# DRIVE TRAIN DYNAMIC ANALYSIS

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## ABSTRACT

A method for parametric variations in drive train dynamic analysis is described. The method models the individual components of a drive system, forms the appropriate system interface coordinates and calculates the system dynamic response at particular frequencies. Application of the method for prediction of the dynamic response characteristics of a helicopter transmission and a comparison of results with test data is also included.

## INTRODUCTION

Substructure methods using impedance techniques are a convenient and economical means for dynamic analysis of wind turbine drive trains. Response characteristics of a complex structure, such as a wind turbine drive system, may be effectively evaluated using the technique described herein. The method models the individual components comprising a complex system, forms the appropriate interface coordinates and predicts the total system dynamic response at particular frequencies of interest.

The method has two important features. The first of these relates to reduction in degrees of freedom. Analysis of each basic component is performed with as many degrees of freedom required to accurately predict the motion at the coordinates and frequencies of interest. When the resulting analytical model is used, however, the number of degrees of freedom may be drastically reduced and must include only the following: (1) those which interface other components such as gear mesh line of action coordinates, coupling coordinates and bearing support coordinates, (2) coordinates at which structural modifications are to be included, (3) coordinates at which a force is applied, and (4) coordinates at which dynamic response is desired.

This reduction in the number of coordinates is performed only once at each frequency of interest with no loss in the validity of the analytical model, regardless of the extent of the reduction.

The second important feature relates to the ease with which structural changes may be implemented. Structural modifications such as local mass or stiffness changes, the addition of springs or dampers between components, addition of vibration absorbers and changes in boundary conditions may be cost effectively modeled.

## THEORETICAL BASIS

A linear structure may be represented in the frequency domain as a finite element model using an impedance formulation in terms of the mass, stiffness and damping matrices.

The relationship between the applied force and the structural response is

$$Zy = f \tag{2}$$

where y is a vector representing the response, either displacements or rotations, Z is the impedance matrix, and f is a vector denoting the excitation, either force or moment. Except for an undamped system at resonance or for $\omega = 0$, Z will not be a singular matrix. Thus

$$y = Z^{-1}f = Yf \tag{3}$$

where Y is referred to as the mobility matrix and may be determined by inversion of the impedance matrix. Elements of the mobility matrix are not amenable to analytical modeling, however, they have physical significance and are measurable quantities representing the deflection at a coordinate due to a unit force applied at that or some other point. Since each element may be directly measured on the actual structure, they are independent of the number and location of the other degrees of freedom.

The criterion for a valid impedance matrix is that the elements of its inverse correctly represent the true response characteristics of the structure. Thus, a direct method of obtaining a valid reduced impedance matrix is as follows: (1) perform a structural analysis using conventional methods to obtain a valid, full size, impedance matrix, $Z(\omega)$, at each frequency of interest, (2) invert $Z(\omega)$ to obtain a valid full size mobility matrix $Y(\omega)$, (3) select elements from $Y(\omega)$ corresponding to the coordinates to be retained, which form a new reduced mobility matrix, $Y_R(\omega)$, (4) finally, the reduced impedance matrix is formed by inversion of $Y_R(\omega)$:

$$Z_R(\omega) = Y_R^{-1}(\omega) \tag{4}$$

The reduced impedance matrix $Z_R(\omega)$ is valid only at the frequency at which it was formed. There is no special interpretation of $Z_{R(\omega)}$ in terms of mass stiffness and damping matrices. The physical system represented by $Z_{R(\omega)}$, at the frequency $\omega$, behaves precisely as the system under study (Ref 1).

