A NASTRAN IMPLEMENTATION OF THE DOUBLY ASYMPTOTIC APPROXIMATION FOR UNDERWATER SHOCK RESPONSE*

Gordon C. Everstine
David W. Taylor Naval Ship Research and Development Center

SUMMARY

A detailed description is given of how the decoupling approximation known as the doubly asymptotic approximation (DAA) can be implemented with NASTRAN to solve shock problems for submerged structures. The general approach involves locating the nonsymmetric terms (which couple structural and fluid variables) on the right-hand side of the equations. This approach results in coefficient matrices of acceptable bandwidth but degrades numerical stability, requiring a smaller time step size than would otherwise be used. It is also shown how the structure's added (virtual) mass matrix, a necessary ingredient to DAA, can be calculated with NASTRAN. The version of NASTRAN used is NASA's standard level 16 with one program modification, velocity-dependent nonlinear loads, for which the FORTRAN changes are listed.

STATEMENT OF THE PROBLEM

The general class of problems known as fluid-structure interaction problems includes the special case of determining the shock response of submerged structures. This is of particular interest to naval engineers concerned with the dynamic structural response of submarines (including the hull, appendages, and internal equipment).

Consider the idealized situation consisting of a ring-stiffened cylindrical shell with flat end caps which is deeply submerged in water, initially at rest, and subjected to the shock of a distant underwater explosion (fig. 1). The general problem is to compute the time-dependent elastic structural response of the cylinder. We will further simplify the problem with the following assumptions:

1. The shock wavefront in the vicinity of the cylinder is planar, a reasonable assumption whenever the source of the shock is located far away.

2. The time history of the free-field incident pressure is a step function. This assumption can be made without loss in generality since the

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structural response to an arbitrary time variation of the pressure history can be easily obtained by a convolution integral involving the step response. (See Appendix D.)

3. The shock wave hits the cylinder from the side rather than from the end or from some more general oblique direction. This assumption results primarily in simplifying the data deck which describes the problem. Removing the restriction presents no conceptual difficulty.

BACKGROUND

In either the absence or the presence of the surrounding water, the structure can be modeled with finite elements in the usual way: with plate or shell elements for the unstiffened cylinder, and with beam elements for the ring stiffeners.

The fluid is generally treated as an acoustic medium (e.g., see ref. 1): a compressible, inviscid fluid which undergoes only small amplitude motion and whose pressure p satisfies the wave equation

$$\nabla^2 p = \ddot{p}/c^2 \tag{1}$$

where dots indicate time differentiation and c is the speed of sound. At an interface between the fluid and a solid

$$\frac{\partial p}{\partial n} = -\rho \ddot{u}_n \tag{2}$$

where n is the unit outward normal from the solid at the interface, ρ is the fluid mass density, and u_n is the outward normal component of displacement of the interface.

In principle, the fluid part of the problem can be handled by modeling a portion of the fluid with finite elements (refs. 2, 3). In reference 3, for example, the analogy was drawn between the scalar wave equation (1) and the elasticity equations so that a standard structural analysis computer program like NASTRAN (refs. 4, 5) can be used to solve problems involving the wave equation, Poisson's equation, or Laplace's equation. For finite fluid regions such an approach presents no significant problems. However, for structures submerged in infinite fluids, the analyst is faced with the additional problem of truncating the fluid and applying a radiation condition at the artificial boundary in order to absorb outgoing waves. Even if a reasonable radiation condition could be formulated, the cost of explicit fluid modeling could be prohibitive.

An attractive alternative to such modeling is provided by approximations (refs. 6, 7) which uncouple the structural response from the fluid response in the sense that the fluid pressure at the fluid-structure interface is determined (approximately) from a knowledge of only the interface motion. Although

several such decoupling approximations have been formulated, one which is currently attracting attention is the doubly asymptotic approximation (DAA) of Geers (ref. 6). This paper shows how the DAA scheme can be cast in a form which can be conveniently solved by NASTRAN. The version of NASTRAN used is NASA's standard level 16 with one program modification: velocity-dependent nonlinear loads. The less efficient earlier version of NASTRAN, level 15, can also be used, although at greater computer expense. The addition of velocity-dependent nonlinear load capability is convenient, but not crucial, for the NASTRAN implementation of DAA.

THE DOUBLY ASYMPTOTIC APPROXIMATION (DAA)

A submerged structure subjected to an underwater shock wave experiences, at any given time t, a total dynamic pressure p which can be considered to comprise two components: an incident pressure p_i which would occur if no obstacle (the structure) were present, and a scattered pressure p_s which is the difference between the total pressure and the incident pressure. Thus,

$$p = p_i + p_s \tag{3}$$

The scattered pressure p_s is sometimes further decomposed as

$$p_s = p_{rs} + p_r \tag{4}$$

where p_{rs} is the scattered pressure which would result if the structure were rigid and stationary and p_r, the radiated pressure, is the remainder. Of the three components of pressure, only p_r depends on the structural motion, whereas both p_i and p_{rs} can be computed as if the structure were rigid and stationary.

For the submerged ring-stiffened cylinder of interest (fig. 1), the plane wave incident pressure p_i is taken to be a step function (with wavefront moving to the left):

$$p_i(x,t) = p_0 H(x - x_0 + ct) \tag{5}$$

where H is the dimensionless Heaviside unit step function (zero for negative argument and unity for positive argument), x_0-ct is the location of the wavefront at time t, c is the speed of sound, and p_0 is a constant.

