PERFORMANCE ANALYSIS OF FLEXIBLE AIRCRAFT WITH ACTIVE CONTROL*

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SUMMARY

The small-perturbation equations of motion of a flexible aircraft with an active control technology (ACT) system were developed to evaluate the stability and performance of the controlled aircraft. The total aircraft system was formulated in state vector format and the system of equations was completed with fully unsteady and low-frequency aerodynamics for arbitrary, complex configurations based on a potential aerodynamic method. The ACT system equations have been incorporated in the digital computer program FCAP (Flight Control Analysis Program) which can be used for the analysis of complete aircraft configurations, including control system, with either low-frequency or fully unsteady aerodynamics. The application of classical performance analyses including frequency response, poles and zeros, mean-square response, and time response in FCAP in state vector format was discussed.

INTRODUCTION

The integrated study of the interactive effects of the flight control system in the active control of flexible aircraft has received considerable attention in recent years. In particular, Active Control Technology (ACT) is being investigated for improving ride quality, decreasing structural deformation, extending the fatigue life of the aircraft, relaxing static stability requirements, suppressing flutter, and reducing structural loads.

A new computer program, Flight Control Analysis Program (FCAP) has been developed for NASA to analyze ACT systems (refs. 1 and 2). The program was designed in a modular fashion to incorporate aircraft dynamics, aerodynamics for complex configurations, and sensor, actuator, and control logic dynamics, as well as analysis methods for determining stability and performance of the ACT system. The formulation of the total aircraft dynamic system for FCAP was unified by casting all the equations in state space format. This paper presents the state-vector formulation of the ACT system, and discusses its application in FCAP for the performance analysis of ACT systems.

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SYMBOLS

A, B, C, D	state-space matrices of ACT system and subsystems
A_1	matrix defined by Equation (17)
F	matrix defined by Equation (16)
H	matrix of transfer functions
$i\omega$	imaginary part of complex frequency, s
M_R	generalized mass/inertia matrix
q	dynamic pressure
q_N	Lagrangian generalized coordinates
R(0)	covariance matrix of ACT system outputs
r	output vector for ACT system and subsystems
s	complex frequency
U(0)	covariance matrix of ACT system inputs
U_{RA}, U_{RD}, U_{RR}	generalized aerodynamic force coefficients
U_{SD}, U_{SR}, U'_{SR}	coefficient matrices of aircraft dynamics equations at sensor locations
u	input vector for ACT system and subsystems
u'_L	matrix of pilot and guidance system commands
u_R	generalized aerodynamic forces in uniform flow
u'_R	matrix of aerodynamic forces due to turbulence
X(0)	covariance matrix of ACT system state variables
x	state vector

Subscripts:

A	actuator
D	aircraft displacement
L	control logic
R	aircraft rate
S	sensor

Superscripts:

T	transpose
(0), (1)	coefficients of power series expansion

A tilda (~) over a symbol indicates that it is designated in the Laplace domain. A dot over a variable indicates time differentiation.

ACT ANALYSIS

The ACT system formulated in FCAP is shown in Figure 1. External disturbances to the ACT system are seen to be atmospheric turbulence and gusts contributing to the aerodynamic forces and moments, and pilot or guidance system commands introduced through the control logic. The aircraft dynamic system includes both rigid-body and flexible-body dynamics.

The analysis of ACT systems is unified by casting all system equations in either the time domain or the frequency domain, and in similar format. In state space methods, the motion of a given dynamic system is described by the following pair of matrix equations (ref. 3):

$$\left.\begin{aligned} \dot{x} &= Ax + Bu \\ r &= Cx + Du \end{aligned}\right\} \tag{1}$$

where x is the state vector, u is the input (or control) vector, r is the output vector, and A, B, C, and D are the matrix coefficients. The equations for the dynamics of the state variables, and for the outputs of the aircraft dynamics, sensors, logic, and actuators are given in the following sections. The equations are then combined with equations for aerodynamics of the aircraft, and the total system matrix equations are formulated using the compatibility relationships among the dynamic systems.

