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THE PITCH-HEAVE DYNAMICS OF TRANSPORTATION VEHICLES

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SUMMARY

The analysis and design of suspensions for vehicles of finite length using pitch-heave models is presented. Dynamic models for the finite length vehicle include the spatial distribution of the guideway input disturbance over the vehicle length, as well as both pitch and heave degrees-of-freedom. Analytical results relate the vehicle front and rear accelerations to the pitch and heave natural frequencies, which are functions of vehicle suspension geometry and mass distribution. The effects of vehicle asymmetry and suspension contact area are evaluated. Design guidelines are presented for the modification of vehicle and suspension parameters to meet alternative ride quality criteria.

I. INTRODUCTION

A fundamental problem in the development of high performance transportation vehicles is insuring adequate passenger ride quality when operating over economically feasible guideway surfaces and structures. While passenger comfort has long been an important factor in vehicle design, recent developments in high speed ground transportation vehicle technology have identified ride quality as a primary constraint to successful implementation of systems capable of speeds of 200 to 500 km/hr.

Prediction of passenger ride quality is based on three components: quantification of guideway irregularities and aerodynamic conditions that act as disturbances to the vehicle; analysis of the dynamic response of the vehicle body, suspensions, and other subsystems to external disturbances; and finally comparison of the resultant vibrational environment to be experienced by the passengers to suitable measures of ride quality.

At present significant deficiencies exist in all three components of ride quality analysis when applied to the development of new transportation systems. No measure of passenger ride quality has yet gained wide acceptance due to the differences that exist between typical applications in transportation and the controlled experiments conducted to date [1]*. Ride quality is a subjective evaluation of a random, multidirectional, vibrational environment that may vary among individuals in a population and may be altered by exposure times and environmental factors other than motion. Currently available data does not systematically represent the above variables, either quantifying subjective human evaluations of one-dimensional, single frequency steady sinusoidal motion [2] or limited samples of vibrational environments on specific vehicles [3]. Existing ride quality measures or specifications are useful primarily as a qualitative indication of the nature of human sensitivity to vibration but have limited utility as absolute standards for system design.

*Numbers in brackets refer to the list of references.

Guideway irregularity and aerodynamic disturbances are represented either stochastically, as empirically derived spectra of surface roughness and atmospheric turbulence, or deterministically, as measured contours of specific guideways and computed coupled reactions between vehicles and flexible guideways or between passing vehicles. However, a lack of relevance between the spectral representations of guideway smoothness and meaningful guideway construction specifications has led to recent re-examinations of measures of guideway irregularities [4].

The dynamic response of vehicles to disturbances has been analyzed by two distinct methods. First, simulation models of widely ranging complexity [5,6, 7] have been formulated to represent vehicle response including multidimensional body motions, body flexibility, suspension displacements, and component vibrations. (Even more complicated models have been developed to include guideway flexibility.) In theory such simulations can be made as accurate as desired through the addition of elements to the model; however, the usefulness of these detailed simulations is limited by several factors:

(a) The analysis of vehicle response involves solution or simulation of equations of very high order, with a large number of design parameters. Such complex systems are not amenable to efficient computational and optimization techniques, and can frustrate the use of intuitive design procedures.
(b) Because of the large number of parameters and complex system structure, it may be difficult to generalize simulation results to other vehicles.
(c) It is difficult to validate the model experimentally at a level of detail commensurate with that of the analysis.

The second approach has been to use simple conceptual models of vehicles and suspensions as bases for intuitive design procedures and for closed-form or iterative optimization techniques. One-dimensional vehicle models, supported by either specific or generalized suspensions, subjected to a variety of disturbance, have been studied extensively, yielding mathematical descriptions of optimal suspensions for given sets of disturbances, performance criteria, and design constraints [8,9], or parametric descriptions of specific suspensions that approach the mathematical optima [5,10]. While valuable insights may be obtained by their use, one-dimensional models also have limitations:

(a) The one-dimensional model does not accurately represent the vibration environment experienced by the passenger, and is thus inadequate as a component of the prediction of ride quality.
(b) Suspensions designed to be optimal using the one-dimensional analysis may be suboptimal when the dynamics of the complete vehicle are considered. In turn, specification of non-optimal configurations may lead to costly overspecification of system parameters such as guideway smoothness and stiffness.

The dynamic characteristics of well-designed vehicle systems are such that the analysis and design of the complete vehicle/suspension system can be segmented into several uncoupled subsystems. The dynamics of a complete vehicle/suspension system traveling at constant forward speed can be categorized as

(a) Motion of the vehicle body: Of primary interest are the body heave and pitch in the vertical plane, sway and yaw in the horizontal plane, and roll. The cumulative effect of these motions is the motion perceived by the passengers within the vehicle, thus determining ride quality. These motions will be of low frequency (less than 15 Hz) in well-designed vehicles.

(b) Bending modes of the vehicle body: Due to body flexibility, an infinite number of natural modes can be excited, many at high frequencies. Analysis of these modes can be quite complex for bodies with discontinuous structures, such as transit cars with numerous doors. These vibrations may affect ride quality, passenger compartment noise, and the structural integrity of the body.

(c) Primary suspension motions: The motion of the cushion (or wheel) is primarily perpendicular to the guideway plane. If the cushions are separated from the vehicle by secondary suspensions, the dominant cushion mass oscillation will be of high frequency (greater than 25 Hz), due to the high gap stiffness and low cushion mass (less than 0.1 of the vehicle mass).

(d) Inter-suspension coupling: The motions of powered suspensions may be coupled, via the feeding system in air cushion vehicles or via the magnetic fields of magnetic cushions.

The one-dimensional heave mode suspension model is appropriate for predicting cushion stability and displacements subject to random disturbances. However, representing the vehicle motion as primarily one-dimensional neglects two important effects:

(a) The vehicle is capable of pitch and bending as well as heave. Numerous studies of high speed vehicles traveling over irregular and flexible guideways have shown that the pitch mode has a strong influence on passenger ride quality [11,12,7].

(b) The finite lengths of the cushions produce filtering of guideway irregularities having wavelengths shorter than the length of the pad. The heave mode vehicle model assumes all other wavelengths to be much longer than the vehicle length, which is not true for multicushioned vehicles.

The finite length vehicle model developed here includes both pitch and heave motion of a rigid body vehicle, as well as the effects of guideway irregularities seen along the vehicle length.

The objective of this paper is to help bridge the gap between the well-understood one-dimensional analysis and complex full vehicle models by presenting an analysis of finite length vehicles with pitch and heave* degrees of freedom. The analysis relates vehicle front and rear accelerations to the pitch and heave natural frequencies, which are functions of vehicle suspension geometry, suspension dynamic characteristics, and vehicle mass distribution. The vibrational environment used to determine ride quality is evaluated for vehicles subjected to stochastic and deterministic guideway disturbances. Design guidelines are presented for the modification of vehicle and suspension parameters to meet alternative comfort criteria.