The scattered pressure p_s, which depends on the structural motion and hence cannot be precomputed as a function of time, is determined by the doubly asymptotic approximation (DAA) (ref. 6) from

$$\dot{p}_s + \rho c M_a^{-1} A p_s = \rho c \ddot{u}_s \tag{6}$$

where p_s is the vector of unknown scattered pressures at the wet grid points of

the structure, M_a is the (full) added mass matrix for the structure (see Appendix A), ρ and c are the fluid mass density and sound speed, respectively, A is a diagonal area matrix converting grid point pressures to grid point forces, and $\ddot{\bar{u}}_s$ is the vector of scattered wave particle accelerations normal to the structure's surface. The bar is used to distinguish this vector from the complete acceleration vector $\ddot{u}_s$, which involves all structural degrees of freedom rather than just the normal components at wet points. Surface normals are taken as positive going into the fluid.

Surface normal accelerations, like pressures, are decomposed into incident and scattered components; hence,

$$\ddot{\bar{u}}_s = \ddot{\bar{u}} - \ddot{\bar{u}}_i \tag{7}$$

where $\ddot{\bar{u}}$ is the vector of total normal accelerations at the wet grid points, and $\ddot{\bar{u}}_i$ is the vector of normal components (positive into the fluid) of incident fluid particle accelerations.

Equation (6) was designated "doubly asymptotic" because it exhibits the correct asymptotic behavior at both the low and high frequency limits: at the low frequency limit (which normally corresponds to late time behavior for transient situations), the first term of (6) is dominated by the second term, and (6) reduces to

$$F_s = A p_s = M_a \ddot{\bar{u}}_s \tag{8}$$

in which the fluid loading is due to added (virtual) mass effects alone. At the high frequency limit (early time behavior), the first term of (6) dominates the second term, and (6) reduces to

$$p_s = \rho c \dot{\bar{u}}_s \tag{9}$$

which is the usual radiation damping relation. Equations (8) and (9) are referred to individually as the virtual mass and plane wave approximations, respectively.

In general, the DAA, equation (6), yields better results than either of the special cases, equations (8) or (9). Huang (ref. 8) compared a DAA solution to an exact solution for a spherical shell and found that the DAA solution had slightly faster oscillations and stronger damping. Nevertheless, the DAA provides a good compromise between cost and accuracy for underwater shock problems.

DAA WITH NASTRAN

The differential equation of motion for the ring-stiffened cylinder of interest (fig. 1) is

$$M\ddot{u} + Ku = -\bar{A}p = -\bar{A}(p_i + p_s) \tag{10}$$

where u is the vector of unknown displacements at the grid points, M and K are the structure's finite element mass and stiffness matrices, respectively, p is the vector (of dimension equal to the number of wet grid points) of fluid pressures at the wet grid points, and $\bar{A}$ is an area matrix converting pressures to forces. $\bar{A}$ is not square (and hence not diagonal) because the vectors u and p are of different dimension. $\bar{A}$ contains non-zeros (equal to the area contributions) only at the intersections of rows corresponding to wet structural degrees of freedom with the columns of associated pressure variables. Thus, each row and column of $\bar{A}$ has at most one non-zero entry. $\bar{A}$ reduces to A if the zero rows are deleted and if the wet structural degrees of freedom are sequenced in the same order as the corresponding fluid pressure degrees of freedom. The area matrices defined here are "lumped" rather than "consistent". To switch to consistent loading, one need only change the area matrices A and $\bar{A}$.

In equation (10), the total dynamic fluid pressure p is decomposed into incident and scattered pressures given by equations (5) and (6), respectively. Since (6) is a differential equation, the complete problem involves solving (10) and (6) simultaneously, where the right-hand side of (6) is replaced by its equivalent from equation (7).

The incident fluid particle normal acceleration vector $\ddot{\bar{u}}_i$ is computed as follows: In general, the ratio of the pressure to the volume strain defines the bulk modulus k. Since $k = \rho c^2$ for the acoustic fluid, we have, for a plane wave,

$$p = -k\frac{\partial u_x}{\partial x} = -\rho c^2 \frac{\partial u_x}{\partial x} \tag{11}$$

In particular, for the incident component,

$$p_i = -\rho c^2 \frac{\partial u_{xi}}{\partial x} \tag{12}$$

From (5) and (12), it follows that

$$p_i = -\rho c \dot{u}_{xi} \tag{13}$$

where $\dot{u}_{xi}$ is the x-component of incident fluid particle velocity. The normal component of incident particle velocity $\dot{\bar{u}}_i$ is

$$\dot{\bar{u}}_i = \dot{u}_{ix} \cos\theta \tag{14}$$

Hence

$$p_i \cos\theta = -\rho c \dot{\bar{u}}_i \tag{15}$$

Because p_i as given by (5) is a step function, the $\ddot{\bar{u}}_i$ needed on the right-hand side of (6) is a Dirac delta. This problem can be avoided by defining a new variable q such that

$$\dot{q} = p_s \tag{16}$$

and time integrating equation (6). Equations (10) and (6) then become

$$\begin{aligned} M\ddot{u} + Ku &= -\bar{A}p_i - \bar{A}\dot{q} \\ A\dot{q} + \rho c\, A M_a^{-1} A\, q &= \rho c \bar{A}^T \dot{u} + A p_i \cos\theta \end{aligned} \tag{17}$$

where the second of equations (17) has also been multiplied by the area matrix A to symmetrize the coefficient matrices, and

$$A\dot{\bar{u}} = \bar{A}^T \dot{u} \tag{18}$$

In matrix format, these equations are

$$\begin{bmatrix} M & \\ & 0 \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{q} \end{Bmatrix} + \begin{bmatrix} 0 & \\ & A \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{q} \end{Bmatrix} + \begin{bmatrix} K & \\ & \rho c\, A M_a^{-1} A \end{bmatrix} \begin{Bmatrix} u \\ q \end{Bmatrix} = \begin{Bmatrix} -\bar{A}p_i - \bar{A}\dot{q} \\ \rho c \bar{A}^T \dot{u} + A p_i \cos\theta \end{Bmatrix} \tag{19}$$

which is the form of the equations which NASTRAN uses.