Aircraft Dynamics

The FCAP aircraft dynamics equations for N degrees of freedom (six rigid-body and (N-6) flexible-body degrees of freedom) are restricted to small perturbations which reduce the equations to linear form. This is a reasonable approximation for ACT studies (e.g., refs. 4 and 5). The aircraft equation of motion expressed in state vector form is (ref. 1):

$$M_R \dot{x}_R = A_{RR} x_R + A_{RD} x_D + u_R + u'_R \tag{2}$$

where

x_D = state vector of the displacement variables = $\lfloor x, y, z, \Phi, \Theta, \Psi, q_7, \ldots q_N \rfloor^T$

x_R = state vector of the rate variables = $\lfloor u, v, w, p, q, r, \dot{q}_7, \ldots \dot{q}_N \rfloor^T$

Also, M_R = generalized mass matrix, A_{RR} = Coriolis force/damping matrix, A_{RD} = stiffness/gravity-force matrix, u_R = generalized aerodynamic forces in uniform flow, and u'_R = forces due to turbulence.

The output of the aircraft dynamics at the sensor locations may be expressed linearly in terms of displacements, x_D, rates, x_R, and accelerations, $\dot{x}_R$, and, therefore, it is possible to write

$$r_D = U_{SD}\, x_D + U_{SR}\, x_R + U'_{SR}\, \dot{x}_R \tag{3}$$

where the coefficient matrices U_{SD}, U_{SR}, and U'_{SR} are functions of the types of sensors and their location.

Control System Dynamics

The control system is defined as consisting of sensors, control logic, and actuators for FCAP. Classically, control system dynamics are expressed in the form of a transfer function which can be redefined in the state vector form of Equation (1). In the following sections, let x_S, x_L and x_A be the state vectors for the sensors, logic, and actuators, respectively.

Sensors

The state vector equations for sensor dynamics are given by

$$\dot{x}_S = A_{SS}\, x_S + B_S\, u_S \tag{4}$$

and

$$r_S = C_S\, x_S + D_S\, u_S \tag{5}$$

where u_S is the input to the sensor system from the aircraft.

Control Logic

The state vector equations for the control logic dynamics are expressed

$$\dot{x}_L = A_{LL}\, x_L + B_L\, (u_L + u'_L) \tag{6}$$

and

$$r_L = C_L\, x_L + D_L\, (u_L + u'_L) \tag{7}$$

where u_L is the input to the control logic from the sensors, and u'_L is pilot and guidance system commands (see fig. 1).

Actuators

The equations for the dynamics of the actuators in FCAP are given by

$$\dot{x}_A = A_{AA}\, x_A + B_A\, u_A \tag{8}$$

and

$$r_A = C_A\, x_A \tag{9}$$

where u_A is the input to the actuators.

Note that a term of the type $D_A\, u_A$ is absent in Equation (9). This implies that in the transfer function of the actuators the degree of the numerator is lower than the degree of the denominator. This yields considerable advantage in expressing the low-frequency-aerodynamics closed-loop system.

Aerodynamics

The potential aerodynamic method developed in references 6 to 9 provides a unified approach for both steady and unsteady subsonic and supersonic aerodynamics around complex, three-dimensional configurations. The subsonic portion of this method is incorporated into FCAP. The aerodynamic method of reference 6 is compatible with FCAP in that the generalized aerodynamic forces are proportional to the aircraft dynamic generalized coordinates, x_D, and rates, x_R, and to actuator (i.e., control surface) deflections, r_A. In the time domain, the unsteady aerodynamic forces are expressed as

$$u_R = q(U_{RR}\, x_R + U_{RD}\, x_D + U_{RA}\, r_A) \tag{10}$$

where U_{RR}, U_{RD}, and U_{RA} are operators corresponding to frequency-dependent matrices $\tilde{U}_{RR}$, $\tilde{U}_{RD}$, and $\tilde{U}_{RA}$ usually known as aerodynamic-influence-coefficient matrices.

