EFFECTS OF MEAN FLOW ON DUCT MODE OPTIMUM SUPPRESSION RATES

Robert E. Kraft
General Electric Co.

William R. Wells
University of Cincinnati

SUMMARY

The nature of the solution to the convected acoustic wave equation and associated boundary conditions for rectangular ducts containing uniform mean flow is examined in terms of the complex mapping between the wall admittance and characteristic mode eigenvalues. It is shown that the Cremer optimum suppression criteria must be modified to account for the effects of flow below certain critical values of the nondimensional frequency parameter of duct height divided by sound wavelength. The implications of these results on the design of low frequency suppressors is considered.

INTRODUCTION

The lining of duct walls with acoustic treatment is a standard practice in the jet engine industry for obtaining suppression of turbomachine noise. The design of this acoustic treatment depends upon a number of factors in addition to acoustic performance, including weight, structural integrity, length restrictions, and ability to withstand severe environments. The design goal of obtaining a maximum of suppression with a minimum of panelling requires a thorough knowledge of the acoustic propagation phenomena in ducts in the presence of complex sound sources and mean flow, among other effects. This paper is aimed at increasing the understanding of a vital element in the prediction of sound suppression in ducts with mean flow, the nature of the eigenvalue problem.

In Reference 1 the general problem of the modal solution to acoustic wave propagation in multi-segment ducts with mean flow has been considered. The success of a modal analysis prediction program such as the one developed in Reference 1 is strongly dependent upon the ability to obtain an accurate and complete set of eigenvalues for each section of the duct. It is felt that greater appreciation of the nature of the propagation process can be gained through detailed examination of complex contour plots of the eigenvalue-admittance relationship for particular cases. The basic theory and equations used in this paper are presented in Reference 1.

A useful acoustic treatment design criteria which has been in use for a number of years is the least attenuated mode theory developed by Cremer (Reference 2). Although it is gradually being replaced by the more accurate multi-mode prediction procedures, it is still of practical value for preliminary designs and the evaluation of basic trends. The Cremer theory is based on the location of branch points (or critical points) of the complex eigenvalue-admittance mapping, and the consequences of this criteria, particularly for low values of the frequency parameter (ratio of duct height to sound wavelength)

$$\eta = H/\lambda \tag{1}$$

are examined. It is shown how the theory must be modified for very low η-values.

The results of this study are applied to the specific case of ducts with rectangular cross section. The methods will find direct application to other cross-sectional geometries with the proper generalization of the characteristic duct modes.

SYMBOLS

- c - speed of sound
- f - frequency
- H - duct height
- i - $\sqrt{-1}$
- k - wave number
- M_o - mean flow Mach number
- n - exponent in boundary condition
- t - time
- Z_{opt} - wall impedance (optimum)
- β - wall admittance (dimensionless)
- γ - nondimensional duct eigenvalue
- κ - axial propagation constant
- η - nondimensional frequency parameter, Hf/c
- λ - sound wavelength
- ρ_o - ambient density of air
- ω - circular frequency, $2\pi f$

RECTANGULAR DUCT MODAL SOLUTION

The method of separation of variables is applied to the convected acoustic wave equation under the assumption of uniform mean flow and rectangular duct geometry. In this study, consideration will be limited to duct treatment sections with the same treatment on opposite sides of the duct.

Substitution of the general solution of the differential equation into the boundary condition leads to two different expressions

$$\beta k H = \frac{i\gamma}{\left(1 - M_o \frac{\kappa}{k}\right)^n} \frac{1 - \cos\gamma}{\sin\gamma} \tag{2}$$

and

$$\beta kH = \frac{-i\gamma}{\left(1 - M_o \frac{\kappa}{k}\right)^n} \frac{1 + \cos\gamma}{\sin\gamma} \tag{3}$$

where β is the acoustic admittance at the wall, based on $e^{-i\omega t}$ time dependence. These are complex, transcendental equations for the eigenvalue, γ. Their solution leads to two distinct sequences of eigenvalues, the symmetric mode eigenvalues and the antisymmetric mode eigenvalues, respectively. For simplicity, the two sequences can be combined into a single set of eigenvalues.

