ACOUSTOELASTICITY*

Earl H. Dowell
Princeton University

INTRODUCTION

We consider internal sound fields. Specifically the interaction between the (acoustic) sound pressure field and the (elastic) flexible wall of an enclosure will be discussed. A good introduction to this subject is given in "Sound, Structures and their Interaction" by Junger and Feit (ref. 1). Also the author has briefly discussed this subject in his book, "Aeroelasticity of Plates and Shells" (ref. 2). This paper is a highly condensed version of reference 3.

Such problems frequently arise when the vibrating walls of a transportation vehicle induce a significant internal sound field. The walls themselves may be excited by external fluid flows. Cabin noise in various flight vehicles and the internal sound field in an automobile are representative examples.

Briefly considered are mathematical model, simplified solutions, and numerical results and comparisons with representative experimental data. An overall conclusion is that reasonable grounds for optimism exist with respect to available theoretical models and their predictive capability.

MATHEMATICAL MODEL

Here the essentials of the mathematical noise transmission model will be summarized. No mathematical derivations are included, however. A complete description of the analysis is contained in reference 3. A modal representation of the structural wall(s) and acoustic cavity(ies) is used. For the structural wall

$$w(x,y,t) = \sum_m q_m(t)\, \psi_m(x,y) \tag{1}$$

w - physical wall deflection

q_m - modal coordinate; function of time

ψ_m - natural mode shape (in vacuum); defined over an appropriate area with coordinates x, y

Associated with the ψ_m are natural mode frequencies, ω_m, and generalized masses, M_n.

$$M_m \equiv \iint m(x,y)\, \psi_m^2 \, dxdy \qquad m \text{ - structural mass per unit area} \tag{2}$$

*This work was performed under NASA Grant NSG 1253, Langley Research Center.

For the acoustic cavity

$$p(x,y,z,t) = \rho c^2 \sum_n \frac{P_n(t)}{M_n^A} F_n(x,y,z) \tag{3}$$

p - physical acoustic pressure

ρ - air density

c - air speed of sound

P_n - acoustic modal coordinate

F_n - acoustic natural mode (for rigid walls)

Z_a - absorbent wall impedance

Associated with the F_n are acoustic natural frequencies, ω_n^A, and generalized masses

$$M_n^A \equiv \frac{\iiint F_n^2}{V} dxdydz \qquad V \equiv \text{total cavity volume} \tag{4}$$

The external pressure field, p^E, is represented in terms of its generalized forces

$$Q_m^E \equiv -\iint_{\text{on area, } A_{EW}} p^E(x,y,t)\, \psi_m(x,y)\, dxdy \qquad A_{EW} \equiv \text{(external) structural wall area} \tag{5}$$

The minus sign arises from the sign convention that w is positive outward and p^E is positive inward with respect to the cavity. See Figure 1.

The data given are:

p^E for the external sound field

m, ψ_m, ω_m for the structural wall

M_m is determined from (2)

Q_m is determined from (5)

ρ, c, F_n, ω_n^A, Z_a for the cavity(ies)

M_n^A is determined from (4)

q_m, P_n are then determined from the modal equations of motion, i.e.

$$M_m \left[\ddot{q}_m + 2\zeta_m \omega_m \dot{q}_m + \omega_m^2 q_m\right] - \rho c^2 A_{EW} \frac{\sum_n P_n L_{nm}}{M_n^A} = Q^E{}_m \tag{6}$$

$$\ddot{P}_n + \omega_n^{A^2} P_n + A_A \rho c^2 \sum_r \frac{\dot{P}_r C_{nr}}{M_r^A} = \frac{-A_{EW}}{V} \sum_m \ddot{q}_m L_{nm} \tag{7}$$

where

$$L_{nm} \equiv \frac{\iint_{\text{over } A_{EW}} F_n \psi_m \, dxdy}{A_{EW}}, \quad C_{nr} \equiv \frac{\iint_{\text{over } A_A} \frac{F_n F_r}{Z_a} dA}{A_A}, \quad A_A \equiv \text{absorbent cavity wall area}, \ \zeta_m \text{ - modal damping}$$

These are two coupled systems of spring-mass-damper-oscillators and (6) and (7) are familiar and computationally efficient descriptions of their dynamics.

