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INVERSE BOUNDARY-LAYER TECHNIQUE FOR AIRFOIL DESIGN

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SUMMARY

Presented is a description of a technique for the optimization of airfoil pressure distributions using an interactive inverse boundary-layer program. This program allows the user to determine quickly a near-optimum subsonic pressure distribution which meets his requirements for lift, drag, and pitching moment at the desired flow conditions. The method employs an inverse turbulent boundary-layer scheme for definition of the turbulent recovery portion of the pressure distribution. Two levels of pressure-distribution architecture are used - a simple roof top for preliminary studies and a more complex four-region architecture for a more refined design. A technique is employed to avoid the specification of pressure distributions which result in unrealistic airfoils, that is, those with negative thickness. The program allows rapid evaluation of a designed pressure distribution off-design in Reynolds number, transition location, and angle of attack, and will compute an airfoil contour for the designed pressure distribution using linear theory.

Comparison of pressure distributions and corresponding airfoil geometries resulting from different specifications of recovery-region boundary-layer form parameter will be presented.

INTRODUCTION

Airfoil design has traditionally been a trial-and-error process. Before large computers were available the airfoil designer had little option other than to make small changes to existing sections, guided by linear theory, in the hope that he would improve the section's performance in the area he wanted. Or, if he could afford the time and expense, the designer could undertake the testing of a large number of sections to try to find a good one for his application. Understandably, airfoil progress was a rather hit-or-miss proposition. Certainly, airfoil designers were guided by the physics of the flow around their sections but the mathematics required to describe the flow field adequately was too complex to solve in closed form and without computers, and it was too time consuming to solve numerically.

When relatively cheap computing equipment became available, methods for solving the inviscid flow field about an arbitrary airfoil evolved, and, soon after, methods for computing boundary-layer characteristics were developed and are still evolving. These were powerful tools for the airfoil designer. He could now look at many more variations in his design variables and could be bolder in his departure from what had been previously tested. However, as good as these tools were, he was still using the cut-and-try method to design an airfoil to new requirements. Because of the highly nonlinear way the different variables in the airfoil-design problem interact there is no guarantee that cut-and-try will produce an overall improvement in performance; it is all too easy to design a section which is superior in one aspect of performance only to find it is totally unacceptable in another.

To get around the inherent problems of cut and try, the inverse of the airfoil-analysis problem needed to be solved more rigorously. Instead of computing the performance characteristics of a given section, one needed to specify the performance and compute the section. At present, this inverse airfoil problem is being attacked in two parts, as was airfoil analysis, that is, the inverse inviscid and the inverse boundary-layer problems.

The inverse inviscid problem, that is, specifying a pressure distribution and computing an airfoil contour, has been well developed recently as evidenced by several excellent papers on the subject given in this volume. However, while the techniques for performing the inverse inviscid computations are seemingly well in hand, the use of these techniques for airfoil design is not so well off. The reason is that although there is a real, unique inviscid pressure distribution for every airfoil at every angle of attack, the reverse is not true in a practical sense. One can easily specify a pressure distribution which results in an airfoil with negative thickness. Although such an airfoil is a perfectly valid solution to the inverse inviscid problem in a mathematical sense, one might have a little trouble building one. The fact is that most arbitrarily defined pressure distributions result in unrealistic airfoil contours if present inverse inviscid techniques are applied blindly. Future programs which allow specification of both pressure distribution and thickness by using weighting and advanced solution techniques (such as least-squares or Newton's method) to resolve the over-specified nature of the problem may aid the design. However, these programs have yet to appear in general use. So, one must be very careful in the choice of pressure distribution if the existing inverse invsicid programs are to be used to advantage.

How should the pressure distribution be defined? With a little thought one can see that at low Mach numbers the performance of an airfoil is either defined by or limited by the boundary layer and the requirement of a reasonable (or buildable) thickness form. In order to proceed with a rational design process one must have a boundary-layer technique that, given boundary-layer parameters, will compute a pressure distribution and, hopefully, is constrained to produce only pressure distributions that are realistic.