It is interesting to observe that the new variable q defined by equation (16) is, in essence, the (scattered) velocity potential, since for an acoustic fluid the velocity potential ϕ is related to the pressure p by (ref. 9)

$$p = -\rho \dot{\phi} \tag{20}$$

Thus, as a consequence of trying to avoid the numerical problem of a Dirac delta, the fundamental unknown for the fluid is switched from the pressure to the velocity potential, thus returning to the established convention of fluid dynamicists.

In equation (19), the unknowns u and q are defined using GRID cards. For the variables q, only one degree of freedom per point is retained. The usual finite element modeling of the structure yields M and K. The damping matrix is created by attaching dashpots (CDAMPi) between each interface fluid point and ground. The fluid matrix $\rho c A M_a^{-1} A$ can be assembled either by supplying it directly (on DMIG or DMI cards) or by letting NASTRAN compute it using an explicit finite element model of a portion of the fluid region. In Appendix A, it is shown that

$$\rho c A M_a^{-1} A = c H \tag{21}$$

where H is the fluid stiffness matrix condensed to the wet degrees of freedom. This condensation (using OMIT cards) is not necessary for the calculation but may result in a faster integration. The multiplicative constant c in the term cH, equation (21), can be automatically incorporated by setting the shear modulus on the MAT1 card equal to c rather than unity.

The right-hand side of (19) consists of both time-dependent and velocity-dependent loads, which are supplied using TLOAD1 and NOLIN1 cards, respectively. (See Appendix C.) The input of the incident pressure is particularly simple since it is a step function. The input data can be further simplified by using the DELAY card to indicate that the incident wavefront (which is traveling at speed c) does not reach all points at the same time.

In equation (19), M and A are diagonal matrices, and K and H are positive definite and symmetric. K is also large and banded. H can be either large and banded, or small and full, depending on whether a static condensation (with OMIT's) is applied to it.

The unknowns in equation (19) are arranged so that the structural and fluid variables are uncoupled on the left-hand side, the only coupling occurring on the right. Thus the grid points should be sequenced to maintain the indicated partitioning and to give both K and H the smallest possible matrix wavefront (refs. 10, 11).

The time step size needed to achieve numerical stability when the velocity-dependent terms are on the right was found to be about 1/10 of the transit time (the time required for a wave to travel one radius of the cylinder at speed c).

ALTERNATIVE DAA APPROACHES

Since the velocity-dependent loads in equation (19) are linear, they can be moved to the left-hand side and incorporated in the damping matrix. Symmetry (but not positive definiteness) is then retained by dividing the second equation in (19) by $-\rho c$. Since this formulation causes fluid-structural coupling on the left, the unknowns have to be sequenced taking into account this new connectivity. This approach is practical only if the fluid stiffness matrix $\bar{H}$ is not condensed but left large and banded, so that the overall system can be made banded. Otherwise, the added mass matrix coupling causes non-zeros far off the matrix diagonals.

The principal advantage in placing the velocity-dependent terms on the left is numerical stability, so that a larger integration time step can be used. With those terms on the right, as in equation (19), the matrix bandwidth (and hence wavefront) is smaller, and the user has the option of condensing the fluid "stiffness" matrix $c\bar{H}$ into the smaller, but full, matrix $\rho c A M_a^{-1} A$.

Another possible way to formulate the DAA is to make use of the decomposition of scattered pressure p_s into rigid body scattered and radiated components (eq. (4)). Since only the radiation pressure p_r depends on structural motion, the rigid body scattered pressure p_{rs} can be precomputed and combined with the incident pressure p_i. In that case, the DAA (eq. (6)) must supply only p_r, which satisfies

$$\dot{p}_r + \rho c M_a^{-1} A p_r = \rho c \ddot{\bar{u}}_r \tag{22}$$

where the normal component of fluid particle acceleration at the fluid-solid interface is decomposed into

$$\ddot{\bar{u}} = \ddot{\bar{u}}_i + \ddot{\bar{u}}_{rs} + \ddot{\bar{u}}_r \tag{23}$$

For rigid stationary structures, equation (23) simplifies to

$$\ddot{\bar{u}}_i + \ddot{\bar{u}}_{rs} = 0 \tag{24}$$

at the interface, so that, in general,

$$\ddot{\bar{u}} = \ddot{\bar{u}}_r \tag{25}$$

at the interface. Thus, (22) is equivalent to

$$\dot{p}_r + \rho c M_a^{-1} A p_r = \rho c \ddot{\bar{u}} \tag{26}$$

The advantage of this general approach is that the rigid body scattered pressure p_{rs} can be computed in advance to whatever accuracy one wants, so that the only approximation remaining involves the radiation pressure p_r. The disadvantage, however, is that the pre-calculation (a nontrivial one) has to be done at all. The decision of whether to use equation (6) or (26) also depends on the relative sizes of p_{rs} and p_r, since if p_{rs} were small it would not make sense to compute it accurately. Unfortunately, the relative sizes are problem-dependent and hard to estimate.

EXPLICIT FINITE ELEMENT FLUID MODELING

The problem of computing the linear shock response of submerged structures can, in principle, be solved by explicit finite element modeling of a portion of the fluid volume. The purpose of this section is to formulate the problem sufficiently so that it can be solved by NASTRAN once the user has picked a suitable radiation condition to apply at the outer fluid boundary.