Moreover multiple walls or cavities may be readily included in a similar fashion. For two connected cavities, see Figure 1, the common wall between the two cavities may be treated as a (common) structural wall and in an obvious notation (where we have cavities a and b) (6) and (7) are replaced by

$$M_m[\ddot{q}_m + 2\zeta_m\omega_m\dot{q}_m + \omega^2_m q_m] - \rho c^2 A_{CW} \sum_n \frac{P^a_n L^a_{nm}}{M_n^{Aa}} + \rho c^2 A_{CW} \sum_n \frac{P^b_n L^b_{nm}}{M_n^{Ab}} = 0 \quad (8)$$

$$\ddot{P}^a_n + \omega_n^{Aa^2} P_n^a + A^a_A \rho c^2 \sum_r \dot{P}_r^a \frac{C^a_{nr}}{M_r^{Aa}} = \frac{-A_{CW}}{V_a} \sum_m \ddot{q}_m L^a_{nm} \quad (9)$$

$$\ddot{P}^b_n + \omega_n^{Ab^2} P_n^b + A^b_A \rho c^2 \sum_r \dot{P}_r^b \frac{C^b_{nr}}{M_r^{Ab}} = + \frac{A_{CW}}{V_b} \sum_m \ddot{q}_m L^b_{nm} \quad (10)$$

where $L^a_{nm} \equiv \iint \frac{F_n^a \psi_m}{A_{CW}} dxdy$, etc. and the subscript CW denotes common wall. For simplicity we have considered the external walls rigid. However, clearly (6), (7), (8), (9), (10) can be combined to allow for both external and (internal) common wall motion with multiple cavities.

Once q_m and P_n are determined, the physical deflection, w, and sound pressure, p, are known from (1) and (3). The flexibility in the model is in treating ω_m, ψ_m and ω^A_n, F_n as given. For simple shapes, these are known analytically. In some cases it will be possible to approximate the structural wall and cavity by a simple shape or several component simple shapes. In other cases it will be necessary to determine the natural modes by numerical methods (finite element analysis) or experiment (scale models).

Before leaving the mathematical model, two of its widely applicable consequences should be noted. Firstly, the coupling of the acoustic cavity-structural wall is gyroscopic. This can be seen directly by replacing the acoustic pressure, p, by the corresponding velocity potential. These are related through Bernoulli's equation (ref. 2 and 3). The importance of recognizing that the coupling is gyroscopic is that one can then invoke Meirovitch's algorithm for determining the eigenvalues of the acoustic-structural system using standard numerical techniques (ref. 4). This is preferable to alternative formulations which lead to trial and error solutions to transcendental equations, e.g. ref. 5. For additional detail, see reference 3. Fortunately the coupled acoustic-structural wall natural frequencies are normally little changed from their rigid wall acoustic mode and in vacuum structural wall mode counterparts. This simplifies matters considerably, of course, and will often permit one to avoid a completely coupled analysis altogether. More will be said of this in the next section.

The second theoretical consequence and one of more practical importance is the direct way in which two interacting cavities can be treated. (Recall

Fig.1.) If there is a pure opening between two cavities, i.e. one of zero mass, damping, and stiffness, then (8) becomes

$$\sum_n \frac{P_n^a L_{nm}^a}{M_n^{Aa}} - \sum_n \frac{P_n^b L_{nm}^b}{M_n^{Ab}} = 0 \qquad (11)$$

To determine the natural frequencies of this two cavity system, one assumes simple harmonic motion, solves for P_n^a, P_n^b from (9), (10) in terms of q_m and substitutes the result into (11) to obtain (for $Z_a \to \infty$)

$$\sum_m q_m Q_{rm} = 0 \quad \text{where} \quad Q_{rm} \equiv \frac{1}{V_a} \sum_n \frac{L_{nr}^a L_{nm}^a}{M_n^a[-\omega^2+\omega_n^{Aa^2}]} + \frac{1}{V_b} \sum_n \frac{L_{nr}^b L_{nm}^b}{M_n^b[-\omega^2+\omega_n^{Ab^2}]} \qquad (12)$$

The natural frequencies are determined by the condition that the determinant of Q_{rm} must vanish. This is a non-standard eigenvalue problem because of the form that ω^2 takes in Q_{rm}, see (12). However it has one overwhelming advantage: The size of the matrix is determined by the number of <u>two-dimensional</u> pure opening modes, ψ_m, rather than the number of <u>three-dimensional</u> cavity modes, F_n^a, F_n^b. The former will be much smaller in number than the latter for a given desired accuracy. This advantage will persist even when the opening is a structural member of finite stiffness, etc., or there are more than two cavities (ref. 3).