NATURE OF THE BOUNDARY CONDITIONS

The boundary condition expressions (2) and (3) must be solved by numerical methods if the eigenvalues are desired for given admittances. The admittance, however, can be isolated as a function of the eigenvalue, making it susceptible for plotting contours of constant magnitude and phase of the quantity βkH in the complex eigenvalue plane. The graphical representation of the relationship between the admittance and the eigenvalue is considered in detail in Reference 3, in which detailed contour plots are shown for a variety of conditions. From these plots, it is possible to obtain the sequences of eigenvalues which determine the characteristic duct modes for a given wall admittance.

It is shown in Reference 3 that the boundaries separating eigenvalue regions for different modes in the eigenvalue plane are branch cuts of the admittance-eigenvalue contour mapping. One point on the branch cut is a branch point, or critical point, of the mapping, at which the eigenvalues for two adjacent modes coalesce, giving a double-value. By considering plots of lines of constant attenuation superimposed on the eigenvalue-admittance mapping, it has been shown by Cremer (Reference 2), and is illustrated in Reference 3, that adjacent modes, in particular the first and second modes, attain nearly the same attenuation rate at the branch point. Cremer proposed the choice of the admittance at the branch point of the first mode as a design criteria which optimizes suppression for the least attenuated mode.

For symmetric modes in the duct with the same liner on both sides, at Mach 0.0, the optimum admittance for the least attenuated mode is (in polar form)

$$\beta kH = (5.28, -38.7^o) \tag{4}$$

Transforming this to an impedance ($e^{+i\omega t}$ convention) design criteria, we get

$$\frac{Z_{opt}}{\rho_o c} = (0.93 - 0.74i)\eta \tag{5}$$

Although useful for preliminary design, the attenuation rate of a single mode or even a pair of modes is not sufficient to predict attenuation or to design optimum treatment for finite length ducts with arbitrary sources at higher η-values. In these cases, the impedance must be chosen to maximize suppression for a particular combination of modes, and may turn out to be nowhere near the classical Cremer optimum.

The assumption of uniform mean flow requires that a physically unrealistic slip-flow condition must be postulated to occur at the wall surface. It can be shown that the surface flow convection effect leads to an anomaly in the boundary conditions, such that two different conditions can be obtained depending on whether continuity of particle displacement or continuity of particle velocity is assumed to hold at the wall. Based on the analysis of Reference 4, the current most widely accepted condition is that of particle displacement continuity. For this reason, and since it causes the more drastic effect of the two conditions, particle displacement continuity is assumed in this study. The most fortuitous choice of these conditions for any given flow, frequency, or duct height is yet to be resolved.

The effect of flow on the modal maps is to cause a distortion of the βkH magnitude and phase contours from the Mach 0.0 case. Since the propagation constant κ is a function of kH as well as γ, the eigenvalue relationships can no longer be made independent of kH, and a separate mapping must be made at each Mach number and value of η (since $\eta = kH/2\pi$).

A branch point of the mapping of the complex function βkH on the complex γ-plane is the point at which the derivative of βkH with respect to γ is zero. Equation (2) was used to determine the location of the branch point for arbitrary values of kH and mean flow Mach number. The desired value of γ is the root which corresponds to the branch point between first and second modal regions for each case. These roots were extracted using a simple Newton-Raphson iteration scheme. The branch points for the Mach 0.0 case were used as initial values to provide accuracy in the fourth decimal place.

When the values of the eigenvalue at the branch points are determined, the optimum admittance (or impedance) can be found from Equation (2) and the optimum suppression rate can be found from the axial propagation constant. Figures 1 and 2 show the dependence of the optimum specific resistance R and reactance X, respectively, on η with Mach number M_o as a parameter. The impedance components have been divided by the η-value, which makes the Mach 0.0 curve a straight line with zero slope, that is, independent of η. Figure 3 shows the optimum attenuation rate for each of the impedances as a function of η.

The optimum suppression rates appear to be independent of Mach number for η-values higher than about 2, but diverge from the Mach 0.0 case below $\eta = 2$, as the region of high suppression rates is entered. Higher optimum attenuations can be obtained for propagation against the flow for these low η-values than for propagation with the flow.