With such a computational tool in hand the designer might take advantage of the airfoil design process shown in figure 1. The most notable aspect of this process is that it proceeds from performance requirements to initial contour entirely in the inverse mode - first to compute a desirable pressure distribution and then to compute an initial airfoil contour. The last block, the detailed analysis and refinement stage, is the last remnant of trial and error; and for the small changes involved in refinement this is probably desirable (or at least inevitable).

Assuming one has the capability for computing an airfoil from a pressure distribution (which is, after all, primarily a problem in mathematics, not aerodynamics) the primary task of the aerodynamicist is to define a desirable pressure distribution, hopefully the best pressure distribution, for the job at hand. In the remainder of this paper I will discuss a computer technique which I believe will greatly aid the designer in this task.

The symbols used herein are defined in an appendix.

TECHNIQUE

The Inverse Boundary-Layer Equations

An inverse boundary-layer technique is a solution to the boundary-layer equations where boundary-layer parameters are <u>specified</u> and a pressure distribution is computed. For the program I will discuss in this paper, I have used the boundary-layer momentum equation, Garner's equation for form parameter variation, and the Ludwig-Tillman equation for the wall shear stress (ref. 1). These equations have been arranged to solve for a velocity distribution given a variation in the form parameter $H = \delta^*/\theta$. The solution technique is shown as follows:

Solving Garner's equation for the velocity derivative gives

$$\frac{d(u/u_\infty)}{d(s/c)} = -\frac{dH/d(x/c)}{(u/u_\infty)e^{4(H-1.4)}} - \frac{0.0135(H-1.4)(u/u_\infty)}{(R_\theta)^{1/6}\,(\theta/c)} \tag{1}$$

From the momentum equation solve for

$$\frac{d(\theta/c)}{d(s/c)} = \frac{\tau_o}{\rho u^2} - \frac{(H+2)(\theta/c)\,d(u/u_\infty)/d(s/c)}{u/u_\infty} \tag{2}$$

The wall shear stress coefficient $\tau_o/\rho u^2$ is given by Ludwig-Tillman:

$$\frac{\tau_o}{\rho u^2} = 0.123(R_\theta)^{-0.268} \times 10^{-0.678H} \tag{3}$$

The momentum thickness ratio θ/c and velocity ratio are given by integrating equations (1) and (2) numerically with a known $H(x/c)$ (and thus a known $dH/d(x/c)$):

$$\theta/c = \frac{d(\theta/c)}{d(s/c)} \Delta s/c \tag{4}$$

$$u/u_\infty = \frac{d(u/u_\infty)}{d(s/c)} \Delta s/c \tag{5}$$

where $\Delta s/c$ is the integration step in arc length. Equations (1) to (5) should be relaxed at each step for average values of u/u_∞ and θ/c.

Pressure-Distribution Synthesis

With this inverse turbulent boundary-layer technique and an appropriate pressure-distribution architecture one may quickly design a pressure distribution on one surface of an airfoil. To explain the way the pressure distribution is synthesized consider the simpler of the two architectures available - the roof top (fig. 2). This architecture is characterized by an acceleration region starting at the leading edge and terminating at an input fair point. Constant pressure is assumed from the fair point to the input beginning of turbulent recovery (recovery point). The turbulent recovery spans the remainder of the foil and facilitates pressure recovery from the roof-top pressure to the desired trailing-edge pressure. The roof-top pressure is not input and is found by the inverse boundary-layer equations by employing an iterative procedure explained subsequently.

Iterative Procedure for Determining Minimum Cp

Given a fair point, a recovery point, a trailing-edge pressure, and a guess at roof-top pressure $C_{p_{min}}$, the pressure distribution is assembled up to the recovery point. This pressure distribution is then analyzed to provide the starting values of H and θ/c for the inverse turbulent boundary-layer module. Having these values, the recovery pressure distribution is computed from $C_{p_{min}}$ at the recovery point to the trailing edge by using the desired variation of form parameter in the recovery region. If the computed trailing-edge pressure is not that desired, $C_{p_{min}}$ is incremented, the pressure distribution up to the recovery point is reassembled, and the process is repeated to convergence.