For system stability and performance analyses, low-frequency aerodynamics is often adequate. Therefore, in the rest of this paper only low-frequency aerodynamics is considered. In this case, the equations for the aerodynamics become linear with constant coefficients. The low-frequency aerodynamics equations are expressed in the time domain as

$$u_R = q(U^{(0)}_{RR}\, x_R + U^{(0)}_{RD}\, x_D + U^{(0)}_{RA}\, r_A) + q(U^{(1)}_{RR}\, \dot{x}_R + U^{(1)}_{RD}\, \dot{x}_D + U^{(1)}_{RA}\, \dot{r}_A) \tag{11}$$

where, for example, $U^{(0)}_{RR}$ and $U^{(1)}_{RR}$ are the first two terms of the MacLaurin-Taylor series of $\tilde{U}_{RR}$. All of these coefficients are frequency-independent.

ACT System

The state vector equations for each of the ACT subsystems are given by Equations (2) to (9). Using the low-frequency aerodynamics given by Equation (11) and recognizing from Figure 1 that the input for each subsystem is the output of the previous subsystem, the ACT system can be cast in the form of Equation (1) as

$$\dot{x} = Ax + Bu \tag{12}$$

where

$$x = \lfloor x_D \; x_R \; x_S \; x_L \; x_A \rfloor^T \tag{13}$$

$$A = F^{-1} A_1 \tag{14}$$

and

$$B = F^{-1} \tag{15}$$

with

$$F = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & M_R & 0 & 0 & 0 \\ 0 & -B_S U'_{SR} & I & 0 & 0 \\ 0 & -B_L D_S U'_{SR} & 0 & I & 0 \\ 0 & -B_A D_L D_S U'_{SR} & 0 & 0 & I \end{bmatrix} + q \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ U^{(1)}_{RD} & U^{(1)}_{RR} & 0 & 0 & U^{(1)}_{RA} C_A \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{16}$$

$$A_1 = \begin{bmatrix} A_{DD} & A_{DR} & 0 & 0 & 0 \\ A_{RD} & A_{RR} & 0 & 0 & 0 \\ B_S U_{SD} & B_S U_{SR} & A_{SS} & 0 & 0 \\ B_L D_S U_{SD} & B_L D_S U_{SR} & B_L C_S & A_{LL} & 0 \\ B_A D_L D_S U_{SD} & B_A D_L D_S U_{SR} & B_A D_L C_S & B_A C_L & A_{AA} \end{bmatrix} + q \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ U^{(0)}_{RD} & U^{(0)}_{RR} & 0 & 0 & U^{(0)}_{RA} C_A \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{17}$$

$$u = \begin{Bmatrix} 0 \\ u'_R \\ 0 \\ B_L u'_L \\ B_A D_L u'_L \end{Bmatrix} \tag{18}$$

The input u to the ACT system depends upon the gust forces, u'_R, and the pilot and guidance system commands, u'_L.

FCAP PERFORMANCE ANALYSIS

Performance analysis routines are available in FCAP to compute frequency response, transfer function poles and zeros, mean-square response to random inputs and time response. The theoretical basis for each of these techniques is well founded in the literature; therefore, the following discussions will emphasize the nature of the technique as applied to FCAP equations.