The total dynamic fluid pressure satisfies the wave equation (1) in the field. This pressure can be decomposed into the sum of incident and scattered pressures, p_i and p_s, as in equation (3). Since p_i is defined to satisfy (1),

p_s must also satisfy the wave equation (1). At a fluid-solid interface, the boundary condition (2) becomes

$$\frac{\partial p_s}{\partial n} = - \frac{\partial p_i}{\partial n} - \rho \ddot{\bar{u}} \tag{27}$$

where, for the finite cylinder of figure 1 subjected to a plane wave incident pressure, equation (5), we have

$$\frac{\partial p_i}{\partial n} = \nabla p_i \cdot \underset{\sim}{n} = \frac{\partial p_i}{\partial x} \cos \theta \tag{28}$$

and

$$\frac{\partial p_i}{\partial x} = \frac{1}{c} \dot{p}_i \tag{29}$$

Thus, from (27),

$$\frac{\partial p_s}{\partial n} = - \frac{1}{c} \dot{p}_i \cos \theta - \rho \ddot{\bar{u}} \tag{30}$$

The above Neumann boundary condition is equivalent to specifying a "load" on each interface pressure variable p_s equal to

$$A\left(\frac{1}{c}\dot{p}_i \cos \theta + \rho \ddot{\bar{u}}\right) \tag{31}$$

where A is the area associated with the interface point, so that the resulting finite element equations take the form (ref. 3)

$$\begin{bmatrix} M & \\ & cQ \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{q} \end{Bmatrix} + \begin{bmatrix} K & \\ & c\bar{H} \end{bmatrix} \begin{Bmatrix} u \\ q \end{Bmatrix} = \begin{Bmatrix} -\hat{A} p_i - \hat{A} \dot{q} \\ \tilde{A} p_i \cos \theta + \rho c \hat{A}^T \dot{u} \end{Bmatrix} \tag{32}$$

where here q, defined as in equation (16), includes all fluid points, not just interface points. The area matrices also have to be redefined slightly to reflect the change in dimension of the vector q. The above formulation is consistent with the definitions of fluid inertia Q and stiffness $\bar{H}$ given in Appendix A, which differ from the definitions of ref. 3 by a constant factor ρc^2.

Equation (32) is complete except for a radiation condition on the pressure variable q. Once the user decides what radiation condition to use, it can be incorporated into the matrix equation (32).

It is interesting to observe the similarity between the explicit finite element formulation, equation (32), and that which arises from the doubly asymptotic approximation, equation (19). The right-hand sides and the overall stiffness matrices are the same in both cases. In (32), the overall mass

matrix now includes the fluid inertia, which NASTRAN computes whenever the user supplies a non-zero mass density, in this case equal numerically to 1/c. The damping matrix in (19) also appears in (32) if a radiation condition involving dashpots is used, although in (32) the dashpots connect the outer boundary points, rather than the interface pressure points, to ground.

Another approach using explicit fluid modeling was recently described by Newton and Atchison (ref. 12), who elected to use the full fluid pressure p (rather than p_s) as the fundamental pressure unknown. In that case, the time-dependent part of the right-hand side of the equations of motion is replaced by a non-zero initial condition on p and $\dot{p}$ throughout the fluid region.

The main impediments to solving the shock problem by these approaches are the potentially high cost of modeling a three-dimensional region of fluid and the difficulty in determining the radiation condition. For one-dimensional problems, the correct radiation condition merely involves attaching grounded dashpots to the outer fluid boundary (ref. 2). However, for general three-dimensional situations, the mathematically exact radiation condition is a more complicated relation (which cannot be modeled using only masses, springs, and dashpots) coupling all pressure variables at the outer boundary (ref. 13). Since the implementation of such a condition is impractical, the analyst must resort to approximate radiation conditions which will not absorb 100% of outgoing waves. It is for reasons like these that decoupling approximations such as DAA are being used.

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APPENDIX A - ADDED MASS MATRICES

Consider an elastic structure submerged in a finite acoustic fluid, whose pressure p satisfies the wave equation

$$\nabla^2 p = \ddot{p}/c^2 \tag{A1}$$

where c is the speed of sound in the fluid. If both structure and fluid are modeled with finite elements, the resulting matrix equations take the general form (refs. 2, 3)

$$\begin{bmatrix} M & 0 \\ -\rho\hat{A}^T & Q \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{p} \end{Bmatrix} + \begin{bmatrix} K & \hat{A} \\ 0 & \bar{H} \end{bmatrix} \begin{Bmatrix} u \\ p \end{Bmatrix} = \begin{Bmatrix} f \\ 0 \end{Bmatrix} \tag{A2}$$

where M and K are the usual structural mass and stiffness matrices, Q and $\bar{H}$ are the inertia and "stiffness"matrices for the fluid, $\hat{A}$ is the area matrix converting pressure to force at the fluid-structure interface nodes, and ρ is the fluid's mass density.

In equation (A2), $\bar{H}$ can be assembled from standard 3-D elasticity finite elements (ref. 3) if only the x-component of displacement at each point is retained (to represent the scalar quantity p) and Hooke's law is specified as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} 1 & -1 & -1 & & & \\ -1 & 1 & -1 & & & \\ -1 & -1 & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{A3}$$

In terms of the usual engineering constants, equation (A3) is equivalent (numerically) to choosing the shear modulus G and Young's modulus E as

$$G = 1$$

$$E = \alpha G, \quad \alpha >> 1$$

whose α must be large enough so that $\alpha+1$ is indistinguishable (numerically) from α. On most computers, $\alpha = 10^{20}$ suffices. Equation (A4) applies only in three dimensions. In two dimensions (plane stress), the corresponding constants are

$$G = 1$$

$$E = \beta G, \quad \beta << 1 \tag{A5}$$

where β should not be so small that $1+\beta$ is indistinguishable (numerically) from unity. On most computers, $\beta = 10^{-4}$ suffices.

Equation (A5) also applies to axisymmetric problems formulated in cylindrical coordinates with axisymmetric elements such as NASTRAN's CTRAPRG. However in this case only the z-component of displacement can be used to represent pressure, in contrast to Cartesian coordinates in which any of the three translation components can be used.