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*The analysis in this paper applies to motion in the vertical plane (heave and pitch) and the lateral plane (sway and yaw). The discussion refers only to heave and pitch, but the extension to the lateral case is always implied. The analysis assumes that body motions in the vertical and lateral planes are uncoupled.

II. PREVIOUS RESEARCH

An extensive literature has developed on the dynamics of vehicle suspensions. Physical models of both wheeled and tracked levitated suspensions have been synthesized and in most cases experimentally verified [13 to 16]. Suspension characteristics have been commonly quantified in terms of primary and secondary suspension stiffness and damping, sprung and unsprung mass, and suspension/guideway contact area [17,10]. For levitated and some wheeled vehicles, suspension motion is primarily one-dimensional, so that the suspension force transmitted to the vehicle body is a function of the relative displacements and velocities between the suspension attachment point on the vehicle and the adjacent guideway location. (If the suspension mass is large, its acceleration must also be included in the model.)

Research conducted in parallel to the physical modeling studies exploited the commonality between the various suspension types by developing generalized suspensions that were mathematically optimal for specified sets of input disturbances and performance measures. Inputs from surface roughness, wind gust loading, and guideway flexibility have been considered, with performance indices composed of weighted RMS body accelerations* and relative body-guideway displacements [8,9]. Transfer functions of the optimal suspensions are obtained as a result. These functions indicate the limits of performance of suspensions under various conditions as well as optimal parametric values for suspension stiffnesses, damping, and mass.

Virtually all suspension designs resulting from both optimal suspension formulations and parametric studies of the physical models have been based on one-dimensional vehicle models. Both vehicle body accelerations and suspension excursions are inadequately represented by the one-dimensional analysis, as shown in this paper; at the same time both variables are of critical importance in vehicle/suspension/guideway design.

Coupled pitch and heave vehicle motions have been studied principally via simulation; however, some analytical results have been developed for special cases. As described above, the detailed complete vehicle simulations that have been performed have seen limited use because of their complexity. In-depth studies of automobile and transit vehicle ride quality [12,7], vehicle/flexible guideway dynamic interactions [11], and rail vehicle stability [16] have shown the importance of including the pitch mode in vehicle dynamic analyses. Pitch-heave, finite length vehicle models were used in flexible guideway studies to improve the ride quality prediction and to account for the distribution of the traveling load of the vehicle along the guideway. Body hunting in rail vehicles results from coupling between yaw and sway body modes, plus suspension (truck) motions; most attention in this area has been focused on elimination of instabilities below top operating speed, so that the effects of yaw on rail vehicle ride quality have not been fully explored.

Since the input seen by the front suspension propagates back along the vehicle to succeeding suspensions, the suspension inputs are correlated via pure time delays. The time delay formulation can appear in transfer function

*Many suspension parametric studies have used comfort criteria to develop standards or weighting functions [3,18] rather than RMS acceleration as measures of ride quality.

[19,20] or state variable [21] forms. Several studies have neglected the correlation between these guideway inputs, with significant degradation in predictive accuracy [7,21]. Hedrick, et al. [21] has shown that the effects of correlated inputs can be included in the Lyaponov's equation method for computing RMS vibration levels. However, no systematic study has previously determined the properties of pitch-heave, finite length vehicles.

III. GOVERNING EQUATIONS

Model Description and Basic Assumptions

The dynamic model for the finite length vehicle model differs fundamentally from the simple heave model in that the input disturbances are distributed spatially over the vehicle length. In this paper only the guideway inputs are considered. In general aerodynamic loading will be spatially distributed also, but very little information is available on the correlation between front and rear loading, and a quantified description of this input is likely to be dependent on the specific body shape.

The pitch and heave natural frequencies and damping ratios (i.e. the roots of the system transfer functions) are determined by the vehicle mass geometry and the suspension stiffness and damping. The spatial distribution of the guideway input affects the phase relationship between the motion of each point in the vehicle and the ground motion below it (i.e., all spatial terms appear in transfer function numerators). The phasing of the inputs and the vehicle motion depends on vehicle geometry and forward speed and the wavelengths of the guideway irregularities.

The basic model as shown in Figure 1 consists of a rigid body vehicle of mass M and inertia I about a center of gravity of arbitrary location. The coordinate system employed in the analysis is detailed in the figure. The governing equations are based on the following assumptions:

(a) All cushion dynamics are neglected; this is valid when the cushion mass natural frequency is much higher than the pitch-heave frequencies.
(b) The vehicle height is small compared to its length, and pitch angles are small so that each suspension is essentially in heave motion.
(c) The guideway displacement profile can be represented by a static description of either random or deterministic irregularities. This is the case for a guideway characterized by statistical roughness and for supported guideways not subject to dynamic excursions under transient vehicle loading.

Suspension Inputs

Each suspension exerts a force on the vehicle as a function G(s) of the relative suspension displacement,

$$\Delta\overline{F}_n = -G(s)\ (\Delta\overline{y}_{2,n} - \Delta\overline{y}_{o,n}) \qquad (1)$$

In general G(s) represents the cumulative effects of the primary and secondary suspensions, and unsprung mass. For the analysis of the vehicle pitch-heave motions, only the secondary suspension is assumed to be important. Since the dynamics of the cushion are neglected, Equation (1) is equivalent to assuming that the pad follows the guideway. This assumption is verified by detailed simulation in Section V.

Each secondary suspension is defined to have stiffness and damping equal to the total vehicle heave secondary suspension divided by the number of suspensions N, attenuated by the finite pad length function ψ_v,

$$G(s) \approx \frac{1}{N}(b_b s + k_b) \cdot \psi_v \tag{2}$$

The guideway input to the nth suspension is correlated with the input to the first suspension, and is expressed using equivalent time delay operators as a function of the distance, $x_{1,n}$, from the center of the first cushion to the center of the nth cushion,

$$\Delta\overline{y}_{o,n} = e^{-T_n s}\Delta\overline{y}_{o,1} \equiv e^{-(x_{1,n}/V)s}\Delta\overline{y}_o \tag{3}$$

This time delay formulation preserves the proper phase relationships between successive pads in a manner consistent with the finite pad length analysis described below.

Vehicle Pitch-Heave Motion

The heave motion of the center of gravity $\Delta\overline{y}_2$ is found by summing the suspension forces $\Delta\overline{F}_n$ *

$$Ms^2\Delta\overline{y}_2 = -G(s)\sum_n(\Delta\overline{y}_{2,n} - e^{-T_n s}\Delta\overline{y}_o) \tag{4}$$

From geometry $\Delta\overline{y}_{2,n} = \overline{y}_2 - \theta \cdot x_{2,n}$, so that

$$(Ms^2 + b_b s + k_b)\,\Delta\overline{y}_2 = \frac{1}{N}(b_b s + k_b) \cdot \psi_v \cdot$$

$$[-N\eta\, L_v\,\overline{\theta} + \sum_n(e^{-T_n s}\Delta\overline{y}_o)] \tag{5}$$

where η defines the location of the c.g.