The Lower Surface Pressure Distribution

Since the total pressure distribution must represent a realistic airfoil and the upper surface pressure distribution is defined by boundary-layer considerations alone, the lower surface pressure distribution must be defined by the thickness requirements specified by the designer. In the present program, a standard thickness form is used, the NACA 00XX, which is scaled to the designer's desired maximum thickness. So with a single input t/c_{max}, the designer gets a lower surface pressure distribution which will result in an airfoil which has the upper surface shape required to give him his designed upper surface pressure distribution and a lower surface contour which results from an NACA 00XX thickness form of the desired maximum t/c. Linear airfoil theory is used to accomplish this and I will not discuss it further in this paper.

The Four-Region Architecture

Although the roof-top architecture is effective and simple to use for the preliminary stages of pressure-distribution optimization, it allows very little flexibility for controlling the laminar and transitional portion of the boundary layer. As will be shown later, the transition-point position is often of first-order importance to the airfoil-design problem. To allow for more precise control of the laminar boundary layer, the four-region pressure distribution was devised.

This architecture is shown in figure 3 and is characterized by an acceleration region (1), a region of constant pressure gradient (II), a laminar stressing region (III), and finally the turbulent recovery region (IV). The fair point is input as before; and, in addition, the desired value of pressure gradient is given for region II, and a desired point of initiation of laminar stressing and a desired value of laminar form parameter are given for region III. A simple, inverse, laminar boundary routine is used to produce the pressure distribution required to produce the desired laminar form parameter. Region III may be used for either stressing the laminar boundary layer to achieve rapid transition without separation or may be used to avoid transition - depending, of course, on the value of H specified. The equations used in this region are

$$u/u_\infty = C(s/c)^{\left(\frac{1 - H/2.55}{0.94}\right)}$$

$$C = (u/u_\infty)_o \;/\; (s/c)_o^{\left(\frac{1 - H/2.55}{0.94}\right)}$$

where $(\;)_o$ refers to the values at the beginning of region III.

In the derivation of these equations some liberty was taken in dropping terms for simplicity; however, practice has shown this relationship is remarkably accurate. The symbol $C_{p_{min}}$ is defined as the pressure at the beginning of region III. Region IV, the turbulent recovery, is defined by the inverse turbulent boundary-layer technique as described earlier.

Optimization Using an Interactive Program

The pressure-distribution design technique I have described has been implemented in an interactive optimization program. That is, the program user is operating the computer program in a conversational mode where input is requested by typed messages and given to the program, real time, by responding with typed input at a terminal. Output is printed or plotted immediately at the terminal at the request of the user.

Why an interactive program? Pressure-distribution design is a highly overdefined optimization problem. Constraints and secondary requirements are tough to quantify and the weight given them often reflects the judgement of the designer alone. In deciding how to implement this technique I adopted the philosophy that it is desirable to leave the judgement in the hands of the user and that a computerized pressure-distribution design tool should eliminate the computation in the design process and amplify the user's judgement by quickly and clearly illuminating the important relationships between the variables and constraints of the design problem at hand.

An interactive program is a natural outcome of this philosophy as it allows an adaptive flexibility in the optimization process impossible using batch-type computing. Furthermore, and not a bit less important, we have found that being a part of the optimization process is very educational for the user, especially when he is designing in a flow regime where he has little experimental experience. Lastly, it is one heck of a lot more fun to compute this way - as anyone who has tried it will agree. While this may seem a trivial concern - the fact that the computation is now pleasant, rather than a chore, often means one will tend to stick out a really tough problem longer.

The options available in this optimization program are illustrated in figure 4. I will just summarize them here.

The design is initiated by giving the program the flow conditions and details of the architecture chosen. The type of form-parameter variation is then chosen - this program allows a constant form parameter or linear, quadratic, or exponential variation to be specified with very little input. An arbitrary H variation may be used if a file of the desired values of H vs. x/c has been previously generated. This complete, the program enters the design module and displays C_ℓ, C_d, C_m, and a plot of the designed pressure distribution. This whole process takes about 30 seconds. At this point the user may redefine the H variation, or any other of the design parameters, or if he is satisfied with the design-point pressure distribution he may analyse the pressure distribution at an off-design Reynolds number or trip location or he may analyse the pressure distribution off-design in lift (accomplished by adding an angle-of-attack velocity distribution to the design distribution using linear theory). The results of any of these off-design analyses are printed and plotted immediately at the terminal. In the event that a desirable pressure distribution is generated, a preliminary airfoil contour may be obtained simply by request. This is accomplished by a simple linear inverse program due to Trukenbrodt (ref. 2), and the airfoils produced should be considered as starting points for more accurate design methods. They are, however, excellent bench marks to check the resulting airfoil's fidelity to structural restraints (such as a restraint on maximum camber or compound surfaces).