Once the natural frequencies of the multiple cavities have been determined, they may be treated as an equivalent single cavity so far as determination of interior sound levels is concerned.

SIMPLIFIED SOLUTIONS FOR INTERNAL SOUND LEVELS

The mathematical model may be solved numerically without further approximation. Indeed one of its advantages is that the calculation would be no more (nor less!) tedious than is frequently performed today for structural vibration response. However it is of interest to make further simplifications <u>if</u> little accuracy is lost and/or substantial computational reduction is possible. Here a summary of highlights from analytical and numerical studies is provided.

It is usually true that complete coupling between the structural wall and acoustic cavity can be neglected. Hence it is normally permissible to first calculate the external wall motion due to an external pressure loading (neglecting the acoustic cavity) and then determine the internal acoustic cavity pressure due to the now known wall motion.

There are two known circumstances where the complete coupling must be taken into account (see ref. 3 for details):

(1) If the fundamental wall resonant frequency is well below the fundamental acoustic resonant frequency (in the direction perpendicular to the wall), the Helmholtz mode of the cavity will provide a spring stiffness which may

substantially raise the panel wall mode frequency above its in vacuum value. But then only the single Helmholtz mode of the cavity need be considered. An example is discussed in the following section.

(2) If a structural wall mode and acoustic cavity resonant frequency are in close proximity, then again a fully coupled analysis may be required. But then only the two closely coupled modes need be examined.

Assuming that the more usual situation applies, one may make further progress analytically if one considers simple harmonic external excitation at either a structural wall or cavity resonant frequency. Also for broad band random excitation, similar results may be obtained by invoking power spectral analysis since for small damping the internal cavity response will be dominated by the wall and/or cavity resonances. However if the external field has its own dominant harmonics then the following simple results will not hold and one must return to the full analysis (but hopefully still being able to neglect full wall-cavity coupling). There is one advantage in such a situation, however, and that is a precise knowledge of damping will not be so important in these off resonant conditions and hence the basic mathematical model should be a more accurate representation of the physical system. Here only the simplest type of external excitation will be considered.

External Exciting Frequency, ω^E, = Structural Resonant Frequency, ω_s

The response will be dominated by the s^{th} structural mode and the cavity pressure is given by

$$|p^c| = \frac{\rho c^2 A_F Q_s^E}{2\zeta_s V M_s} \sum_n \frac{F_n}{M_n^A} \frac{1}{[-\omega_s^2 + \omega_n^{A2}]} \tag{13}$$

If $\omega_s < \omega_n^A$ for all $\omega_n^A \neq 0$, then typically $|p^c| > p^E$ and conversely.

External Exciting Frequency, ω^E, = Cavity Resonant Frequency, ω_c^A

The cavity response will be dominated by the c^{th} cavity mode, and if in addition there is a dominant structural mode (say s^{th}), the cavity pressure is given by

$$p^c = \frac{F_c \int_{\text{on } A_F} p^E \psi_s dA}{\int_{\text{on } A_F} F_c \psi_s dA} \tag{14}$$

From (14), at most $p^c \simeq p^E$. For $p^E \sim F_c$ on A_F, $p^c = p^E$. In particular if F_c and p^c are approximately uniform over A_F, then $p^c \simeq p^E$.

It is interesting to note that neither (13) nor (14) involve the impedance or damping of the cavity. This is true under even broader circumstances, i.e. the wall absorbtion is not important in determining internal sound levels due to <u>external</u> sources (ref. 3).

NUMERICAL RESULTS AND COMPARISONS WITH EXPERIMENTS

For a single cavity with a flexible wall and an external sound source, the theoretical model has been verified experimentally by several authors (refs. 6-11). Hence we first assess the capability of the model to describe accurately the acoustic natural modes in multiply connected cavities. Once the combined natural modes of the multiply connected cavities are determined and verified experimentally, they may be treated as one single cavity. Then the earlier work for a single cavity may be taken as experimental verification for the forced excitation of multiply connected cavities as well.