In the low η regions, the optimum resistance undergoes a rather bizarre behavior. For a given Mach number, there is an η-value below which the optimum resistance tends to negative values. A negative resistance, or active, liner is one which tends to generate energy, as opposed to a passive, positive resistance liner which can only absorb energy. At first sight, this phenomenon appears to be physically unreasonable, possibly indicating a basic flaw in the theory.

It must be kept in mind that the optimum impedance criteria to this point has been based on the branch point criteria developed by Cremer. In the no-flow or high η-value cases, this criteria is unambiguous, but for low η-values it will be shown that the flow-induced distortion of the modal maps is so severe as to cause significant changes in the nature of the problem. It will be shown that, if one will admit the existence of active (negative resistance) wall liners, two suppression criteria must be provided, one for passive liners and one for active liners. The strange behavior is caused by the

$$\frac{1}{\left(1 - M_o \frac{\kappa}{k}\right)^2}$$

factor in the particle displacement continuity boundary condition, when κ/k begins to get large in magnitude.

MODIFIED OPTIMUM CRITERIA FOR LOW η-VALUES

Choosing a Mach number of -0.4, modal maps of the lowest order symmetric mode region were plotted for successively lower η-values, according to the following list:

Figure 4	η	= 0.36
Figure 5	η	= 0.3
Figure 6	η	= 0.15

Note in Figure 4 that the -90° phase lines have left the real and imaginary γ-axes and are converging on the branch point. The branch cut which defines the region of passive impedance for the lowest order mode now consists of just a short length of line of constant magnitude of βkH ($|\beta kH| = 1.37$), with the rest of the cut being comprised of $\pm 90^\circ$ phase lines. This η-value is just above the value for which the optimum impedance goes negative. In Figure 5, the -90° lines have passed through the branch point, which now has a phase of less than -90°, giving a negative resistance. The lowest order mode region has become isolated from the second order mode region, and contains no branch point. The optimum impedance for the lowest order mode is now defined as the point at which the highest valued curve of constant attenuation touches the boundary of the modal region.

At the branch point, a higher positive suppression is predicted than in the modal region, in spite of the negative resistance. This implies that an active liner would provide more suppression than a passive liner, if designed with the branch point impedance components. This unexpected behavior may be possibly understood in terms of the modal "cut-off" phenomenon, for which modes below their cut-off frequency decay exponentially in the duct. Apparently the effects of cut-off are so strong that even an active impedance leads to strong decay in the presence of uniform flow.

At sufficiently low η, the modal regions reunite on the right-hand-side of the real γ axis, where a new branch point appears, as shown in Figure 6, providing a new optimum criteria. Figure 7 shows the revised optimum impedance and suppression curves for Mach -0.4 and n = 2 where only passive liners are allowed. The region between $0.20 < \eta < 0.36$ is where the lowest order mode exists in isolation of the second mode. Note the substantial drop-off in optimum attenuation below $\eta = 0.36$. Figure 8 shows the revised optimum resistance, reactance, and suppression curves for the Mach +0.4 case. Note the decrease in suppression below $\eta = 0.2$.

CONCLUSIONS

The Cremer optimization theory for least attenuated modes has been modified to account for the effects of mean flow at low η-values. It is seen that the branch point criteria no longer holds below certain critical η-values, and the optimum passive liner impedance must be determined from the modal maps. Revised optimum impedance and suppression curves have been presented for Mach ±0.4. In future studies, it would be useful to provide experimental verification of the revised optimum criteria. In particular, investigation of the active liner concept might prove of practical value, if such a device can be shown to exist.

REFERENCES

1. Motsinger, R.E., Kraft, R.E., and Zwick, J.W., "Design of Optimum Acoustic Treatment for Rectangular Ducts with Flow", ASME paper 76-GT-113, March, 1976.

2. Cremer, L., "Theory of Sound Attenuation in a Rectangular Duct with an Absorbing Wall and the Resultant Maximum Attenuation Coefficient", McDonnell-Douglas Corp. Report No. MDC-J6630, July, 1974, translated from Akustische Beihefte of Acustica, Hef 42, 1953, pp. 249-263.