To indicate the power of this approach, consider that to define a pressure distribution, it should be checked at several Reynolds numbers and off-design angles of attack, and to produce a preliminary airfoil takes about 7 minutes on a PDP 11-70 or about 8 minutes on a CDC Cyber 175. The reason the more powerful machine takes longer is that the data-transmission rate to our terminal is 300 band on the 175 and 9600 band on the 11-70. Most of the time is used during transmission and user head scratching in deciding what to do next.

Some Illustrative Results

Figure 5 displays a comparison of pressure distributions and their corresponding airfoil geometries resulting from different specifications of recovery-region boundary-layer form parameter (H). The 'C', 'L', and 'E' denote constant H, linearly varying and exponentially varying H, respectively. Each pressure distribution was designed to a lift coefficient of approximately 1.6. The impact of the variation of the form parameter on both airfoil performance and shape is striking. The L/D (ε^*) ranges from 172 for constant H (2.06) to 225 for the exponentially varying H (1.45 at x/c = 0.3 to 2.0 at x/c = 1.0). Pitching moment varies from 0.057 to -0.186 for the same two examples. The airfoil contours, I think, speak for themselves. Again, the values of H for each type of variation were picked so that the pressure distributions all produced a design-point lift coefficient of about 1.6.

In the designs of figure 5, natural transition occurred near the fair point (x/c = 0.1) due to very high design-point Reynolds number (Rn = 30×10^6). In figures 6 and 7 I have tried to illustrate the powerful effect that transition has on the design of a pressure distribution. In figure 6 a design was again undertaken at a Reynolds number of 30×10^6 with a roof-top architecture and natural transition. The result was a pressure distribution of modest performance with transition occurring very near the fair point.

Using the four-region architecture, an attempt was made to maintain laminar flow as far aft on both surfaces as possible. The result was a pressure distribution that produces significantly better performance. The L/D at the design point went from 215 for the roof top to 317 for the four-region pressure distribution. Design C_ℓ was increased from 1.42 to 1.68 for the same specification in form parameter, whereas the pitching-moment coefficient remained relatively unchanged. Although the final pressure distribution in this figure shows remarkable improvement and impressive performance, work still needs to be done to achieve acceptable off-design performance as one might expect considering the section's rather sharp leading edge.

Figure 7 displays the effect of tripping the laminar boundary layer on design lift and L/D at a Reynolds number of 4×10^6 for several recovery locations. Laminar flow is extensive when no trip is specified due to the lower Reynolds number, even though the roof-top architecture is used throughout, so one would expect tripping to have a significant impact. The curves show it does. The peak design C_ℓ is reduced from 2.25 to 1.3 by tripping and the L/D is reduced from 177 to 77. The explanation for this sharp reduction in performance is that early transition produces a much thicker boundary layer at the recovery point due to the more rapid growth of a turbulent boundary-layer relative to the laminar layer. This thicker boundary layer is able to withstand less adverse pressure gradient and so must recover from a lower velocity. Thus, there is a loss in lift and L/D.

Figure 8[1] summarizes the performance of some of the pressure distributions we have designed to date. With the exception of points 3 and 4 all were low C_ℓ designs with heavy restrictions placed on the maximum camber and all used the simple roof-top type architecture with tripping to produce a turbulent boundary layer at the recovery point where necessary. Because of this, one should not interpret the results as an attempt to plumb the limits of L/D at each Reynolds number but rather consider these results only as an example of the type of information obtainable from this program with a relatively modest amount of time and expense. Points 3 and 4 were attempts at optimizing L/D at their design Reynolds numbers, and although they do perform credibly, more effort is required to refine the designs and guarantee good off-design performance.