The pitch motion θ is found by summing the moments about the center of gravity $\Delta F_n \cdot x_{2,n}$, where $x_{2,n}$ is the distance from the pad attachment point (pad center) to the body center of gravity.

$$Is^2\overline{\theta} = \sum_n G(s)x_{2,n}[(\Delta\overline{y}_{2,n} - e^{-T_n s}\Delta\overline{y}_o)] \tag{6}$$

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*All summations in this paper are from n = 1 to n = N. Vehicles with missing midbody suspensions, such as wheeled vehicles, are treated by modifying the summation; unless otherwise stated the results in this paper generalize to both vehicle classes.

or

$$(Is^2 + \frac{\overline{Y^2}}{N} b_b s + \frac{\overline{Y^2}}{N} k_b)\ \overline{\theta} = \frac{1}{N} (b_b s + k) \cdot \psi_v \cdot \quad (7)$$

$$[-N\eta L_v \Delta\overline{y}_2 - \sum_n x_{2,n} (e^{-T_n s} \Delta\overline{y}_o)]$$

where $\frac{\overline{Y^2}}{N}$ is the mean squared value of the distance $x_{2,n}$.

Equations (5) and (7) are dynamically coupled through their cross terms in $\Delta\overline{y}_2$ and $\overline{\theta}$. Transfer functions relating accelerations in the vehicle to the guideway input have fourth-order characteristic equations. For inputs that have wavelengths much longer than the vehicle length, these transfer functions are equivalent to those of the one-dimensional heave model.

The vertical accelerations along the vehicle above the nth suspension are given by

$$s^2\Delta\overline{y}_{2,n} = s^2\Delta\overline{y}_2 - x_{2,n}s^2\overline{\theta} \quad (8)$$

Finite Pad Length

Finite pad length analysis is based on a description of the guideway profile, either random or deterministic, as a sum of sinusoidal profiles of the form

$$y_o(x) = y_o \left(\sin \frac{(Vt + x_o)}{\lambda/2\pi}\right) \quad (9)$$

where x is the coordinate along the guideway, and the guideway profile is assumed to be uniform across the pad area. By direct integration the effects of changes in exit gap in air cushions, in active volume in air cushions and magnetic suspensions, and in deformation in the contact patch of flexible wheels can be determined. The distribution of the guideway input over the finite contact area causes a frequency dependent filtering of wavelengths λ shorter than the pad length L_p, given by the filtering function,

$$\psi_v = \frac{\lambda}{\pi L_p} \sin \left(\frac{\pi L_p}{\lambda}\right) = \frac{2V}{L_p\omega} \sin \left(\frac{L_p\omega}{2V}\right) \quad (10)$$

for the active volume and contact patch cases [17]. (A similar expression exists for the exit gap case.)

IV. PROPERTIES OF PITCH-HEAVE MODELS

Symmetric Vehicles

For arbitrary location of the center of gravity Equations (5) and (7) are coupled. They become decoupled, and hence considerably simplified, for symmetric vehicles ($\eta = 0$). The heave mode natural frequency ω_h and damping ratio ξ_h for a vehicle of mass M with suspensions as previously described are identical to those of the one-dimensional model:

$$\omega_h = \sqrt{\frac{k_b}{M}} \qquad \xi_h = \frac{b_b}{2\sqrt{k_b M}} \tag{11) (12}$$

The pitch mode natural frequency ω_p and damping ratio ξ_p are

$$\omega_p = \sqrt{\frac{\gamma^2 k_b}{NI}} \qquad \xi_p = \frac{b_b \frac{\gamma^2}{N}}{2\sqrt{\frac{\gamma^2 k_b}{N} I}} \tag{13) (14}$$

The inertia I of the vehicle can be expressed as

$$I = r^2 I_o = r^2 \left(\frac{1}{12} M L_v^2\right) \tag{15}$$

where I_o is the inertia of a uniform bar of length L_v. If the vehicle has its mass concentrated near its center, $r < 1$; $r > 1$ for vehicles with mass concentrated at the body ends. Then,

$$\frac{\omega_p}{\omega_h} = \frac{\xi_p}{\xi_h} = \frac{1}{r}\sqrt{\frac{12\gamma^2}{NL_v^2}} \tag{16}$$

As the number of pads N goes to infinity, $\frac{\gamma^2}{N} \rightarrow \frac{L_v^2}{12}$. For a vehicle with only four pads, $\frac{12\gamma^2}{NL_v^2} = 0.985$; for symmetric vehicles with four or more suspensions along the length the pitch frequency and damping are given by

$$\frac{\omega_p}{\omega_h} = \frac{\xi_p}{\xi_h} \approx \frac{1}{r} \tag{17}$$

The relative magnitudes of the pitch and heave natural frequencies are important in determining the responses of the front and rear of the vehicle. If the pitch and heave modes are in phase, then the heave and pitch motion seen at the front of the vehicle will add, while they will tend to cancel each other in the rear. The reverse is true if the heave and pitch modes are 180° out of phase. At the center of the vehicle the distance $x_{2,n}$ equals zero, so that the acceleration is determined by the heave motion alone.

The relative phase of the heave and pitch motions is determined by examination of Equations (5) and (7) for symmetric vehicles, rewritten here in transfer function form,

$$\frac{\Delta \bar{y}_2}{\Delta \bar{y}_o} = \frac{\frac{1}{N}(b_b s + k_b)\ \psi_v \sum_n (e^{-T_n s})}{(Ms^2 + b_b s + k_b)} \tag{18}$$

$$\frac{\bar{\theta}}{\Delta \bar{y}_o} = - \frac{\frac{1}{N}(b_b s + k_b\ \psi_v \sum_n (x_{2,n} e^{-T_n s})}{(Is^2 + \frac{\gamma^2}{N} b_b s + \frac{\gamma^2}{N} k_b)} \tag{19}$$

The $(b_b s + k_b)$ and ψ_v terms in (18) and (19) will always contribute identical phase angles at all frequencies. The phase angles due to the quadratic denominators will be in the range $(0, -\pi)$, passing through $-\frac{\pi}{2}$ at their respective natural frequencies.

The phase difference between the delay terms in (18) and (19) can be shown to be constant for all frequencies $s = j\omega$ for any symmetric vehicle,

$$\angle \Sigma\ (e^{-jT_n \omega}) - \angle \Sigma\ (x_{2,n}\ e^{-jT_n \omega}) = + \frac{\pi}{2} \tag{20}$$

The relative phase difference between heave and pitch is then a function only of the natural frequencies and damping ratios. For the limiting $(N \geq 4)$ case, the behavior as a function of normalized frequency ω/ω_h can be shown parametrically by the inertia factor r^2. The phase difference is shown in Figure 2 and the amplitude responses of the front, rear, and c.g. body positions to sinusoidal guideway inputs are shown in Figure 3. For comparison the heave model response is also plotted.