In equation (A2), Q can be assembled from standard elasticity finite elements (ref. 3) if the mass density assigned to the material is numerically equal to $1/c^2$.

For an incompressible fluid, $c \to \infty$ (or the frequency $\omega \to 0$) and the wave equation (A1) reduces to Laplace's equation

$$\nabla^2 p = 0 \tag{A6}$$

Also, $Q = 0$, so that p can be eliminated from (A2) to yield

$$(M + \rho \hat{A} \bar{H}^{-1} \hat{A}^T) \ddot{u} + Ku = f \tag{A7}$$

thus defining the added mass matrix

$$\bar{M}_a = \rho \hat{A} \bar{H}^{-1} \hat{A}^T \tag{A8}$$

for the submerged structure.

We observe that the area matrix $\hat{A}$ is non-square since the vectors u and p are of different dimension. In addition, $\hat{A}$ is such that each row or column has but one non-zero entry. This entry is the area assigned to a particular node and located at the row corresponding to the nodal outward normal displacement and column corresponding to the associated pressure variable. Thus, since $\hat{A}$ involves only the interface variables, the fluid stiffness matrix $\bar{H}$ in equation (A8) can be reduced by static condensation (Guyan reduction) prior to performing the matrix product in (A8). If the condensed stiffness matrix is denoted H, then the corresponding added mass matrix is

$$M_a = \rho A H^{-1} A \tag{A9}$$

where A is the diagonal area matrix and M_a involves only wet degrees of freedom rather than all structural degrees of freedom as in $\bar{M}_a$.

A physical interpretation of a particular ij entry in M_a or $\bar{M}_a$ is that it is the normal fluid force induced at point i (on the fluid-structure interface) due to a unit normal acceleration at interface point j, with all other points held fixed. Thus, it is clear that M_a is a fully populated matrix. Since the interface boundary condition is

$$\frac{\partial p}{\partial n} = - \rho \ddot{u}_n \tag{A10}$$

the specification of acceleration at the interface is equivalent to a Neumann boundary condition. Hence, the calculation of the added mass matrix is mathematically equivalent to solving Laplace's equation (A6) in the fluid region with Neumann boundary conditions. For uniqueness, p must be specified somewhere.

This Neumann problem is also equivalent to a steady-state heat conduction problem in which one seeks the temperatures at all mesh points (on the fluid-solid interface) due to a unit heat source at each such point in turn. The matrix $\bar{H}$ in equation (A8) is exactly the heat conduction "stiffness" matrix computed by finite element heat transfer computer programs if the thermal conductivity is specified as unity.

Since, in heat conduction problems, the extreme temperatures must occur on the boundary, and uneven temperature distributions can be maintained only by supplying heat at the warmest point on the boundary and removing heat from the coolest point (ref. 14), it follows that the individual elements of the added mass matrix are always positive.

Thus far, this discussion of added mass matrices has assumed the fluid region to be finite. Of more interest in naval applications is the infinite region. In this case one can define and model a finite region of fluid whose outer boundary (where p=0) is "sufficiently far" from the structure. The major problem facing the analyst is deciding where to locate this outer boundary. For a given problem, one approach to insure that the outer boundary is distant enough is to compute the added mass matrix M_a (condensed to include only wet degrees of freedom) with two different locations of the outer boundary and look for convergence of the dominant terms in the matrix.

The calculation of added mass matrices for structures submerged in infinite fluids would be more appealing if it did not involve the explicit modeling of a portion of the fluid. Since the problem to be solved is a Neumann problem in the infinite region surrounding the structure, it can also be formulated in terms of simple sources distributed over the fluid-solid interface (ref. 15). For economy, the source density distribution is usually assumed constant over each surface element. Consequently all matrices (including the added mass matrix) refer to element centroids rather than to the finite element grid points. One possible approach for transforming an added mass matrix from element to grid point values is as follows: For simplicity assume a rectangular mesh of surface elements (fig. 2), where a typical element

(number $\hat{4}$) is shown connecting grid points 5, 6, 15, and 16. The simplest relationship between the central and nodal displacements is the arithmetic average

$$\hat{u}_4 = (u_5 + u_6 + u_{15} + u_{16})/4 \tag{A11}$$

A more complicated relation taking into account the element shape function can also be written. The complete transformation involving all elements is of the form

$$\hat{u} = \Gamma u \tag{A12}$$

where $\hat{u}$ is the displacement vector for the element centroids, u is the displacement vector for the grid points, and Γ is the transformation matrix. The added mass matrix can then be computed from

$$M_a = \Gamma^T \hat{M}_a \Gamma \tag{A13}$$

which is the usual transformation relationship for finite element matrices (ref. 16). In equation (A13), $\hat{M}_a$ is the added mass matrix referred to centroidal coordinates. The transformation (A13) may result in converting $\hat{M}_a$, which is non-singular, into a singular matrix M_a.

Virtual Mass

It is of interest to relate the added mass matrix (as used here) to the added mass (virtual mass) defined by hydrodynamicists (e.g., ref. 17). Virtual mass is a scalar quantity defined and computed for rigid structures oscillating in a specific rigid body motion, e.g., heave of a ship hull form. Since the added mass matrix is general enough to allow arbitrary elastic structural motion, virtual mass is merely a special case.

Recall that a physical interpretation of the added mass matrix is that a particular ij entry is the normal fluid force at point i due to a unit normal acceleration at j, with all other points held fixed. In computing virtual mass, the acceleration at all points is specified, and the component of force in a particular direction is desired. For example, if ϕ is a vector describing the amplitude of the desired rigid body motion, the virtual mass m in the same direction is

$$m = \phi^T M_a \phi \tag{A14}$$

where M_a is the added mass matrix and each component of ϕ is equal to the cosine of the angle between the surface normal at a point and the direction of motion (assuming unit amplitude motion). Equation (A14) is identical in form to the definition of generalized mass for vibration mode shapes (ref. 18).

