufficiently large to indicate the need for a careful reappraisal of the role of cirrus clouds in climate.

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