The reason for specifically obtaining the impedance matrix of the reduced system is that the impedance matrix of a complex structure may be obtained by adding the impedance matrices of the separate components at coordinates where the deflections are common. If $Z_a$, $Z_b$, $Z_s$ are partitioned impedance matrices of subsystem a, b and the complete system, respectively, and ($\underline{\hat{\ }}$) refers to interface coordinates, ($\hat{\ }$) refers to noninterface coordinates and ($\bar{\ }$) refers to coupling between interface and noninterface coordinates, then

$$Z_s = \begin{bmatrix} \hat{Z}_a & \hat{\bar{Z}}_a & 0 \\ \hat{\bar{Z}}_a & \bar{Z}_a + \bar{Z}_b & \hat{\bar{Z}}_b \\ 0 & \hat{Z}_b^T & \hat{Z}_b \end{bmatrix} \tag{5a}$$

$$\text{and } Y_s = Z_s^{-1}. \tag{5b}$$

There are several considerations involved in the practical application of this analysis technique as follows: (1) the substructures must be modeled as if they were unrestrained at the interface coordinates, (2) it is not important how the reduced mobilities are computed as long as they are valid, (3) the number of reduced coordinates must not be so large that matrix inversions are prohibitive, (4) local impedance changes due to addition of spring-mass systems or boundary condition changes are simply added to the reduced component impedances, (5) addition of impedance matrices must be performed at corresponding elements representing deflections or rotations in the same direction, (6) the impedance of spring-damper devices separating system components must be added to one of the substructures prior to synthesis.

## SYSTEM OVERVIEW

A computer program was developed to implement the analytical technique. The computer program automatically and conveniently performs the coordinate reductions, coordinate transformations, addition of impedance changes, impedance matrix addition for subsystems and determination of the mobility matrix of the combined system. The program has the ability to accommodate structural modifications including local mass, stiffness or damping changes, addition of springs and/or dampers between components, addition of vibration absorbers and changes in boundary conditions without performing new and costly analyses for each change. Figure 1 presents a schematic of the computer program. Impedance matrices of the various components comprising the structure are formed at the frequencies of interest and stored, with appropriate identification, in the common data bank. Component modifications are performed as desired and the modified impedance matrices with appropriate identification are optionally stored in the data bank or used in the current analysis. The original component impedance matrices are retained in the data bank.

The methodology and associated computer program has the capability to modify the characteristics of any dynamic component as follows:

1. Add Structural Damping

Structural damping may be added in the form igK, where K is simply the component impedance matrix at zero frequency which is stored in the data bank. Thus, the component impedance matrix with the addition of structural damping at a particular frequency of interest, $\omega$, is

$$Z_{Mod}(\omega) = Z_{Orig}(\omega) + ig\, Z_{Orig}(\omega = 0) \tag{6}$$

where g is the structural damping coefficient.

2. Addition of Vibration Absorber or Lumped Mass

The modification of the component impedance matrix due to vibration absorbers attached at particular coordinates involves a change in both the real and imaginary components of the impedance matrix yielding

$$Z_{Mod}(i,i) = \frac{\{[(1 \; \frac{\omega}{\Omega})^{2}] + 4(\frac{\omega}{\Omega})^{2\,2}\}\; \omega^{2} m}{[1 - (\frac{\omega}{\Omega})^{2}]^{2} + 4\xi^{2}} + i\,\frac{2(\frac{\omega}{\Omega})^{3}\,\xi\omega^{2} m}{[1 - (\frac{\omega}{\Omega})^{2}]^{2} + 4\xi^{2}} + Z_{Orig}(i,i) \tag{7}$$

For the addition of a lumped mass the impedance change becomes

$$Z_{Mod}(i,i) = -\,\omega^{2} m + Z_{Orig}(i,i) \tag{8}$$

In the above equations, $\omega$ is the frequency of excitation and $\Omega$, m and $\xi$ are the undamped natural frequency, the mass and the damping ratio, respectively, of the vibration absorber. This change is made in the original impedance matrix to the diagonal element corresponding to the point at which the vibration or lumped mass is attached.