In general, there exist six rigid body modes (three translations and three rotations), each of which induces six components of force. Thus (A14) can be

generalized to define a 6x6 rigid body virtual mass matrix m whose ij entry is

$$m_{ij} = \phi_i^T M_a \phi_j \tag{A15}$$

where ϕ_j is the vector specifying the rigid body acceleration and ϕ_i is the vector describing the direction of the generalized force induced.

APPENDIX B - USE OF STRUCTURAL SYMMETRY

In general, it is economically advantageous to exploit as much structural symmetry as possible when performing a structural analysis. This exploitation is possible whenever the structure, in the absence of loads, possesses geometrical and structural symmetry (ref. 19). Since time-dependent nonsymmetric loads can always be decomposed into the sum of symmetric and anti-symmetric parts, the overall problem can be decomposed in the same way. The purpose of this appendix is to summarize how this decomposition works for the class of wave problems arising in computing submerged shock response.

Consider the cylinder cross-section shown in figure 3 with a typical point number 1 and its image point number 2. The applied loads at the two points can be arbitrary functions of time.

The cylinder possesses numerous planes of symmetry, including the xz- and yz-planes. Thus, only one-fourth of the circumference has to be modeled. (In this particular case, the structure is axisymmetric and thus can be modeled using axisymmetric elements with nonsymmetric loading.) The indicated loading is symmetric with respect to the xz-plane and nonsymmetric with respect to yz. Since the problem is linear, the loading can be decomposed as shown in figure 3. with

$$\begin{aligned} F_s(t) &= (F_1(t) + F_2(t))/2 \\ F_a(t) &= (F_1(t) - F_2(t))/2 \end{aligned} \tag{B1}$$

where the decomposition results in one problem which is symmetric with respect to the yz-plane and another problem which is anti-symmetric with respect to yz. For each component part, it suffices to model but one quadrant (fig. 3) and apply the appropriate boundary conditions (either symmetric or anti-symmetric) for all points in the symmetry planes.

For structural grid points (whose fundamental unknown is displacement) lying in a plane of symmetry, the boundary conditions are that the points can suffer no translation out of the plane of symmetry and no rotation about in-plane lines. The anti-symmetry boundary conditions are that the complementary degrees of freedom are constrained. For example, in figure 3, all points lying in the yz-plane must satisfy

$$u_x = \theta_y = \theta_z = 0 \quad \text{for symmetry}$$

$$u_y = u_z = \theta_x = 0 \quad \text{for anti-symmetry} \tag{B2}$$

where θ denotes rotations.

For fluid grid points (whose fundamental unknown is pressure) lying in a plane of symmetry or anti-symmetry, the boundary conditions are

$$\frac{\partial p}{\partial n} = 0 \quad \text{for symmetry}$$

$$p = 0 \quad \text{for anti-symmetry} \tag{B3}$$

In finite element analysis, the above symmetry condition on p is a natural boundary condition and thus automatically satisfied if the unknown p is left free.

These conditions on fluid pressure grid points are applicable for any pressure points lying in a plane of symmetry or anti-symmetry, including those occurring in an explicit modeling of the fluid volume for the purpose of computing added mass matrices; i.e., the added mass matrix also has to exhibit the proper symmetry.

APPENDIX C - VELOCITY-DEPENDENT NONLINEAR LOADS

The finite element formulation derived to implement the doubly asymptotic approximation (DAA) with NASTRAN involves loads which, at each time step, depend explicitly on the current structural motion rather than on time. The standard versions of NASTRAN (levels 15 and 16) currently allow displacement-dependent loads but provide no convenient way to specify loads which depend on velocity or acceleration. (The implementation of such loads with a combination of transfer function (TF) and nonlinear load (NOLINi) cards is not only inconvenient but also results in nonsymmetric matrices.)

NASTRAN can be easily modified to allow the user to apply velocity- and acceleration-dependent loads using the NOLINi cards now used only for displacement-dependent loads. This appendix summarizes the FORTRAN changes to NASTRAN (level 16) needed to implement such loads.

The approach taken is compatible with that used in MSC/NASTRAN (ref. 20), in which the user indicates velocity dependence by adding 10 to the displacement component number CJ or CK on the NOLINi card. This modification is extended here to allow acceleration dependence, which is indicated by adding 20 to CJ or CK. Acceleration dependence as implemented here, however, is not fully general, since it does not allow a change in the time step size. The velocity dependence is fully general.

The finite difference formulas used to compute velocity and acceleration at the n^{th} time step are

$$\dot{u}_n = (u_n - u_{n-1})/\Delta t \tag{C1}$$

$$\ddot{u}_n = (u_n - 2u_{n-1} + u_{n-2})/\Delta t^2 \tag{C2}$$

where u_n is the displacement vector at the n^{th} time step and Δt is the time step size.

The listing of the FORTRAN changes to NASTRAN (level 16) appears in figure 4, in which the format of CDC's UPDATE utility is used. These modifications were adapted from similar changes made to NASTRAN's level 15 by Messrs. James M. McKee and Myles M. Hurwitz of the David W. Taylor Naval Ship Research and Development Center.

APPENDIX D - RESPONSE TO ARBITRARY TIME-DEPENDENT LOADING BY CONVOLUTION

When the linear shock response of large complex structures is to be computed with NASTRAN, it is often preferable to compute first the response to a step function, because (1) input data preparation for NASTRAN is simplified considerably, and (2) the response for any arbitrary time-dependent loading can be easily computed later by a convolution (superposition) integral (e.g., ref. 21), the formulas for which are summarized here.

Consider the general equation

$$Lw(x,t) = f(t) \tag{D1}$$

where L is a linear differential operator, w is some response variable (e.g., displacement, velocity, stress, etc.), and the forcing function f is considered here to represent the incident free-field pressure which arises in underwater shock problems.