For $r^2 > 1$, the pitch natural frequency is lower than the heave frequency. Between the two frequencies the phase difference between heave and pitch is close to zero, resulting in larger front accelerations, and lower rear ones, than that experienced by the c.g. The converse is true for $r^2 < 1$. At $r^2 = 1$, the phase difference is constant (at $-\pi/2$) and the front and rear experience identical accelerations. Depending on r^2, ξ_h and ω/ω_h the end point accelerations may be greater or less than those predicted by the heave model, as shown in Figure 3.

The acceleration at the c.g. is always less than, or equal to, that predicted by the heave model, and unaffected by the value of the pitch natural frequency. The magnitude of the heave damping ξ_h does not affect the qualitative behavior discussed above for underdamped $(\xi_h < 1)$ suspensions; the principal effect of increased ξ_h is the reduction in the sharpness of the resonant peaks at ω_h and ω_p.

Asymmetric Vehicles: The Effect of Variable c.g. Location

The mass distribution within the vehicle body is a design function subject to engineering constraints. Moving the c.g. along the body away from the geometric center (of a symmetric vehicle) has two principal effects:

(a) By coupling the heave and pitch motions, the characteristic polynomial is modified as follows,

$$0 = D(s) = MIs^4 + (b_b I + \frac{b_b M\gamma^2}{N})s^3 + (k_b M\frac{\gamma^2}{N} + k_b I + b_b{}^2(\frac{\gamma^2}{N} - \eta^2 L_v{}^2))s^2 +$$

$$2b_b\, k_b\, (\frac{\gamma^2}{N} - \eta^2 L_v{}^2)s + k_b{}^2\, (\frac{\gamma^2}{N} - \eta^2 L_v{}^2) \qquad (21)$$

where η is the fraction of vehicle length the c.g. is located aft of the geometric center. Since the c.g. location η appears in (21) only as a squared term, the characteristic polynomial, and hence the pitch and heave natural frequencies, are changed equally for positive and negative η. The effect of η is to cause the roots of (21) to spread for constant M, b_b, k_b, N and I. Whichever pole pair is of higher frequency, pitch, or heave becomes larger in frequency and damping ratio. Causing the higher natural frequency to increase into the 4 to 30 Hz human sensitivity range in many cases will deteriorate ride quality.

(b) The distances between the c.g. and the vehicle front and rear will, of course, change. Whichever distance increases will make that vehicle location more sensitive to pitch. This effect can be used as a design tool to "balance" the ride quality in the rear and front (desirable since ride quality is usually judged by the worst condition in the passenger compartment).

Effects of Pad and Vehicle Length

All previous suspension design studies have concluded that pad length L_p should be as long as possible to obtain the greatest attenuation of the guideway irregularities. Pad length has been principally constrained by geometric considerations such as minimum guideway curvature radii [22].

Finite vehicle length effects in the pitch-heave analysis cause the pad length effect to be much less important. The acceleration of the c.g. of a symmetric vehicle with suspensions uniformly distributed along the body, Equation (18), is independent of the pad length. The magnitude of the time delay term times the finite pad function is independent of pad length (using $L_p = L_v/N$):

$$\left|\frac{1}{N} \cdot \psi_v \cdot \sum_n e^{-T_n s}\right| = \frac{2V}{L_v \omega} \sin\left(\frac{L_v \omega}{2V}\right) \qquad (22)$$

Thus for the heave mode the entire vehicle length filters the guideway input. The expressions for the pitch mode and for the coupled pitch-heave modes of asymmetric vehicles are much more complex; studies of their limiting behavior show that for typical high speed vehicles (see Table 1), ride quality is unaffected by changes in pad length below L_p = 9.15 m (30 feet). The guideway geometry often constrains pads to this length or less. Therefore for vehicles with suspensions distributed along the vehicle body, the individual pad length is not important. However, pad length effects will still be beneficial to vehicles with only front and rear suspensions since the filtering effect will be increased by adding to the total contact area.

V. SIMULATION RESULTS

Verification of Modeling Assumptions

The representation of the suspension dynamics in Equation (2) as being equivalent to the secondary suspension in series with the finite pad length filter is verified by comparison with simulations of complete vehicle/suspension systems. For example, Figure 4 shows a unit step response of two adjacent air cushions coupled through their feeding system on a 134 m/s (300 mph) TACV. Both primary and secondary suspension dynamics are simulated for different pad lengths.

The responses consistent with Equation (2) for this simulation would be two ramps with slopes of V/L_p, the second delayed by (pad separation/V) seconds. The ramp results from the time averaging of the active volume by the cushion as it encounters the step. The simulated responses differ from the approximate representation only in their high frequency content, which is filtered by the secondary suspension. The decoupling of the high frequency primary suspension dynamics and the low frequency secondary suspension/vehicle displacements is demonstrated in Figure 5, again verifying the approximation in (2).

Parametric Effects

The effects of body inertia I, suspension damping b_b, and c.g. location η on ride quality as described in Section IV are demonstrated in Figures 6, 7, and 8. The design example considered is described in Table 1. The vehicle is traversing a guideway with surface roughness spectral density $\pi VC/\omega^2$. Shown for comparison is the U.S. DOT UTACV ride quality specification [17].

The degradation of ride quality with mass concentrated near the vehicle center ($r^2 = 0.5$) is shown resulting from a high pitch natural frequency. Similar degradation can be shown to result from distributing suspensions to vehicle ends by eliminating the middle suspensions, also raising ω_p. Also demonstrated is the dependence of optimum damping ratio ξ_h on pitch and heave natural frequencies, and on the specific ride quality criterion used.

TABLE 1

VEHICLE PARAMETERS - DESIGN EXAMPLE

Mass, M, kg (lb)	54 440 (120 000)
Length, L_v, m (ft)	36.6 (120)
Number suspensions along vehicle length, N	6
Heave natural frequency, Hz	0.75
Forward speed, V, m/s (mph)	134 (300)
Guideway surface roughness, C, m (ft)	36.6×10^{-6} (1.2×10^{-6})

The cumulative effects of variable c.g. location are illustrated in Figure 8, in which $\omega_p < \omega_h$, for values of η equal to -0.2, 0, and +0.2. The shifting of the heave natural frequency from 0.75 Hz to 0.87 Hz for $\eta = \pm 0.2$ is evident, as are the adverse results of an aft location for the c.g. on front acceleration, and front c.g. location on rear acceleration.

From these examples it is clear that the pitch-heave model is necessary, as a minimum level of sophistication, for predicting ride quality.