Acoustic Natural Modes in Multiply Connected Cavities

The experimental studies discussed here were conducted by Smith (ref. 12). A representative configuration consists of two acoustic cavities, one twice the dimensions of the other, with rigid walls and a partial opening between them. In Figure 2, the longitudinal pressure distributions (along with their resonant frequencies) are shown for the first six (symmetric with respect to height) acoustical modes with a full opening between cavities. The agreement between theory and experiment is very good.

In these experiments, $c_o = 343.5$ m/sec, $a = d = 25.4$ cm and the width dimension was 10.16 cm to provide two-dimensional conditions in the frequency range of interest. The thickness of the partition (assumed zero in the theoretical calculations) is 1.27 cm as is the thickness of all external walls. The cavity is constructed from plexiglass.

Forced Response of a Single Cavity with a Flexible Wall

Experimental Arrangement:

For this discussion, Gorman's work (refs. 8, 9) is used; however, also see references 6, 7, 10 and especially 11. The flexible wall panel was 25.4 cm x 50.8 cm x .127 cm aluminum alloy plate that was bonded onto a stiff rectangular frame. By bonding the plate in this way, a clamped edge boundary condition was approximated. A sealed cavity, also 25.4 cm x 50.8 cm, was constructed beneath the panel so that the cavity depth could be varied. The cavity enclosure was made of 1.27 cm thick plexiglass.

The panel was excited acoustically by a Wolverine LS15, 20 watt loudspeaker driven by a B & K Beat Frequency Oscillator, type 1022. By using a single speaker, an external field distribution that was modestly variable in space was obtained. Measurements were made of panel deflections and cavity pressures due to a pure tone. Only the latter are considered here.

Cavity Pressure Measurement:

The sound pressure level within the cavity was measured using a B&K 1/4" microphone, type 4136 with a type 2615 cathode follower with type UA0035 connector

In Figure 3 the cavity pressure is plotted against frequency. This pressure is the difference between the dB level inside the cavity and that outside the cavity on the upper surface of the panel. The dominant features are the three primary resonant peaks occurring at 113 cps, 210 cps, and 518 cps. The first two resonances correspond to the first and third panel modes, and thus indicate that the panel is driving the cavity at these frequencies. The resonance at 518 cps is the fundamental cavity depth mode. Theoretically, if the external pressure were uniform over the flexible panel, it should be motionless, and the pressure level difference between the external and internal measurements should be zero when the external frequency equals the cavity resonant frequency. This is nearly the case, see Figure 3.

Results:

Panel Resonant Frequency Changed by Cavity

In Figure 4, a comparison between theory and experiment is made. The ratio of panel frequency (modified by coupling with the cavity) to "in-vacuo" panel frequency is plotted against panel length to cavity depth ratio, a/d. The "in-vacuo" panel frequencies were computed from Warburton's theory, (ref. 13) and the panel frequencies' variation with cavity depth were computed from Dowell and Voss' theory (ref. 10) which is an earlier version of the present analysis. There is excellent agreement between theory and experiment at the large cavity depths, with some variation from theory occurring at shallow cavity depths. Again it should be emphasized that this interesting change in panel frequency occurs only for flexible panels and stiff (shallow) cavities.

Panel Damping

Three types of damping will be referred to in this discussion: constant damping, frequency damping, and experimental damping. Constant damping is the value measured for a 30.48 cm cavity depth and assumes that there is no variation of panel modal damping ratio with cavity depth. Frequency damping allows for variation of damping ratio with frequency and employs the data measured at a 30.48 cm cavity depth for various panel resonances. Thus, the only effect changing this type of damping is the variation of panel modal frequency with cavity depth (Fig. 4 and ref. 8). Experimental damping is that measured for the exact conditions under study.

Cavity Pressure and Damping Effects

Figure 5 plots the variation of cavity pressure with cavity depth for the three different theoretical damping models, i.e. constant damping, frequency damping, and experimental damping. The exciting external frequency is equal to the fundamental panel resonant frequency. Recall that the damping ratios used in these calculations are those of the panel and not of the cavity; the latter were neglected. Even though cavity damping has not been considered, there is excellent agreement between experiment and the theoretical model using experimental damping.