If $w_s(x,t)$ is the response to a unit step function, then

$$\begin{aligned} w(x,t) &= f(0)w_s(x,t) + \int_0^t f'(\tau)w_s(x,t-\tau)d\tau \\ &= f(t)w_s(x,0) + \int_0^t f(\tau)w_s'(x,t-\tau)d\tau \end{aligned} \tag{D2}$$

where we define

$$w_s'(x,t) \equiv \frac{\partial}{\partial t} w_s(x,t) \tag{D3}$$

Either of the quadrature formulas (D2) can be used to compute the response to an arbitrary forcing function f(t). Since the two relations give different results numerically, the convolution can be computed both ways and averaged. For our work, a short computer program was written to compute w, given tabulated values of f and w_s for non-uniform spacing of the abscissas.

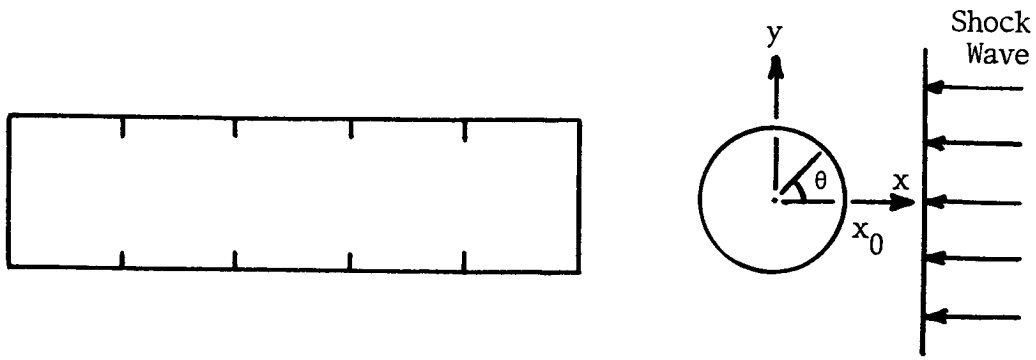

Figure 1. - Ring-Stiffened Cylindrical Shell
Subjected to Plane Wave Shock

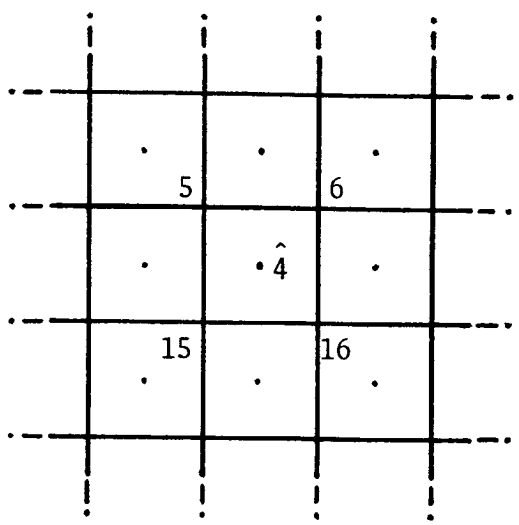

Figure 2. - Rectangular Mesh of Surface Elements
for Added Mass Matrix Transformation