VI. IMPLICATIONS FOR VEHICLE AND SUSPENSION DESIGN

The primary variables to be considered in TLV design include the vehicle weight, shape, and mass distribution, the operating speed, primary suspension characteristics including its unsprung mass and pad length L_p, the secondary suspension dynamics (if used), the guideway geometry, and the external force inputs. A practical design must meet passenger comfort criteria, such as the ISO specifications, with minimal suspension power consumption when operating over economically feasible guideways.

The overall suspension design (secondary, if used, and primary) must support and guide the vehicle along its prescribed path while ignoring local irregularities or errors in the guideway. In the dominant frequency range for vehicle-suspension systems the suspension may be approximated as a stiffness k_b and damping b_b. Lowering k_b lowers the dominant frequencies and improves ride comfort by reducing the acceleration resonant peak. However, limits are placed on low values of k_b by allowable dynamic secondary suspension and gap displacements. Minimizing unsprung mass is beneficial since it reduces dynamic loads on the guideway and usually improves ride comfort by causing the unsprung mass resonance to occur at a frequency that is filtered by the secondary suspension and finite cushion area.

The dominant dynamic motions of TLV in pitch and heave occur with characteristic frequencies ω_p and ω_h which are typically in the range of 0.5 to 2 Hz. Since vibrational accelerations are proportional to frequency squared, it is desirable to make both ω_p and ω_h as low as possible. In most cases ω_p should be made less than ω_h, in which case the worst accelerations will occur at the front of the vehicle. To lower the pitch natural frequency for a given heave stiffness and damping, the following measures can be taken:

(a) Distribute the vehicle mass toward the ends to increase I. Heavy equipment such as axial fans, electric-power conditioning equipment and LIM's are examples of components that could be so located. The upper bound on I is given by $r^2 = 3$ (all mass concentrated at ends) with a realistic limit probably of $r^2 = 2$. (The prototype UTACV recently developed by Rohr Industries has an r^2 equal to about 1.6, with the center of gravity almost exactly at the middle of the vehicle length.)

(b) Reduce rotational stiffness. This would be accomplished by concentrating stiffness near the vehicle center. However, a trade-off between pitch stiffness and allowable endpoint excursions exists, limiting the concentration of stiffness near the center. Added data on transient aerodynamic moments is needed to quantify the bound on pitch stiffness.

Optimum locations of the c.g. will be dependent on the specifications of the vehicle body (mass, flexibility), suspension, and shape of the comfort criteria. At this level of analysis, however, no clear advantage is evident for locating the c.g. away from the geometric center, while the shifting of the natural frequencies and lengthening of the distance to one vehicle end tend to adversely affect passenger ride quality for non-zero values of η. In contrast to these guidelines, a common vehicle analytical model is one consisting of a uniform mass distribution with discrete suspensions at the front and rear only. In this case the pitch frequency ω_p is greater than the heave frequency ω_h by a factor of $\sqrt{3}$; the resulting accelerations in the 4 to 30 Hz ride quality sensitive range may be up to an order of magnitude greater than for a similar vehicle designed according to the above guidelines. A poorly chosen vehicle configuration can lead to costly overspecification of required guideway smoothness and stiffness.

Optimal damping ratios for each suspension are determined by comparison of acceleration spectral densities with frequency dependent ride quality criteria. Considering only heave motion yields optimal values of ξ_h between 0.2 and 0.3 [10]. The reason for the existence of an optimum is that as damping increases the height of the resonant peak at ω_h is reduced, but at the same time more power is transmitted through the suspension to the vehicle at high frequencies. When both pitch and heave motion are considered, optimal suspension damping ratios may range from 0.1 to 0.5, depending on the pitch natural frequency.

When active feedback is used to control a suspension, improved performance can be obtained by sensing absolute vehicle accelerations and vehicle-guideway displacements and using the results to control suspension force. Acceleration feedback increases the effective mass and rotational inertia of the vehicle, reducing the pitch and heave natural frequencies without lowering the respective stiffnesses in these two modes. Displacement feedback is used to alter the stiffness and damping characteristics of the passive suspensions.

The design of support and guidance suspensions differ in several respects. The guidance suspensions support no equilibrium load; the preload is thus a free parameter which can be advantageously used with suspensions with non-linear force-deflection characteristics. Guidance suspensions act in push-pull, effectively doubling the suspension stiffness and damping. Finally, available ride quality data [5] indicates that comfort sensitivity is higher in the lateral plane, resulting in stricter comfort requirements.

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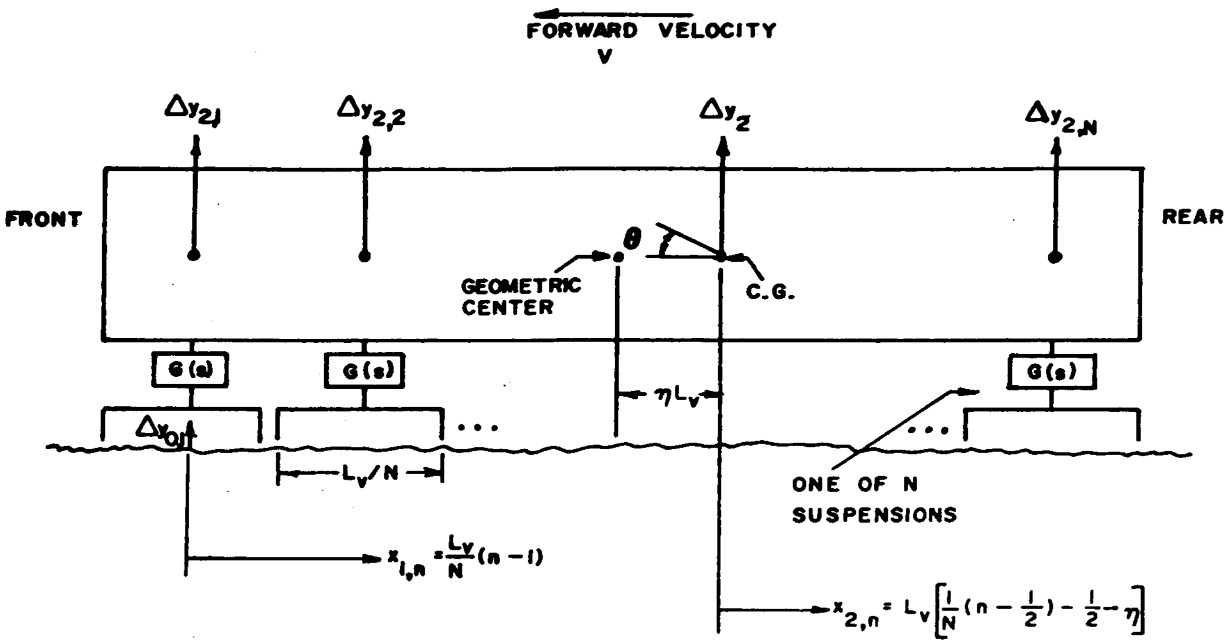

Figure 1.- Schematic of general pitch-heave model.