CONCLUSIONS

We need more experience with this program to say with any certainty whether it can be used to determine an optimum pressure distribution for the flow conditions where the theories are valid. However, the following aspects of this technique are already apparent:

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[1] Dr. John McMasters assisted in preparing this paper and allowed me to use the data in figures 5 and 8 which he generated for a Boeing internal study of the L/D potential of moderately cambered airfoils.

1. The inverse boundary-layer technique is a powerful and efficient device for defining a pressure distribution.

2. The pressure-distribution architecture need not be excessively complex to control the laminar and transitional regions of the pressure distribution.

3. The simple inverse laminar equation is an effective tool for stressing the laminar boundary to transition without causing separation.

4. Allowing a scaled thickness form to define the lower surface pressure distribution does constrain the program to produce pressure distributions which result in realistic sections. However, more flexibility is needed in this area so that aerodynamic considerations may also play a part in definition of the lower surface distribution. This may take form as optional thickness forms or arbitrary thickness forms constrained only in maximum thickness and to positive thickness everywhere.

5. The interactive type of execution seems to have value beyond the obvious advantages of speed and immediate response in the display of intermediate results, that is, it is very educational and pleasant to use.

APPENDIX

SYMBOLS

H	boundary-layer form parameter, δ^*/θ
U	velocity at outer edge of boundary layer
U_∞	free-stream velocity
s/c	airfoil-surface arc length normalized with chord
x/c	distance along airfoil chord, normalized with chord
Rn	Reynolds number based on airfoil chord
R_θ	Reynolds number based on local value of momentum thickness θ and velocity U
θ/c	boundary-layer momentum thickness (normalized), $$\frac{\theta}{c} = \int_0^\infty \frac{u}{u_\infty}\left(1 - \frac{u}{u_\infty}\right) dy/c$$
δ^*/c	boundary-layer displacement thickness, $$\frac{\delta^*}{c} = \int_0^\infty \left(1 - \frac{u}{u_\infty}\right) dy/c$$
ρ	density
C_ℓ	section lift coefficient
C_d	section drag coefficient
C_m	section pitching-moment coefficient
C_p	pressure coefficient, $C_p = 1 - \left(\frac{u}{u_\infty}\right)^2$
t/c	airfoil thickness (normalized)
ε^*	design-point section L/D
ε	maximum section L/D

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1. Schlichting, H.: Boundary Layer Theory. McGraw-Hill Book Co. Inc. pp. 238-251, 391-406, 566-579.

2. Riegals, F. W.: Airfoil Sections, Butterworth and Co., Publishers Ltd., 1961, pp. 80-83.

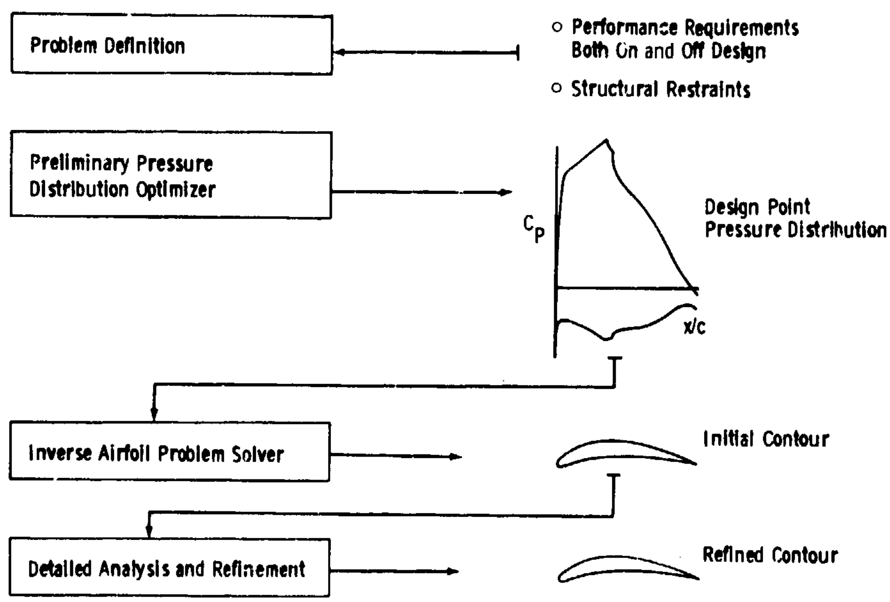


Figure 1.- Airfoil design process.