3. Kraft, R.E., "Theory and Measurement of Acoustic Wave Propagation in Multi-Segmented Rectangular Flow Ducts", Ph. D. dissertation, University of Cincinnati, Department of Aerospace Engineering, June 1976.

4. Eversman, W., and Beckemeyer, R.J., "Transmission of Sound in Ducts with Thin Shear Layers - Convergence to the Uniform Flow Case", Journal of the Acoustical Society of America, Vol. 52, No. 1, pt. 2, 1972.

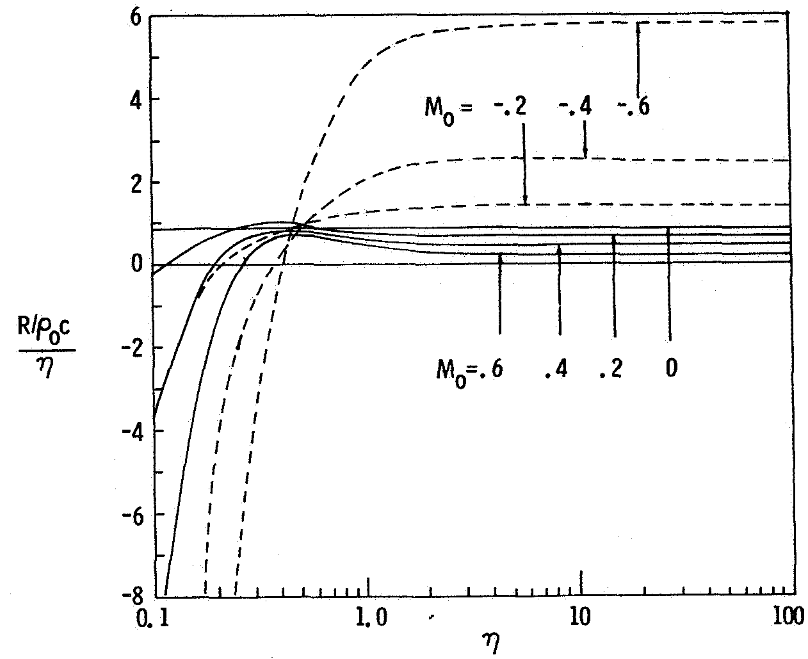

Figure 1.- Optimum resistance for lowest order mode as a function of η, for various Mach numbers, based on Cremer optimum criteria.

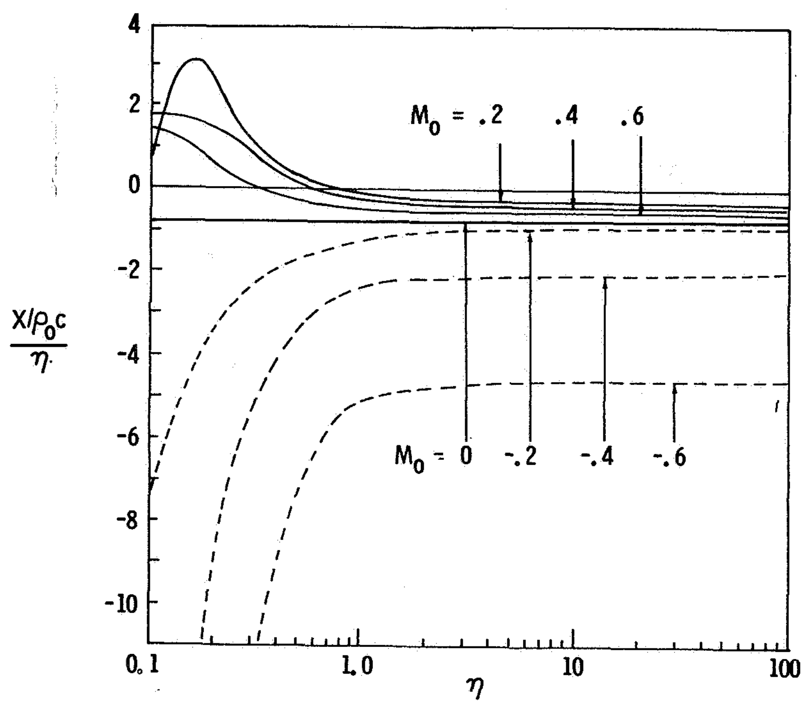

Figure 2.- Optimum reactance for lowest order mode as a function of η, for various Mach numbers, based on Cremer optimum criteria.