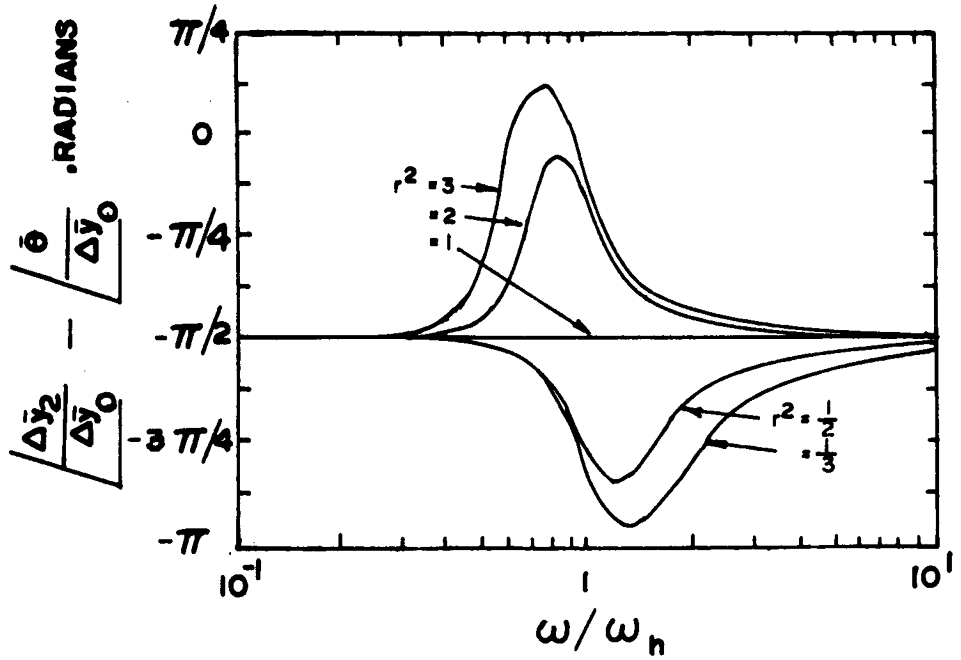

Figure 2.- Phase angle between heave and pitch motions.

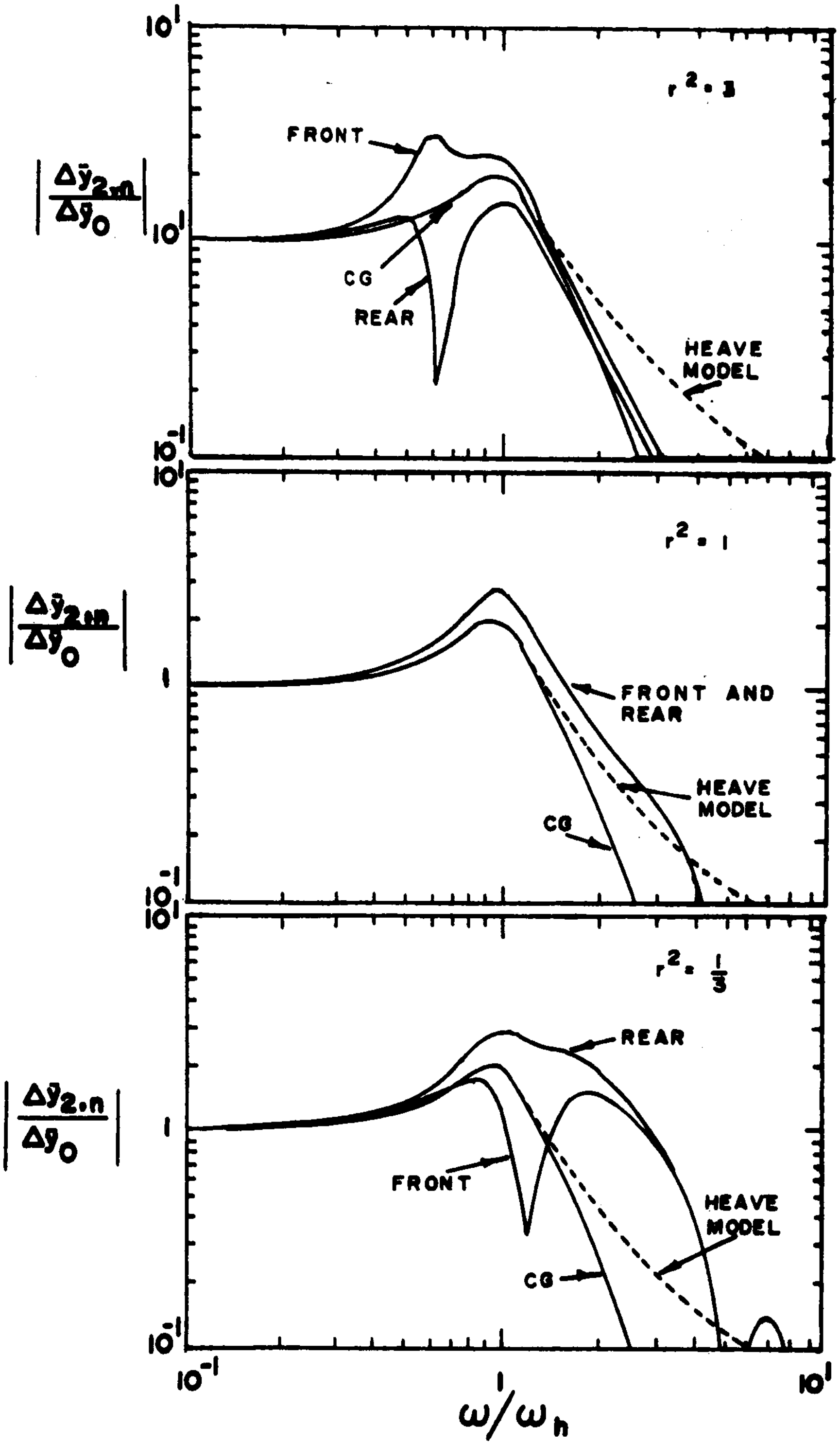

Figure 3.- Magnitudes of transfer functions relating vehicle displacement to guideway input amplitude using pitch-heave models.