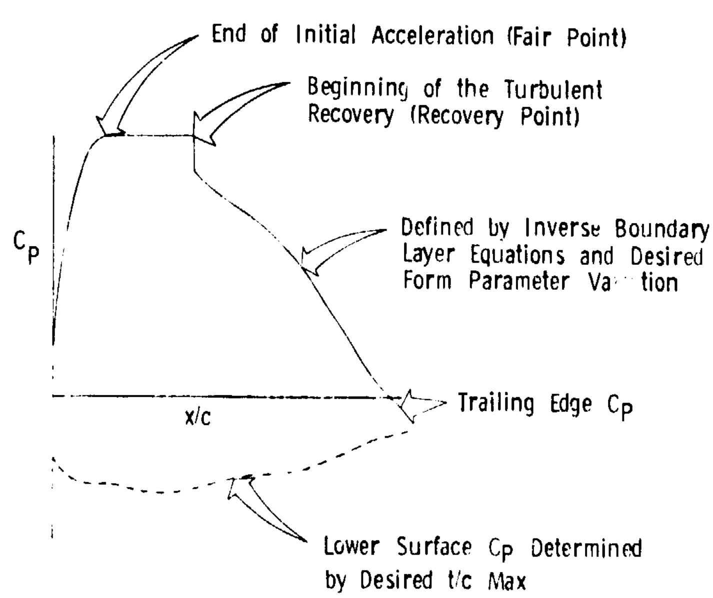


Figure 2.- The roof-top architecture.

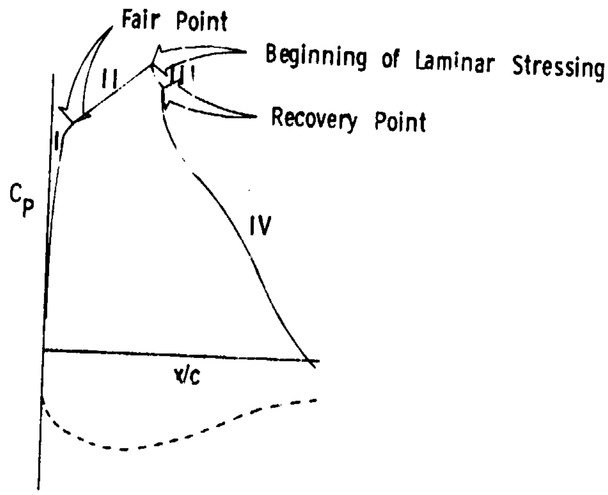


Figure 3.- Four-region architecture.