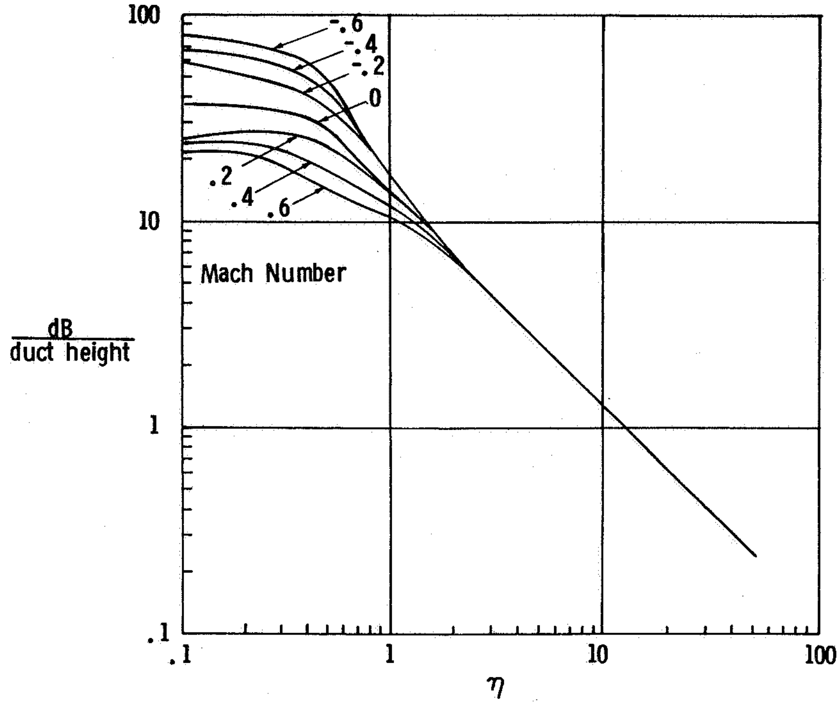

Figure 3.- Optimum attenuation as a function of η for various Mach numbers for lowest order mode, based on Cremer optimum criteria.

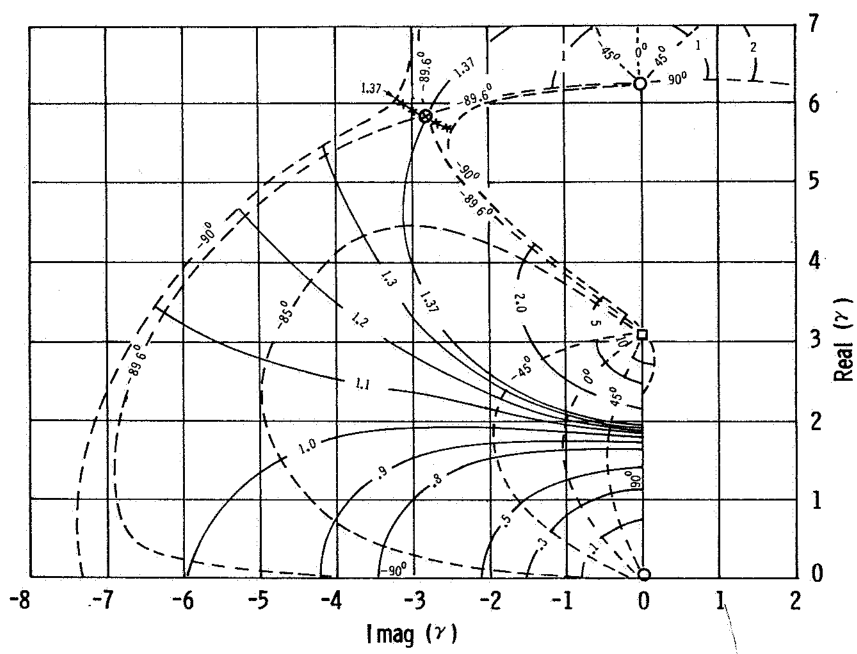

Figure 4.- Complex eigenvalue-admittance mapping for symmetric modes; Mach -0.4; kH = 2.2619; (η = 0.36); Continuity of Particle Displacement. ——— Constant |βkH|, ----- Constant Phase (βkH).