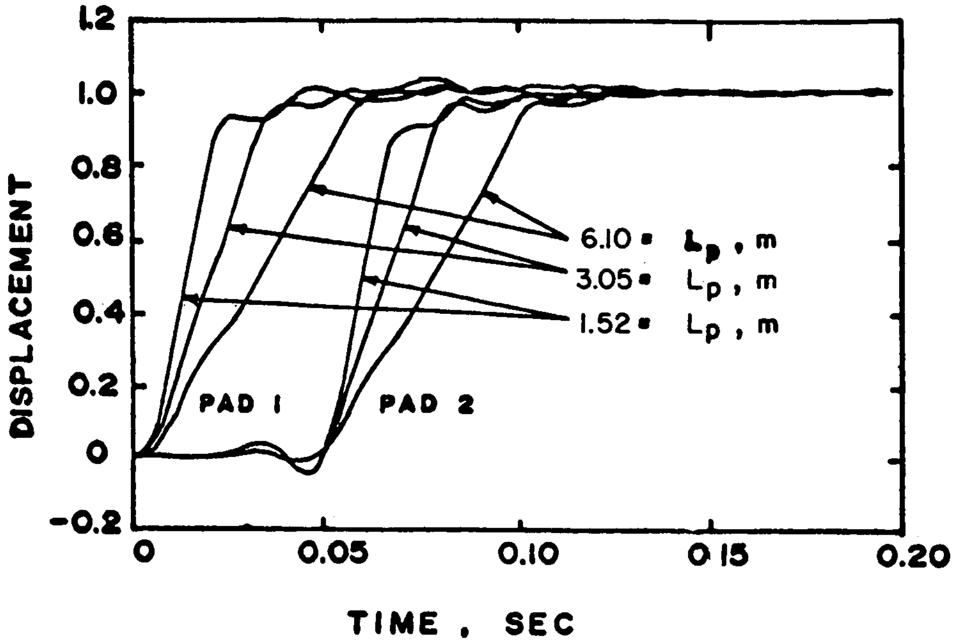

Figure 4.- Unit step response of two cushions, twenty feet apart, for different pad lengths.

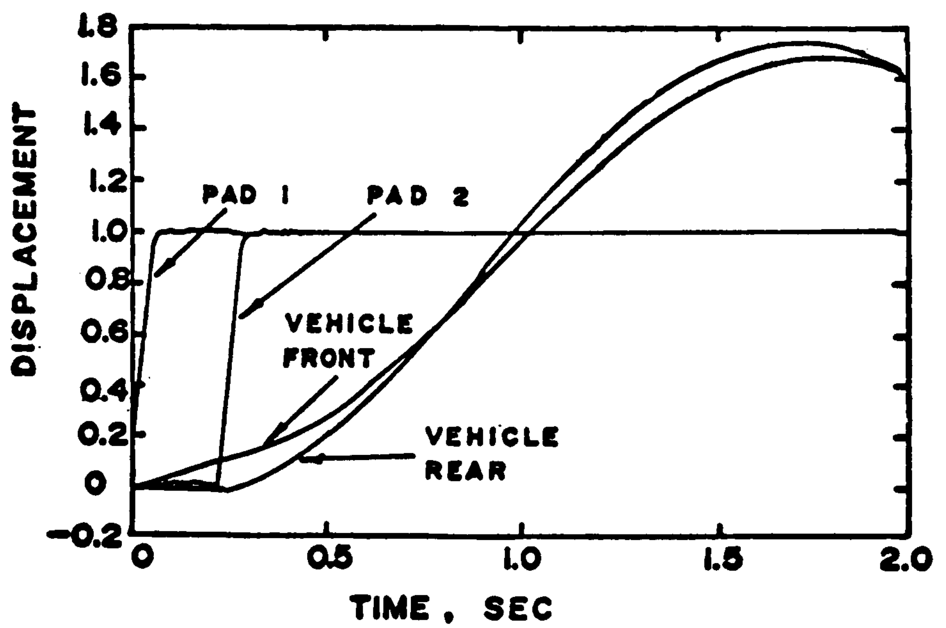

Figure 5.- Unit step response of two cushions, pitch-heave vehicle model.

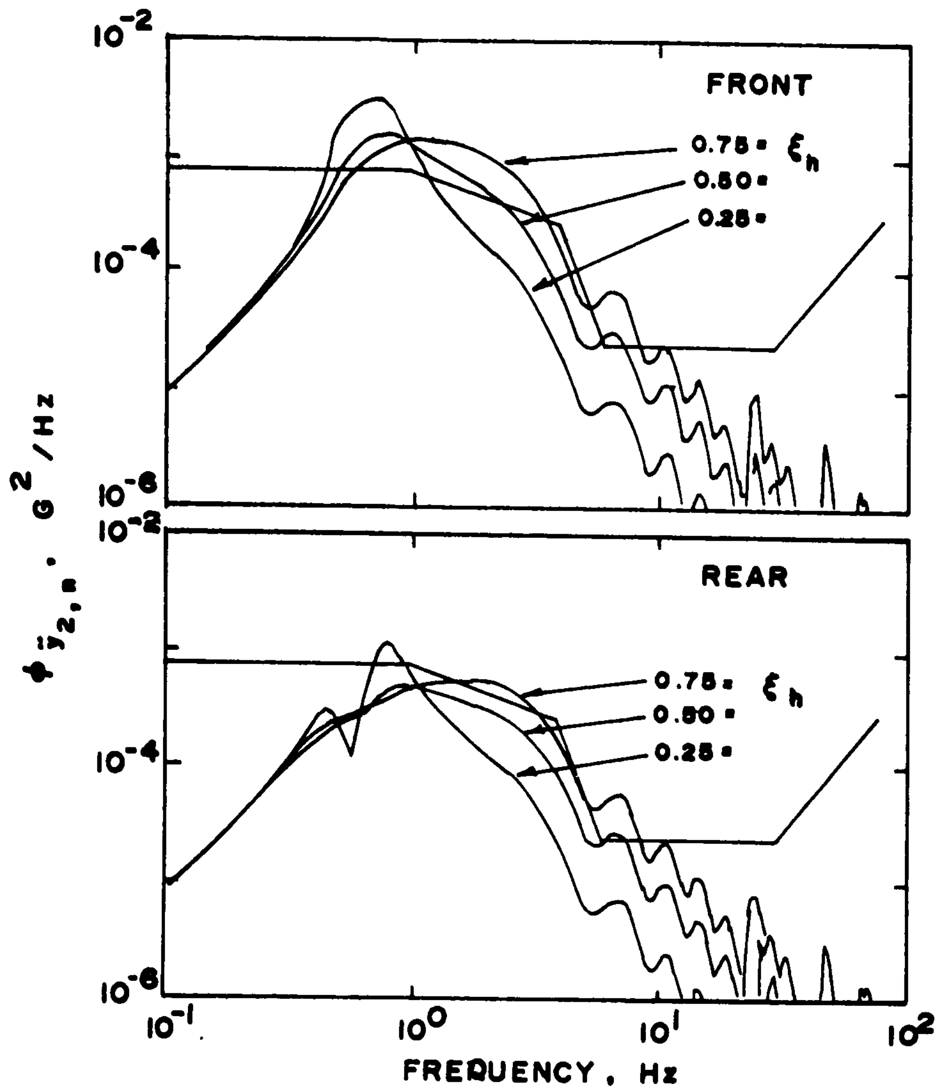

Figure 6.- Pitch-heave model acceleration spectral density as a function of suspension damping for $r^2 = 2$.