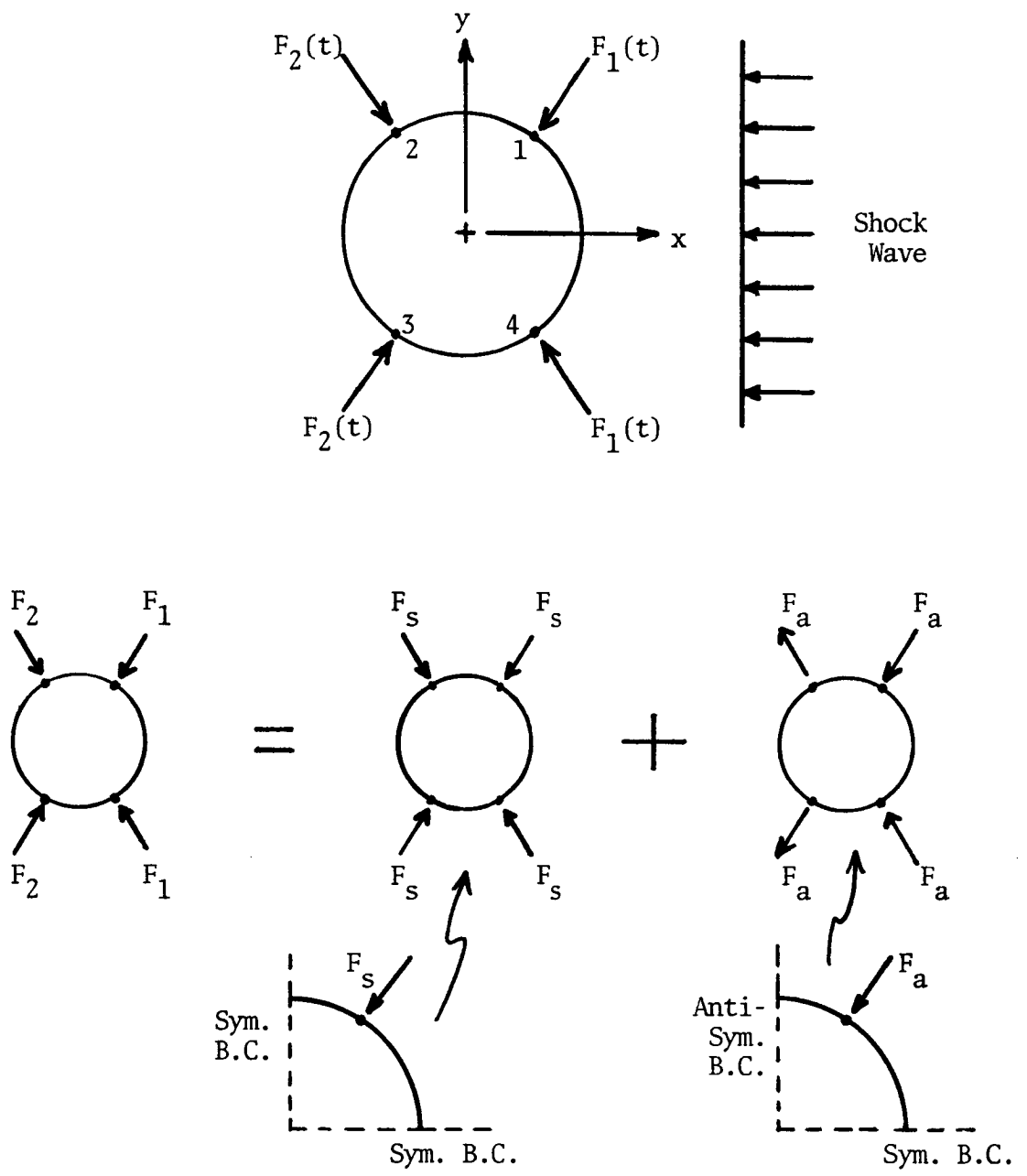

Figure 3. - The Superposition of Symmetric and Anti-Symmetric Solutions for Time-Dependent Problems with Nonsymmetric Loading

```
*IDENT EE842976
*COMPILE OPD4,IFS1P,TRD1C,TRD1D
*DELETE OPD4.78,OPD4.78
C
C     VELOCITY- OR ACCELERATION-DEPENDENT NL LOAD
C
C     LOOK FOR COMPONENTS .GT. 6 FOR INDEPENDENT DOF AND PACK WITH
C                  NOLIN CARD TYPE
C     VELOCITY (OR ACCELERATION) IS INDICATED BY ADDING 10 (OR 20) TO
C                     COMPONENT
C
      IUDOT=0
      IF(BUF(6).LE.6) GO TO 1341
      IUDOT=(BUF(6)/10) * 10
      BUF(6)=BUF(6)-IUDOT
 1341 IF(II.NE.2) GO TO 1345
      IF(BUF(8).LE.6) GO TO 1345
      IGET=(BUF(8)/10) * 10
      BUF(8)=BUF(8)-IGET
      IUDOT=IUDOT+10*IGET
*DELETE OPD4.87
      BUF(2) = II + IUDOT
*DELETE OPD4.163,OPD4.164
      IGET= BUF(1) - (BUF(1)/10) * 10
      IF(IGET.NE.2.AND.KK.EQ.8) GO TO 1396
*DELETE IFS1P.243
      IF( (M(6)/10) .GT.2) GO TO 8
      IF( (M(6)- (M(6)/10) * 10) .GT.6) GO TO 8
*DELETE IFS1P.250
      IF( (M(8)/10) .GT.2) GO TO 8
      IF( (M(8)- (M(8)/10) * 10) .GT.6) GO TO 8
*INSERT TRD1C.20
     2     ,IU3
*INSERT TRD1D.13
     3     ,IU1,DELTAT,IU3
C     IN NL LOAD CALCULATION, IU IS LATEST U, IU1 IS ONE BACK, AND
C                     IU3 IS TWO BACK
*DELETE TRD1D.125
      IGET=IZ(L)-(IZ(L)/10) * 10
      IF(IGET.NE.1) GO TO 110
*INSERT TRD1D.181
C
C     VELOCITY- AND ACCELERATION-DEPENDENT NL LOADS
C
C     STRIP VELOCITY OR ACCEL FLAGS FROM NOLIN CARD TYPE AND COMPUTE
C          VEL OR ACCEL FOR FLAGGED COMPONENTS . . . .
C        UDOT = (U(N)-U(N-1))/DELTAT
C        UDOTDOT= (U(N) - 2 * U(N-1) + U(N-2))/DELTAT**2
C
C*********************************************************************
C
C     NOTE - - -     ACCELERATION-DEPENDENT NL LOADS DO NOT WORK IF A
C                    CHANGE IN TIME STEP SIZE OCCURS.
C
C*********************************************************************
      H=1.0/DELTA T
      HH=H*H
*DELETE TRD1D.185,TRD1D.186
      MM=IU1+IZ(I+2)
      MMM=IU3+IZ(I+2)
      JFLG=IZ(I)/100
      IFLG=(IZ(I)-JFLG*100)/10
      L=IZ(I)-JFLG*100-IFLG*10
      IFLG=IFLG+1
      JFLG=JFLG+1
      GO TO (192,193,194), IFLG
C  DISPLACEMENT-DEPENDENT
  192 X=Z(M)
      GO TO 196
C  VELOCITY-DEPENDENT
  193 X=(Z(M)-Z(MM))*H
      GO TO 196
C  ACCELERATION-DEPENDENT
  194 X=0.
      IF(ICOUNT.GT.1)
     -X=(Z(M)-2.*Z(MM)+Z(MMM))*HH
  195 CONTINUE
*DELETE TRD1D.197
      MM=IU1+IZ(I+4)
      MMM=IU3+IZ(I+4)
      GO TO (212,213,214), JFLG
C  DISPLACEMENT-DEPENDENT
  212 FX=X*Z(M)
      GO TO 240
C  VELOCITY-DEPENDENT
  213 FX=X*(Z(M)-Z(MM))*H
      GO TO 240
C  ACCELERATION-DEPENDENT
  214 FX=0.
      IF(ILOOP.GT.1.OR.ICOUNT.GT.1)
     -FX=X*(Z(M)-2.*Z(MM)+Z(MMM))*HH
```

Figure 4. - Listing of FORTRAN Changes to NASTRAN Level 16 for Velocity- and Acceleration-Dependent Nonlinear Loads