Similar results have been obtained for random external pressure excitation (ref. 9). Pretlove (ref. 7) has made measurements of panel natural frequency variation with cavity depth; Guy and Bhattacharya (ref. 11) have measured cavity pressures and panel natural frequencies. Generally good agreement with theory has also been shown in references 7, 9 and 11.

CONCLUDING REMARKS

A comprehensive theoretical model has been developed for interior sound fields which are created by flexible wall motion resulting from exterior sound fields. Included in the model are the mass, stiffness and damping characteristics of the flexible wall and of the acoustic cavity. Full coupling between the wall and cavity is permitted although detailed analysis, numerical results and experiment suggest that it is the exceptional case when the structural wall dynamic characteristics are significantly modified by the cavity.

Based upon the general theory, an efficient computational method is proposed and used to determine acoustic natural frequencies of multiply connected cavities. Simplified formulae are developed for interior sound levels in terms of in-vacuuo structural wall and (rigid wall) acoustic cavity natural modes.

Comparisons of theory with experiment show generally good agreement. The principal uncertainty remains the structural and/or cavity damping mechanisms. For external sound excitation, cavity damping is demonstrated to be generally unimportant; however it may be of importance for interior sound sources. The results of Wolf, Nefske and Howell (ref. 14) and Petyt, Lea and Koopman (ref. 15) using finite element techniques and Howlett and Morales (ref. 16) using modal analysis also suggest that effective analytical models are available.

REFERENCES

1. Junger, M. and Feit, D., "Sound, Structures and Their Interaction," M.I.T. Press, 1972

2. Dowell, E.H., "Aeroelasticity of Plates and Shells," Noordhoff International Publishing, Leyden, 1974.

3. Dowell, E.H., "Acoustoelasticity," Princeton University AMS Report 1280, May 1976.

4. Meirovitch, L., "A New Method of Solution of the Eigenvalue Problem for Gyroscopic Systems," AIAA Journal, 12, pp. 1337-1342, 1974.

5. Cockburn, J.A., and Jolly, A.C., "Structural-Acoustic Response, Noise Transmission Losses and Interior Noise Levels of an Aircraft Fuselage Excited by Random Pressure Fields," Air Force Flight Dynamics Laboratory Technical Report, AFFDL-TR-68-2, August 1968.

6. Dowell, E.H. and Voss, H.M., "The Effect of a Cavity on Panel Vibrations," AIAA Journal, 1, pp. 476-477, 1963.

7. Pretlove, A.J., "Free Vibrations of a Rectangular Panel Backed by a Closed Rectangular Cavity," J. Sound Vib. 2, pp. 197-209, 1965.

8. Gorman, III, G.F., "An Experimental Investigation of Sound Transmission Through a Flexible Panel into a Closed Cavity," Princeton University AMS Report No. 925, July 1970.

9. Gorman, III, G.F., "Random Excitation of a Panel-Cavity System," Princeton University AMS Report No. 1009, July 1971.

10. Dowell, E.H. and Voss, H.M., "Experimental and Theoretical Panel Flutter Studies in the Mach Number Range of 1.0 to 5.0," AIAA Journal, 3, pp. 2292-2304, 1965.

11. Guy, R.W. and Bhattacharya, M.C., "The Transmission of Sound Through a Cavity-Backed Finite Plate," J. Sound Vib. 27, pp. 207-223, 1973.

12. Smith, D.A., "An Experimental Study of Acoustic Natural Modes of Interconnected Cavities," Princeton University AMS Report No. 1284, August 1976.

13. Warburton, G.B., "The Vibration of Rectangular Plates," Proc. Inst. Mech. Engrs. (London), 1968, pp. 371-384, 1954.

14. Wolf, Jr., J.A., Nefske, D.J., and Howell, L.J., "Structural-Acoustic Finite Element Analysis of the Automobile Passenger Compartment," SAE Paper 760184, Presented at the Automotive Engineering Congress and Exposition, Detroit, Michigan, February 1976.

15. Petyt, M., Lea, J. and Koopman, G.H., "A Finite Element Method for Determining the Acoustic Modes of Irregular Shaped Cavities," J. Sound Vib. 45, pp. 495-502, 1976.