3. Addition of Spring-Damper in Series

Since the addition of a parallel spring-damper system at a coordinate generates a coordinate at the free end of the system, an additional row and column are included in the modified impedance matrix. If k is the spring rate and c the damping rate of the system, the modified impedance matrix becomes

$$Z_{Mod} = \begin{array}{c} \text{Attachment} \\ \text{Coordinate} \end{array} \qquad \begin{array}{c} \text{Additional} \\ \text{Column} \end{array}$$

$$Z_{Mod} = \left[ \begin{array}{c|c} \vdots & 0 \\ \vdots & \vdots \\ \ldots\ (Z_{Orig} + k + i\ c)\ \ldots & (k + i\ c) \\ \vdots & \vdots \\ \vdots & \vdots \\ \hline \ldots 0 \ldots (k + i\ c) \ldots\ldots 0 \ldots\ldots & (k + i\ c) \end{array} \right] \tag{9}$$

$$\text{Additional Row}$$

4. Addition of Spring-Damper to Ground

In this situation the modified impedance matrix is simply the original impedance matrix with each diagonal element altered by the spring-damper system impedance:

$$Z_{Mod}(i,i) = Z_{Orig}(i,i) + k + i\ c \tag{10}$$

5. Coordinate Transformation

Any linear transformation of coordinates may be effected. Rotations, sign changes and combinations of rotation and translation are common applications. Transformation of local coordinates into the global system for component synthesis is a typical application. Additionally, transformation of coordinates on interfacing components to conform to the direction of the line of action of a gear mesh may be implemented. If the transformation matrix is defined as T, the impedance matrix for the transformed coordinates becomes

$$Z_{Mod} = T^T\ Z_{Orig}\ T^1 \tag{11}$$

6. Coordinate Reduction

Coordinate reduction must be performed on the mobility matrix since the elements of this matrix have individual physical significance. Thus, to accomplish a coordinate reduction the impedance matrix for a component must be retrieved from the data bank and inverted to yield the mobility matrix. The coordinates to be eliminated are removed from the mobility matrix and the resulting matrix inverted to form the desired reduced impedance matrix.

$$Y_{Orig} = Z_{Orig}^{\ 1}; \qquad Y_{Mod} = (Y_{Orig})_{Reduced}; \qquad Z_{Reduced} = Y_{Mod} \tag{12}$$

## APPLICATION

The method was successfully applied in a vibration study of a helicopter transmission shown schematically in Figure 2 (Ref 2). The configuration was appropriate to a substructure type analysis satisfying the requirements of a small number of components and a small number of excitation frequencies. The transmission case and each of the shafts were treated as substructures with interfaces at the bearings and gear meshes and which could be reduced to an analytical model with a small number of coordinates. The study included such effects as bearing stiffness, mass and stiffness changes in the shafts, case damping, case vibration absorbers and mounting characteristics.

To validate the dynamic substructure analysis method, analytically derived vibration characteristics were compared to simulated operational test data for a Kaman SH-2D helicopter main transmission. Acceleration response of the transmission case, measured normal to the surface, at fourteen selected points was compared to the respective analytically obtained response. Figure 3 presents a comparison of measured and predicted case surface accelerations in peak g's for excitation applied at the planetary system fundamental frequency of 348 Hz at 80% rotor rpm and a torque loading of 9120 in-lb.

## WIND TURBINE APPLICATION

The method may be cost effectively applied in wind turbine drive train dynamic analyses. A typical wind turbine power system, including a torsional isolation system is presented in Figure 4. Using the rotor, torsional isolation system, gearbox and generator impedance matrices the torque response at the generator may be determined. Thus, parametric studies may be easily conducted to evaluate the effect of torsional isolation system, gearbox modifications and generator structural characteristics on the generator output.

## CONCLUSIONS

1. An analytical method and associated computer program have been developed which have application in the dynamic analysis of a linear complex structure and modifications of it at a small number of discrete frequencies.

2. The method has been successfully demonstrated in the dynamic analysis of a helicopter main transmission where the components consisted of a gearbox case, the shafts and gears and a planetary system.

3. The method has application in the dynamic analyses of wind turbine drive systems.

## REFERENCES


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2. Bowes, M. A., "Development and Evaluation of a Method for Predicting the Vibration and Noise Characteristics of Helicopter Transmissions", AHS Paper No. 77.33-76, American Helicopter Society 33rd Annual National Forum, Washington, D.C., May 1977.