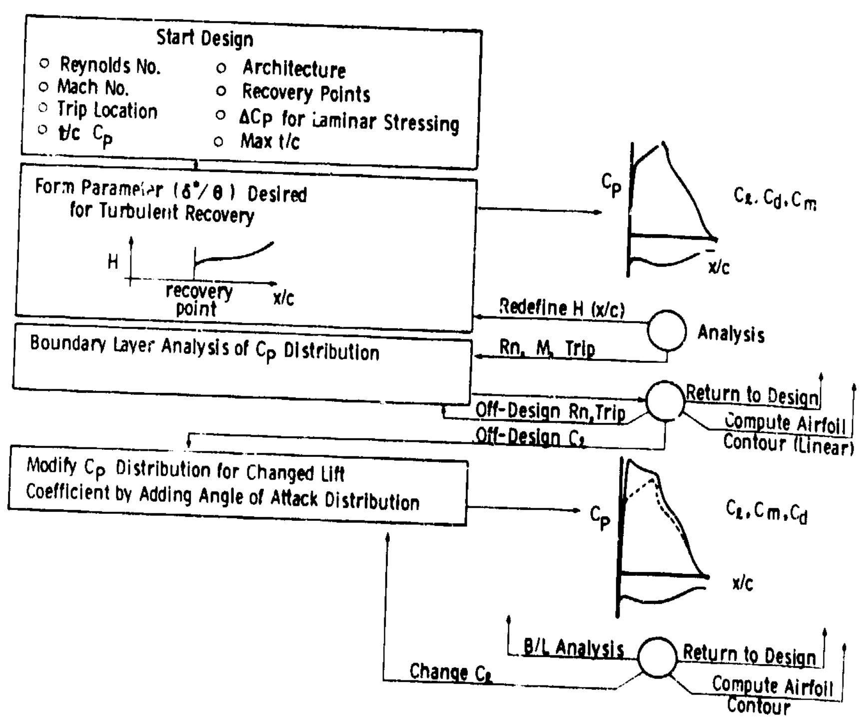
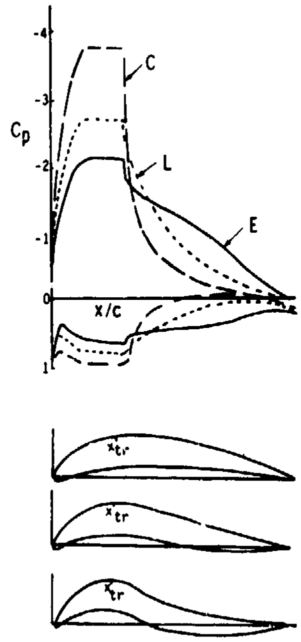
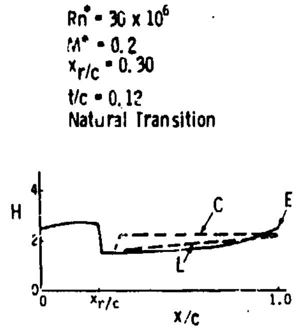


Figure 4.- Block diagram of interactive pressure-distribution optimizer.





BHL 120-300-30/28 (H Variation)

H Variation	C_l^*	C_m^*	ϵ^*	ϵ_{max}	$C_{l_{max}}$
E 1.45/2.4	1.592	-0.186	226	240	2.3
L 1.45/2.0	1.563	-0.058	216	216	2.05
C 2.06	1.603	0.057	172	172	1.97

Figure 5.- Comparison of pressure distributions and corresponding airfoil geometries resulting from different specifications of recovery-region boundary-layer form parameter H.

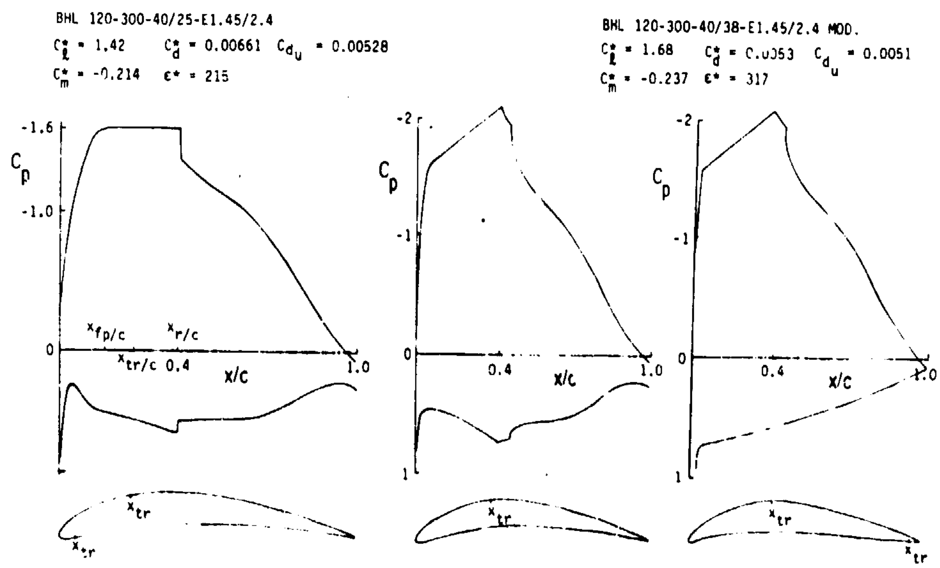


Figure 6.- Examples of basic pressure-distribution architecture.