16. Howlett, J.T. and Morales, D.A., "Prediction of Light Aircraft Interior Noise," NASA TM X-72838, April 1976.

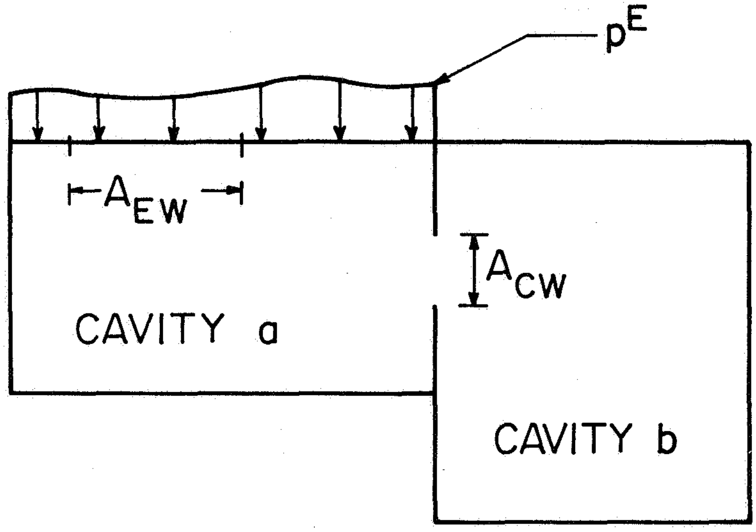

Figure 1.- Acoustic cavity-structural wall geometry.

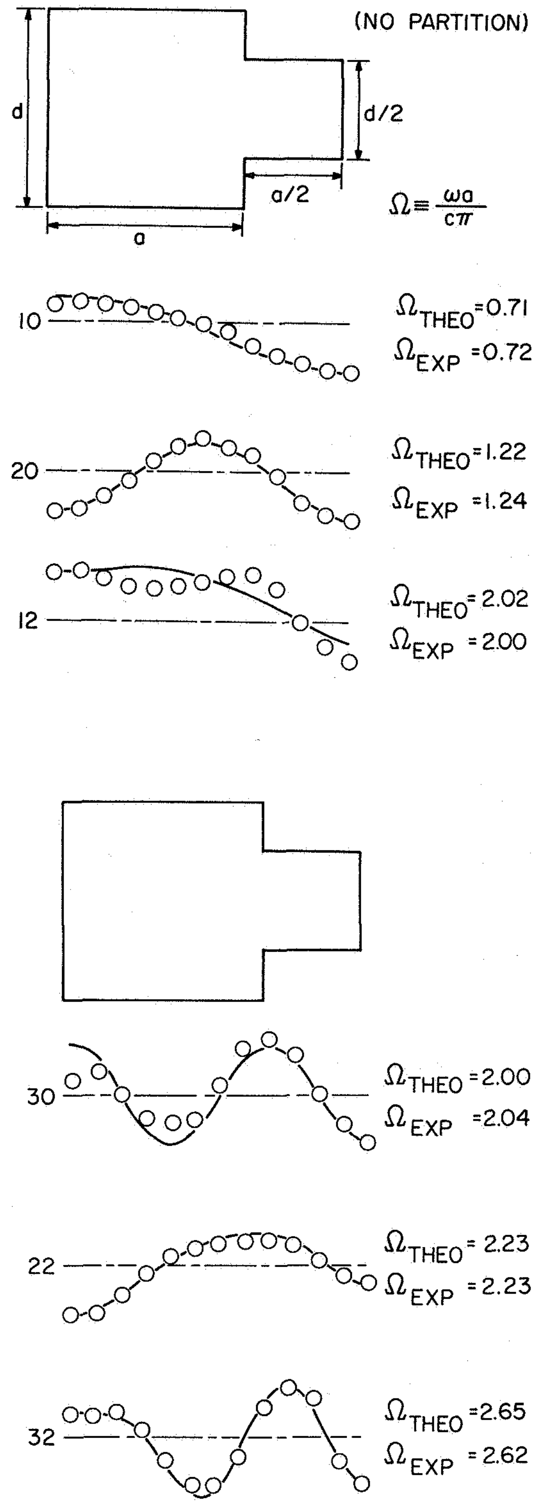

Figure 2.- Comparison of theoretical and experimental cavity acoustic modes.