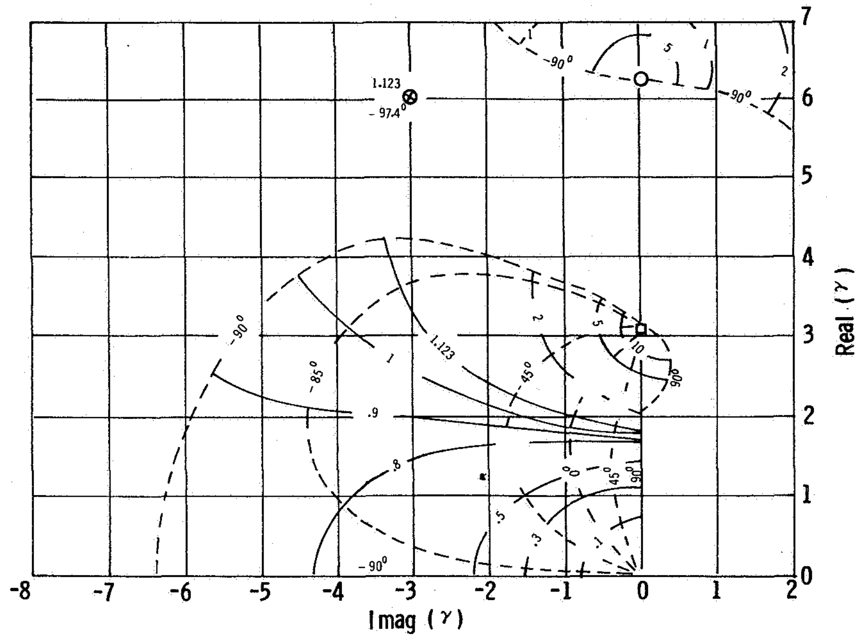

Figure 5.- Complex eigenvalue-admittance mapping for symmetric modes; Mach -0.4; kH = 1.885; (η = 0.3); Continuity of Particle Displacement. ——— Constant $|\beta kH|$, ----- Constant Phase (βkH).

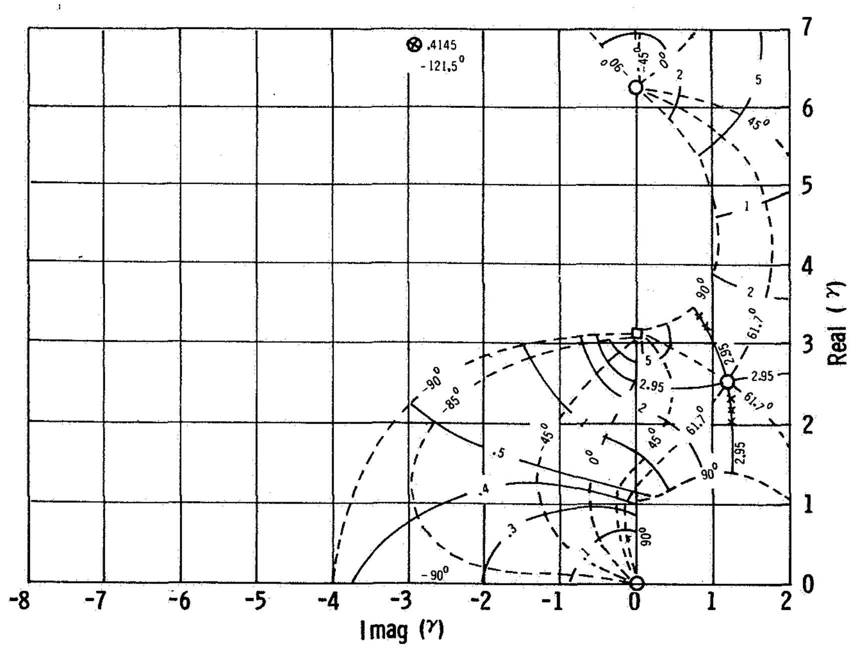

Figure 6.- Complex eigenvalue-admittance mapping for symmetric modes; Mach -0.4; kH = 0.9425; (η = 0.15); Continuity of Particle Displacement. ——— Constant $|\beta kH|$, ----- Constant Phase (βkH).

