Stability of Flow of a Thermoviscoelastic Fluid Between Rotating Coaxial Circular Cylinders

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Abstract

The stability problem of thermoviscoelastic fluid flow between rotating coaxial cylinders is investigated using non-linear thermoviscoelastic constitutive equations due to Eringen and Koh (ref. 1). In the course of the investigation, the solution set for the steady state Couette flow problem is first found. The velocity field is found to be indentical with that of the classical viscous case and the case of the viscoelastic fluid, but the temperature and pressure fields are found to be different. By imposing some physically reasonable mechanical and geometrical restrictions on the flow, and by a suitable mathematical analysis, the problem is reduced to a characteristic value problem. The resulting problem is solved in this paper and stability criteria are obtained in terms of critical Taylor numbers. In general, it is found that thermoviscoelastic fluids are more stable than classical viscous fluids and viscoinelastic fluids under similar conditions.

Introduction

Although stability problems of a wide variety of classical viscous flows have been exhaustively investigated to date, significant qualitative and quantitative analytical studies concerning varieties of non-Newtonian fluid flow stability problems are still lacking. Since non-Newtonian fluids such as high polymer solutions, paints, colloidal suspensions, etc., occur very commonly in many laboratory experiments concerned with technological and biophysical studies, it is of great importance to investigate the stability of flows of such fluids. Among these stability problems of non-Newtonian fluids which have been investigated are those of a viscoinelastic fluid in a Couette flow in the presence of a circular magnetic field by Narasimhan (ref. 2) and that of a pseudoplastic material in a Couette flow by Graebel (ref. 3). In the present investigation we study the stability of Couette flow of a thermoviscoelastic fluid based on the constitutive equations of Eringen and Koh (ref. 1).

Mathematical Analysis

In the course of the analysis, the steady state solution is first obtained for the Couette flow of a thermoviscoelastic fluid. Then by superposing axisymmetric, time-dependent small disturbances on the basic steady flow and linearizing with respect to these small disturbances, the problem is reduced to a characteristic value problem. The requirement that nontrivial solutions exist for this problem leads to a characteristic equation between the parameters of the problem, namely $(T, q, a, \alpha_i, N_i, M_j, G)$, where T is the Taylor number, q, represents the disturbance temperature field, a represents the wave number of the disturbance, α_i represent the material coefficients, N_i, M_j and G represent geometric parameters and thermoviscoelastic properties of the fluid. The resulting characteristic value problem consists in finding those sets of values for these parameters which represent the solution of the system at marginal stability, i.e., determining the mode of unstable motion which will appear at the onset of instability. This would give a sequence of values of T for different values of a, the lowest values of which would be of interest in obtaining marginal stability because the mode described by this value would appear before the others. Thus the characteristic value problem becomes one of finding this minimum lowest positive, real value of T for various values of a. The stability criterion is then determined in terms of these critical Taylor numbers by using the principle of exchange of stabilities and by employing the orthogonal development technique of Chandrasekhar (ref. 4).

Conclusions and Remarks

The numerical results obtained are compared with the existing stability investigations. We find that thermoviscoelastic fluids in a Couette flow are more stable than classical viscous fluids like Bingham plastics and unlike viscoinelastic fluids which have been found to be less stable than viscous fluids. This behavior of the fluid in our case is essentially due to the viscoelastic nature of the fluid under thermal as well as rotational effects.

References

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