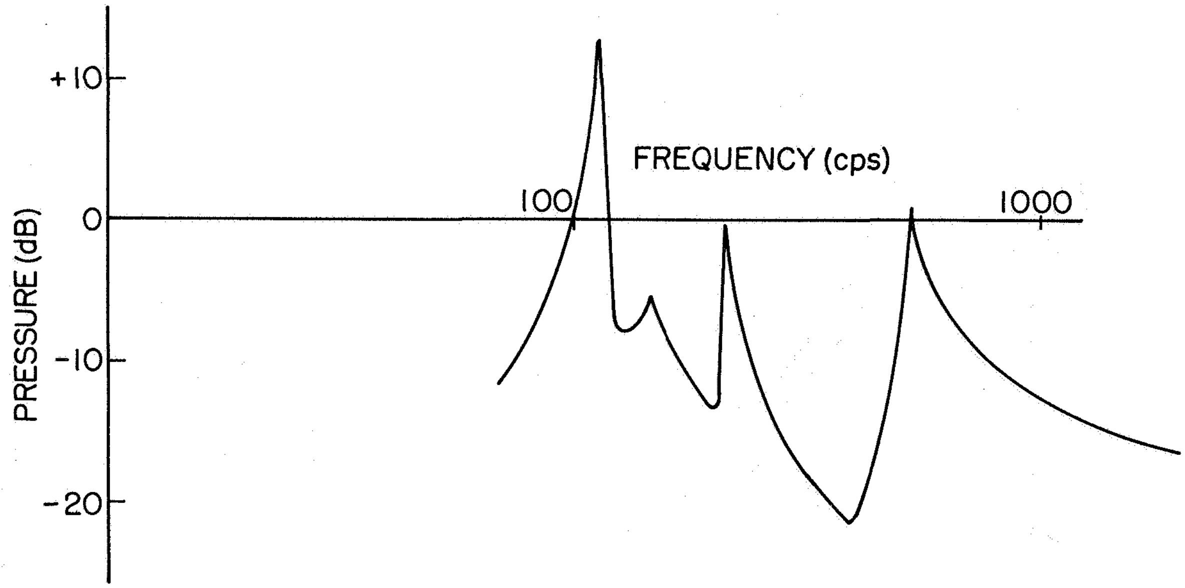

Figure 3.- Cavity response to sinusoidal external field.

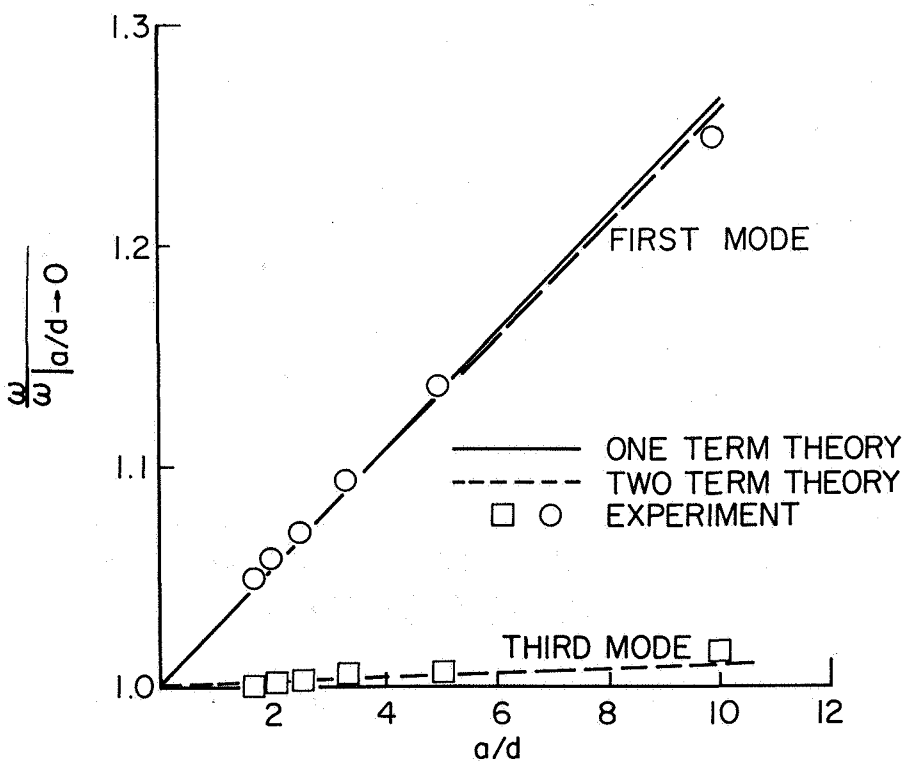

Figure 4.- Cavity effect on panel natural frequencies.