Frequency Response

Since the low-frequency-aerodynamics ACT system dynamics are expressed in state vector format in terms of the (constant) A, B, C, D matrices, the frequency response of the k^{th} output to the ℓ^{th} input can be obtained (in the s-plane with zero initial conditions) from

$$(s\,I - A)\,\tilde{x} = B_\ell\,\tilde{u}_\ell \qquad (19)$$

and

$$\tilde{r}_k = C_k\,\tilde{x} + D_{k\ell}\,\tilde{u}_\ell \qquad (20)$$

Solving Equation (19) for $\tilde{x}$ and substituting into Equation (20) yields

$$\tilde{H}_{k\ell}(s) = \frac{\tilde{r}_k}{\tilde{u}_\ell} = C_k\,[sI - A]^{-1}\,B_\ell + D_{k\ell} \qquad (21)$$

Classical frequency response is obtained for the special case where $s = i\omega$ by computing the amplitude and phase from Equation (21) for a range of frequencies.

Poles and Zeros

Poles and zeros are evaluated in FCAP using a different form of $\tilde{H}_{k\ell}$ than that given in Equation (21). Note that Equation (21) may be rewritten as

$$\tilde{H}_{k\ell}(s) = \frac{\tilde{r}_k}{\tilde{u}_\ell} = \mathrm{Det}\left[\begin{array}{c|c} D_{k\ell} & -C_k \\ \hline B_\ell & sI - A \end{array}\right] \Bigg/ \mathrm{Det}\left[\begin{array}{c|c} 1 & -C_k \\ \hline 0 & sI - A \end{array}\right] \qquad (22)$$

The poles of $\tilde{H}_{k\ell}$ are the zeros of the denominator, i.e., the eigenvalues of the matrix A. The zeros of $\tilde{H}_{k\ell}$ are the zeros of the numerator. The procedure to obtain them is given in references 1 and 10.

Mean-Square Response to Random Inputs

The response of a flexible aircraft to stationary random inputs may be described in the frequency domain in terms of the spectral density matrix $\tilde{R}(\omega)$. The definition used for the outputs are also valid for the inputs and the state variables.

Where the inputs are represented as zero-mean white noise with constant spectral density matrix, $\tilde{U}$, the covariance matrix of the outputs R(0) is given (for D = 0) by

$$R(0) = CX(0)\,C^T \tag{23}$$

where the covariance matrix of the state variables, X(0), is determined from the linear matrix equation (ref. 11, pp. 330-332)

$$AX(0) + X(0)\,A^T + BU(0)\,B^T = 0 \tag{24}$$

Time Response

The time response of a closed-loop system with initial conditions, x(0), and an arbitrary input function can be determined in FCAP using a fourth-order Runge-Kutta numerical integration algorithm. For the special case where the input for $t > 0$ has a rational Laplace transform (i.e., the input can be described as a transfer function), the solution technique described in reference 12 is used.

CONCLUDING REMARKS

The equations of motion for a flexible aircraft have been presented in matrix format and incorporated into a digital computer program FCAP. The objective in the development of FCAP was to model realistically those factors that significantly affect the stability and response of a flexible, ACT-configured vehicle. It should be noted, however, that FCAP is intended primarily for use in the analysis of the performance of an ACT system, rather than in the synthesis of the control system.

The small-perturbation equations of motion for the ACT system were obtained in state-vector format and were completed by the addition of aerodynamics of arbitrary, complex aircraft configurations. Both fully unsteady and low-frequency aerodynamic equations were presented; however, for performance analyses, low-frequency aerodynamics is usually adequate and, therefore, the ACT system equations were presented for low-frequency aerodynamics only. Program FCAP, however, allows the analysis of complete aircraft configurations, including control system, with either low-frequency or unsteady aerodynamics.

The analysis of ACT system performance in FCAP has also been presented. In particular, the application of classical frequency response, poles and zeros, mean-square response, and time response in FCAP in state-vector format has been discussed.

Program FCAP provides a computerized method of integrating multiple systems into a matrix format, and then provides the means for obtaining desired solutions through classical analysis techniques. The program is currently in the final stages of checkout and has been used to solve textbook examples of control system problems. The program thus far has proven to be simple to use and requires a minimum of input.