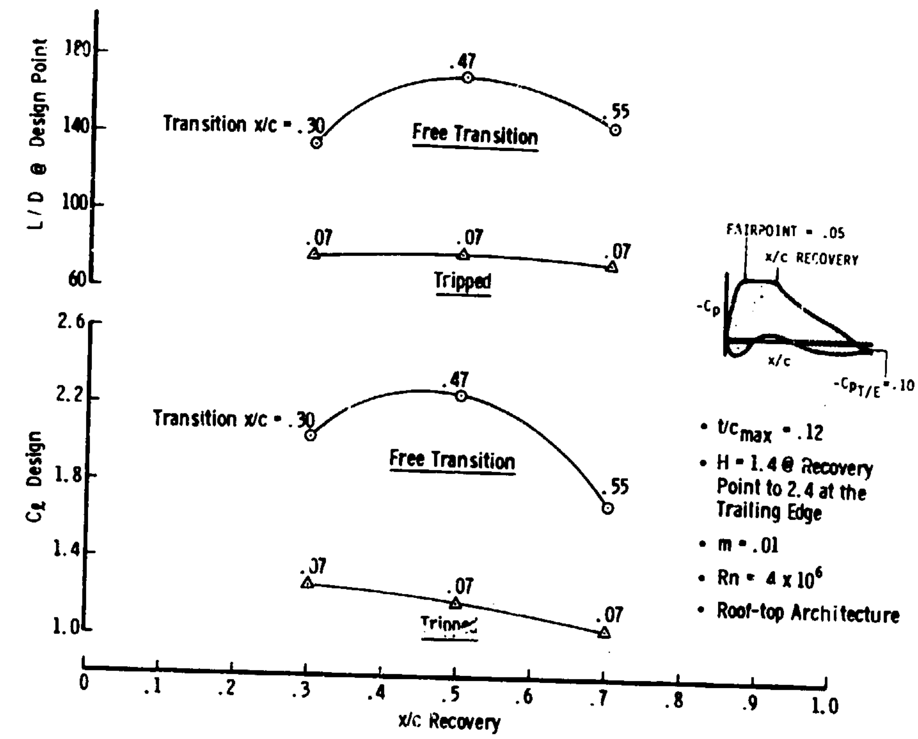


Figure 7.- Effect of transition location on design C_ℓ and L/D.

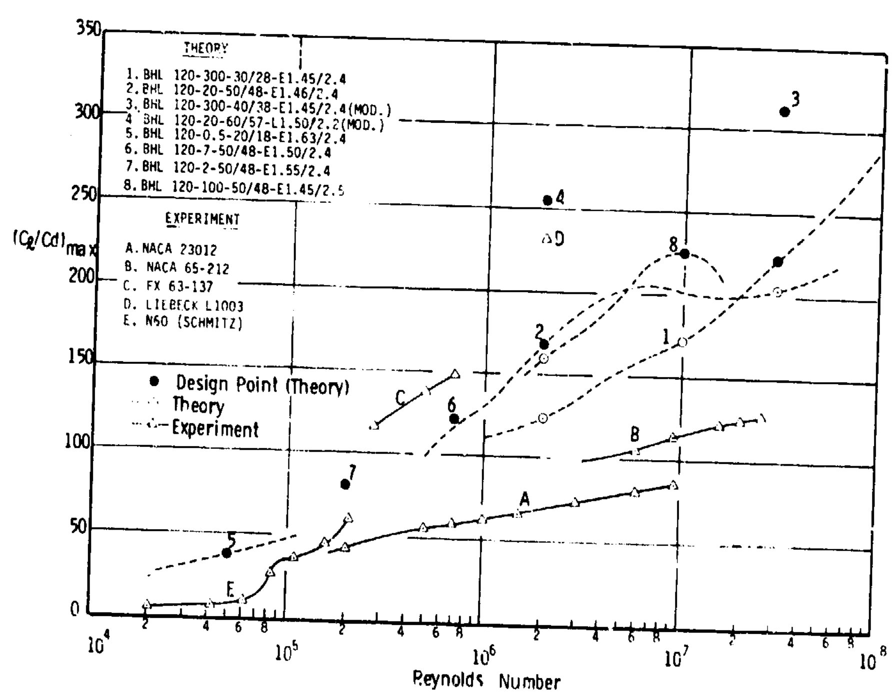


Figure 8.- Maximum L/D plotted against Reynolds number.