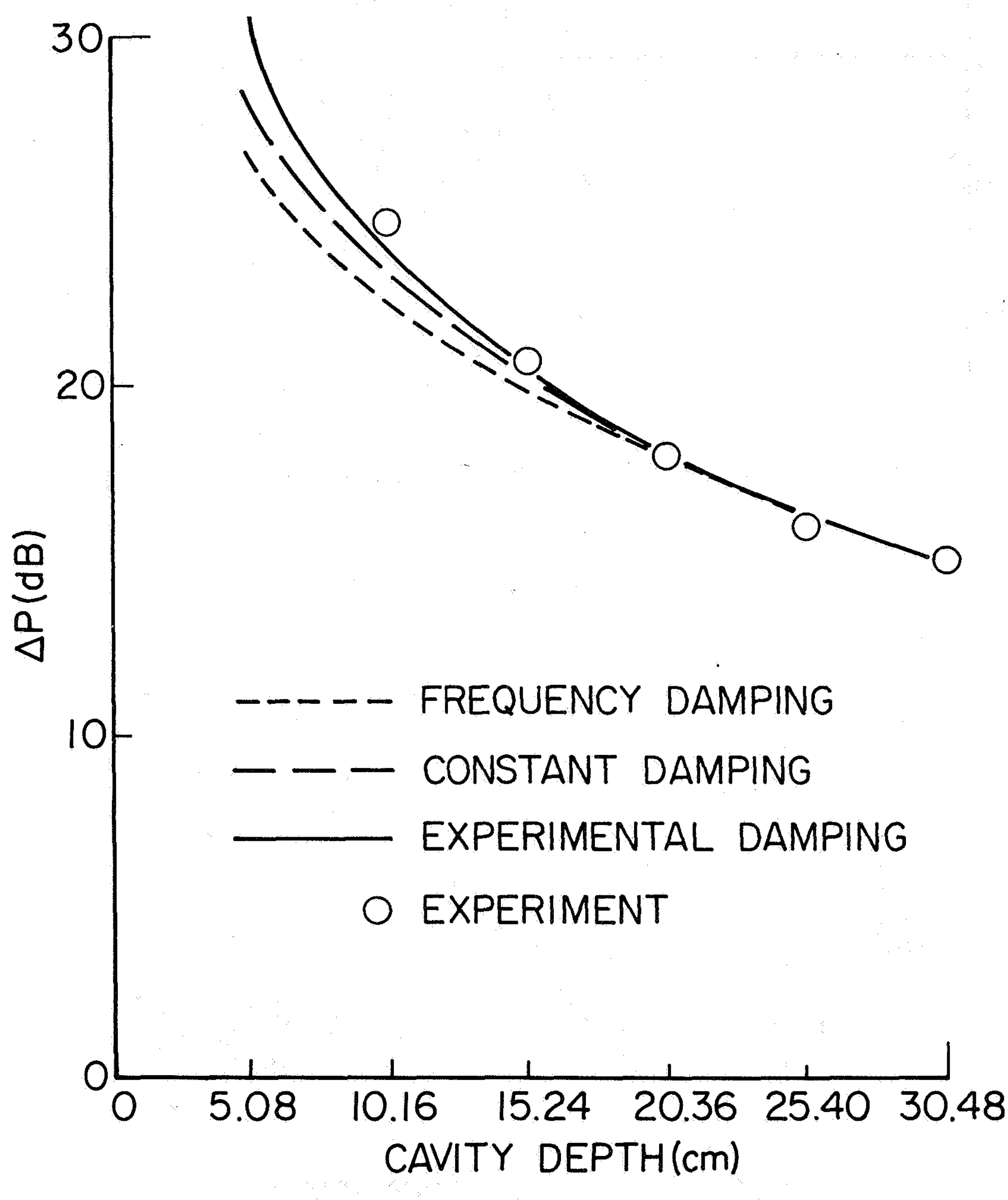

Figure 5.- Cavity pressure versus cavity depth.