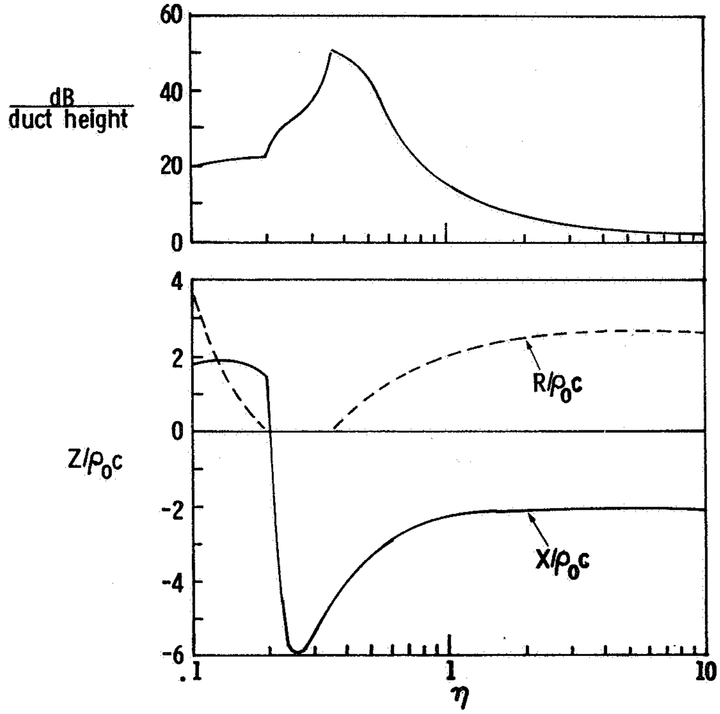

Figure 7.- Lowest order mode optimum suppression rate and impedances as a function of η for passive liners, Mach -0.4.

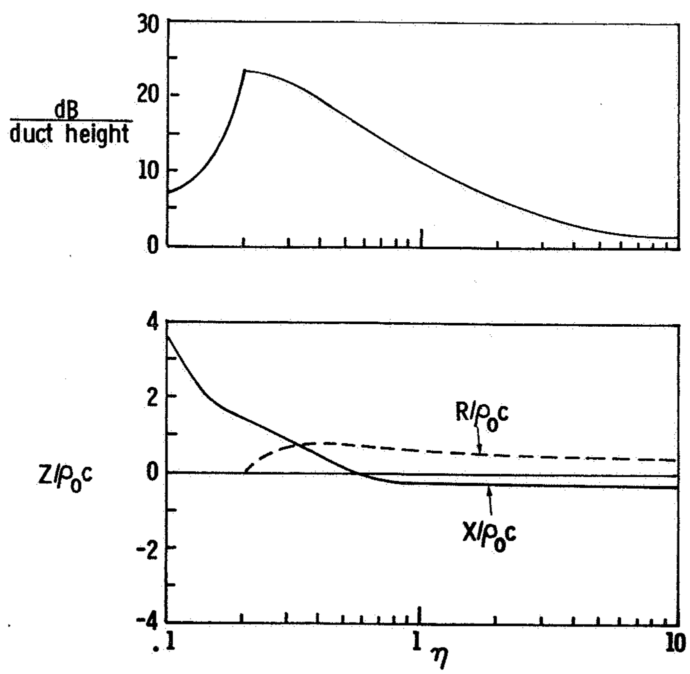

Figure 8.- Lowest order mode optimum suppression rates and impedances as a function of η for passive liners, Mach +0.4.