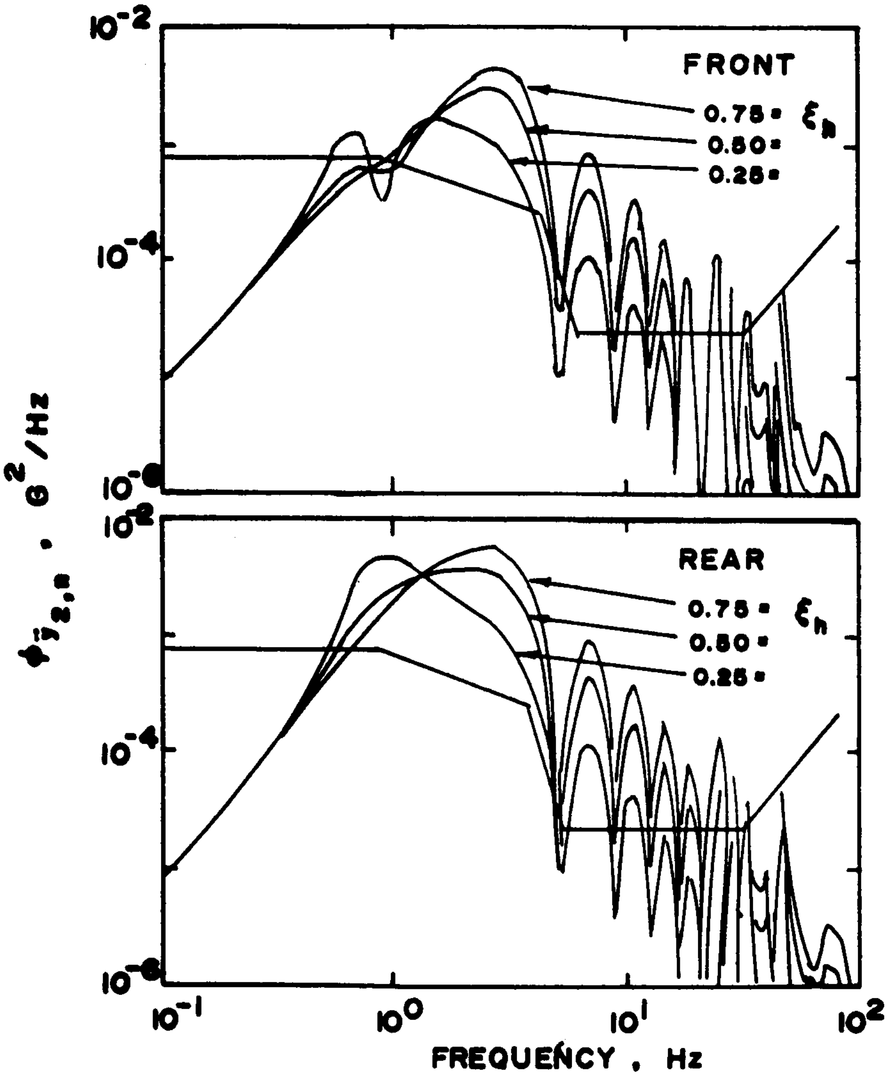

Figure 7.- Pitch-heave model acceleration spectral density as a function of suspension damping for $r^2 = 0.5$.

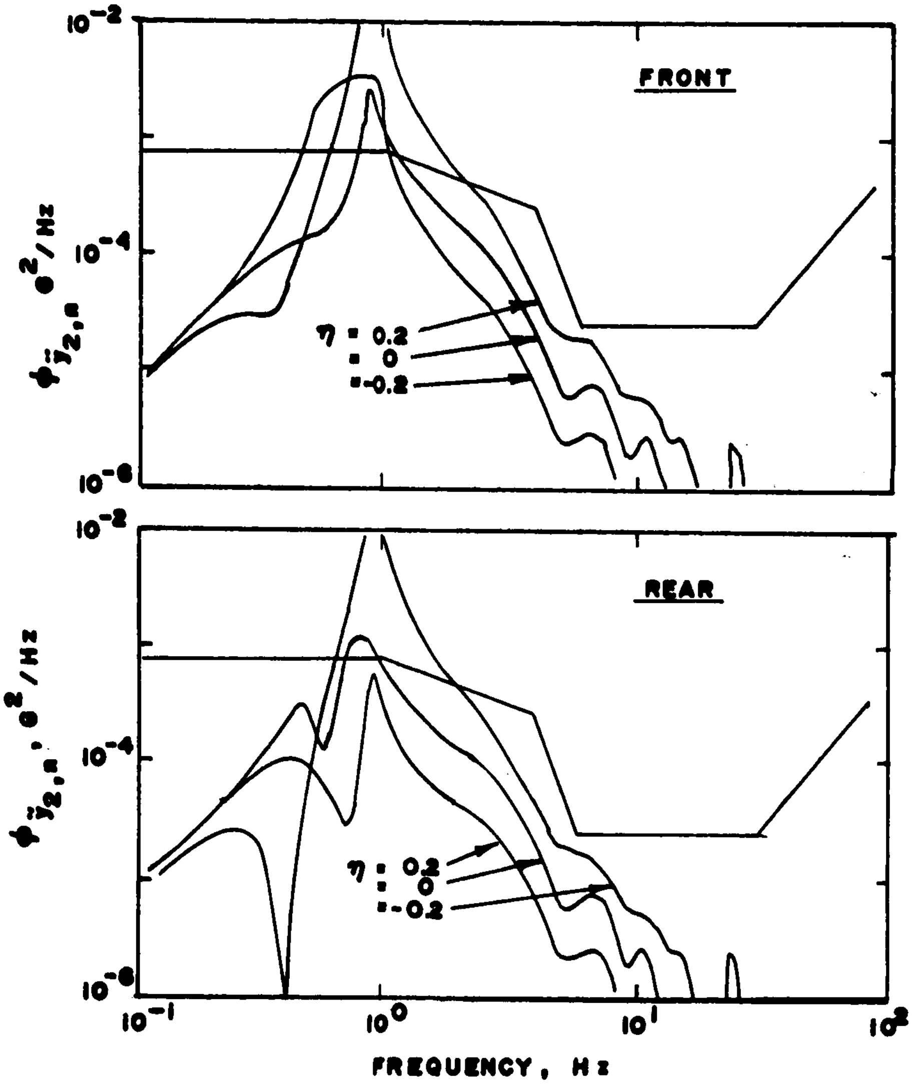

Figure 8.- Effect of variable c.g. location on ride quality $\left(r^2 = 2,\ \xi_h = 0.25\right)$.