3. Berman, A., Flannelly, W. G., "Theory of Incomplete Models of Dynamic Structures", AIAA Journal, Volume 9, August 1971, pp 1481-1487.


## DISCUSSION

Q. Are there problems with inversions if experimental data are used?

A. Information loss can be significant when developing an analytical model from experimental data. Information loss in inversion can be minimized if the ratio of extreme eigenvalues which characterize the structure is minimized. One approach is to form several models, each valid over a limited frequency band, within the complete frequency spectrum. An alternate technique is to use the method of incomplete models of dynamic structures (Ref 3) which combines analytical and test data to yield a model which is valid for the points of interest over a limited frequency range.

Q. Does provision exist in planetary gearing for torsional damping?

A. Torsional damping can be introduced in the planetary gear mesh by adding structural damping at gear mesh coordinates.

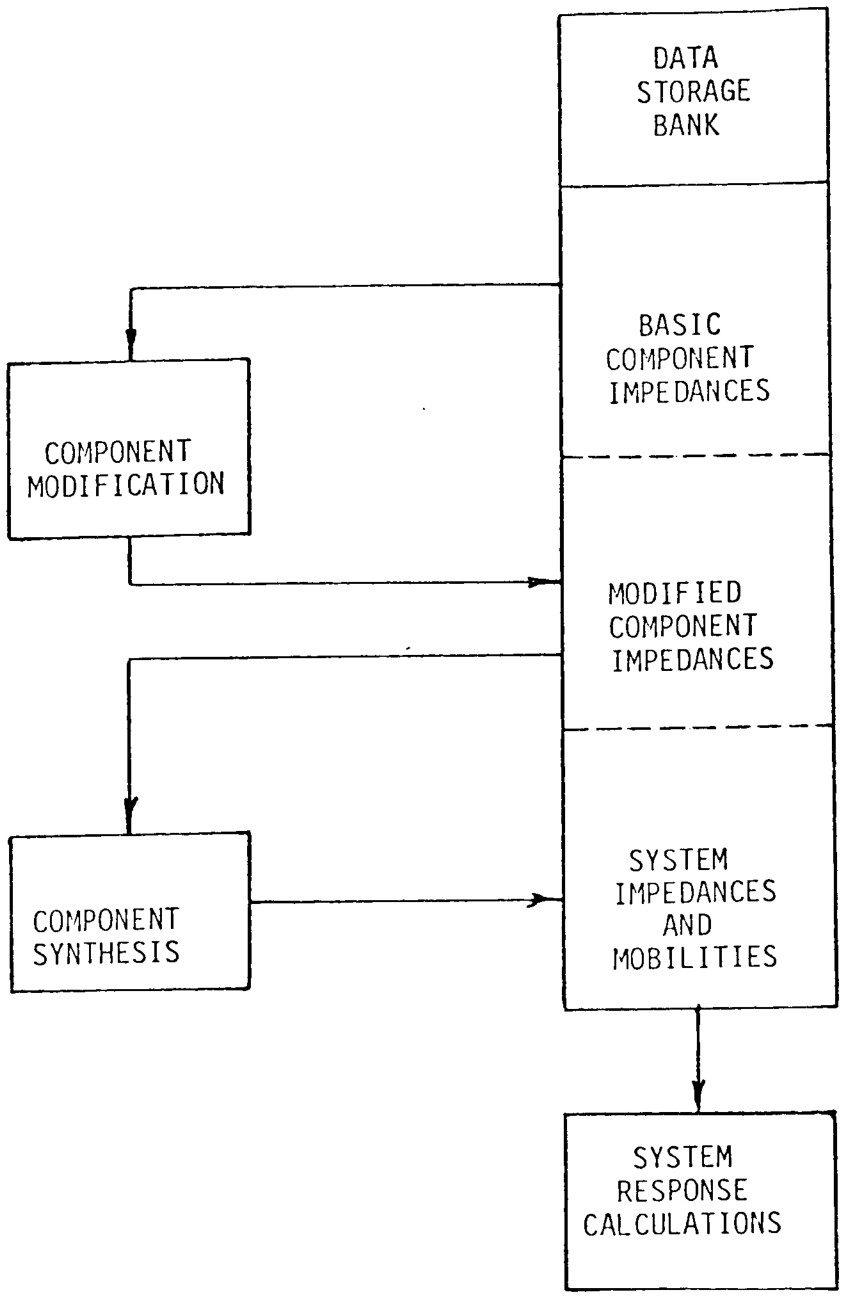


Figure 1. - Schematic of computer program.