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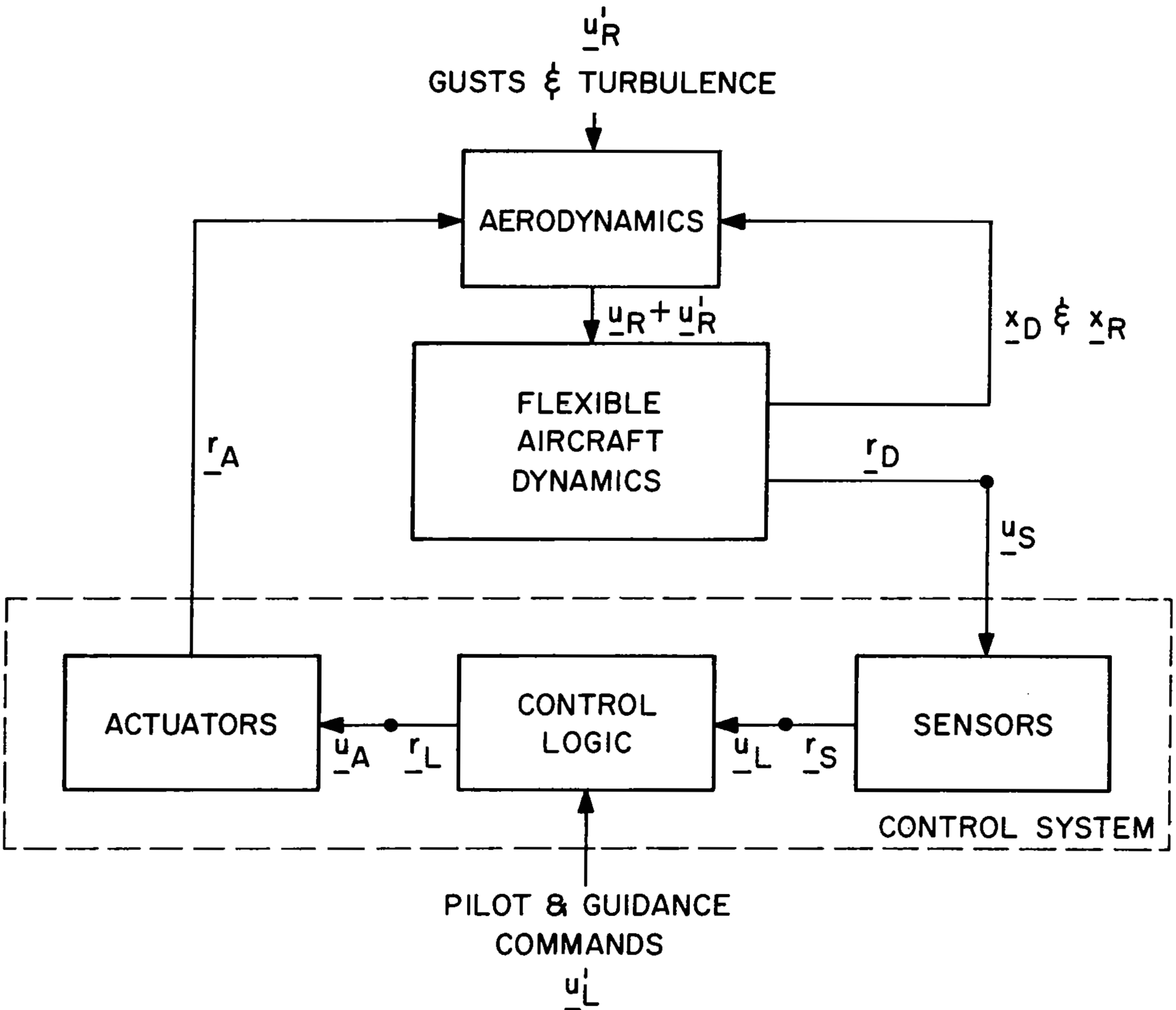

Figure 1. ACT System.