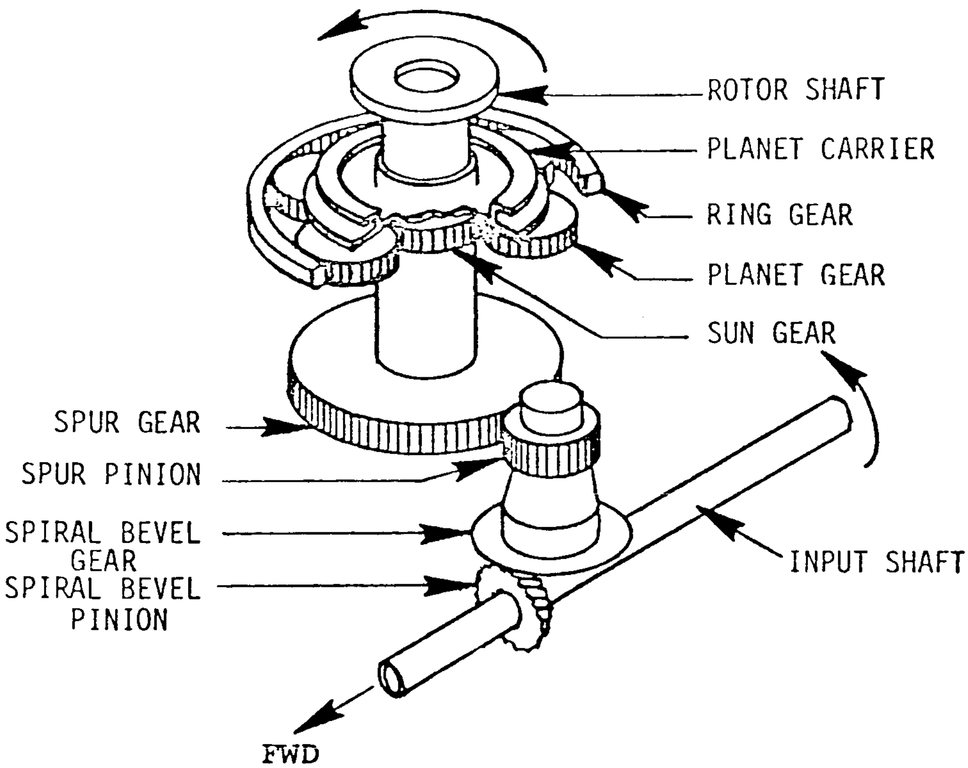


Figure 2. - Schematic of SH2 main transmission.

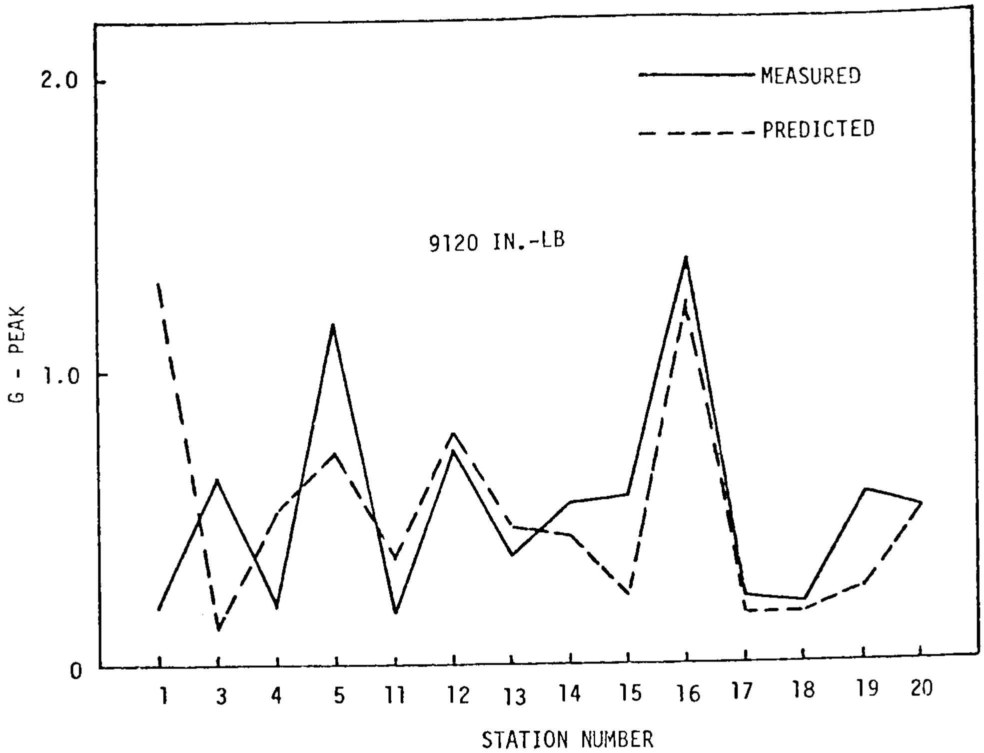


Figure 3. - Measured versus predicted case acceleration for 348-Hz excitation.

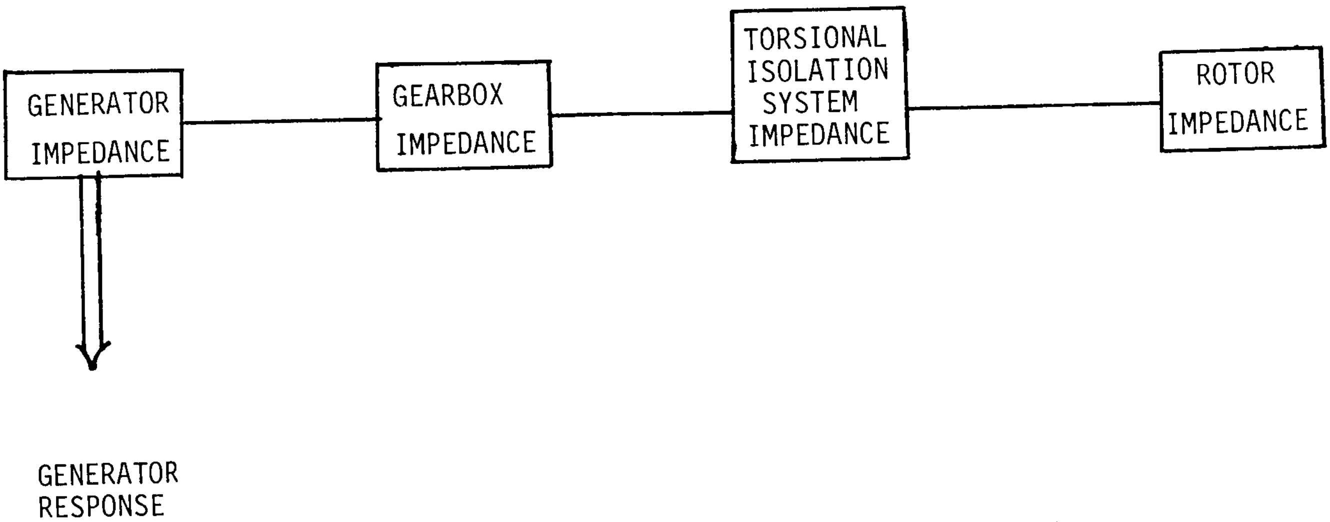


Figure 4. - Schematic of wind turbine power system.