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LONG FLUID FILLED BAGS SUSPENDED BY LINE FORCES

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ABSTRACT: A previous analysis of fluid filled storage bags is extended to the case of a long fluid filled cylindrical membrane supported by uniform line loads. Cross-sectional shape, stiffness of the support system and stress resultants in the membrane are determined. The application of the numerical results to problems arising in the design of non-rigid airships is discussed.

INTRODUCTION

Long fluid filled bags are used for a variety of purposes and examples which have been studied include sausage-like storage bags for oil [1] portable silos [2], inflatable structures including life-rafts [3], suspended cylinders [4] and a variety of non-rigid pressure airships, "blimps", and semi-rigid dirigibles [5].

The long filled cylinder resting on a horizontal flat base was considered by Demiray and Levinson. [1] They obtained a solution for the stress resultants and the shape of the bag in repose. In this present work, their analysis is employed and extended

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to apply to the case in which the long bag is supported by concentrated loads applied to the membrane along lines parallel to the bag axis. It is shown that the results can be summarized by functional relationships of non-dimensional parameters and some numerical results are presented.

The solutions are relevant to the design of non-rigid pressure airships. In these vehicles the principal fixed weight, the car, is attached to the fabric envelope by a so-called "catenary" suspension system inside the envelope as shown in Fig 1.

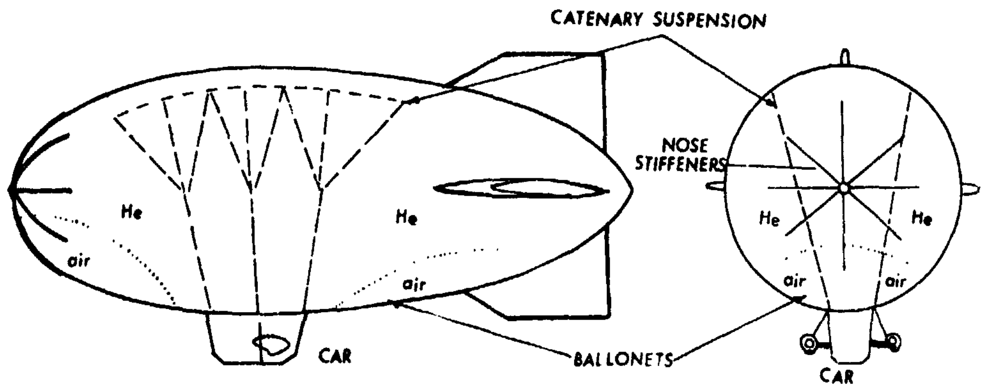

Fig 1 General arrangement of a non rigid pressure airship

The envelope is maintained at a constant differential inflation pressure by pumping air into the ballonets shown. The fabric is reasonably light and woven in such a way as to resist both direct and shear strains. The inflation pressure is sufficient to maintain the shape of the envelope under static and aerodynamic loads. The application of the numerical results of the two dimensional analysis to the case of airship envelopes is discussed.

THE ANALYSIS

The membrane is assumed to be inextensible in all directions, to have zero flexural rigidity and to be weightless. We consider a normal section of an infinitely long uniform bag. Under equilibrium conditions, the cross section is represented by the curve,

$$x = x(s) \qquad x(o) = o$$
$$y = y(s) \qquad y(o) = o \tag{1}$$

where (x,y) is a set of rectangular cartesian coordinates and s is the arc length measured from the lowest point, the origin, in Fig 2.

The stress resultant in the membrane is T, which is constant between loads, and the angles between the tangent to the membrane and the horizontal is $\theta(s)$. For the cases to be considered here, the hydrostatic pressure or more strictly the differential pressure across the membrane, is p(s) which is taken as,

$$p(s) = p_o + wy \qquad (2)$$

where p_o is the inflation pressure at the lowest point, y = o, and w is the difference in specific weight between the fluids inside and outside the membrane. In general w may be either positive or negative and in the problems considered, it is constant i.e. the fluids are incompressible.

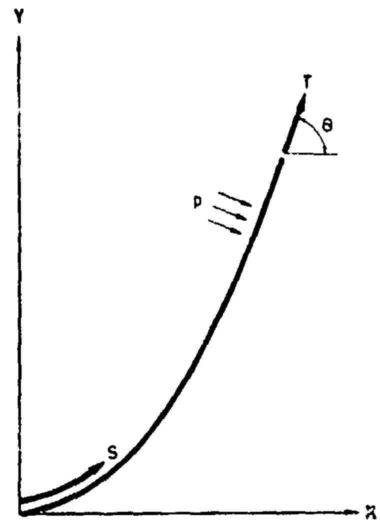

Fig 2 Membrane Coordinates

The general solution to the problem for w < 0 is given by Demiray and Levinson[1]. Define R as $1/2\pi$ times the perimeter of the membrane cross-section. The following dimensionless groupings will be used:

$$\frac{x}{R}, \frac{y}{R}, \frac{s}{R}, \frac{p_o}{Rw}, \frac{T}{p_o R}$$

For convenience in writing the following equations define

$$k^2 = \frac{4Tw}{p_o^{\,2}} = 4\,\frac{(T/p_oR)}{(p_o/Rw)} \qquad (3)$$

For $(4Tw/p_o^2) > 0$, Ref (1) obtains

$$\frac{y}{R} = \frac{p_o}{Rw}\left\{\sqrt{[1+k^2\sin^2(\theta/2)]} - 1\right\} \qquad (4)$$

$$\begin{aligned}\frac{x}{R} = {} & 2\,\frac{T}{p_oR}\,\frac{\sin\theta}{\sqrt{[1+k^2\sin^2(\theta/2)]}} \\ & - \frac{p_o}{Rw}\sqrt{(1+k^2)}\;E\left[\alpha, \frac{k}{\sqrt{(1+k^2)}}\right] \\ & + \left(2\,\frac{T}{p_oR} + \frac{p_o}{Rw}\right)\frac{1}{\sqrt{(1+k^2)}}\;F\left[\alpha, \frac{k}{\sqrt{(1+k^2)}}\right]\end{aligned} \qquad (5)$$

$$\frac{s}{R} = 2\,\frac{T}{p_oR}\,\frac{1}{\sqrt{(1+k^2)}}\;F\left[\alpha, \frac{k}{\sqrt{(1+k^2)}}\right] \qquad (6)$$

where $F[\alpha,\rho]$ and $E[\alpha,\rho]$ are the elliptic integrals of the first and second kind respectively, and α is defined by

$$\alpha = \text{arc sin} \left[\frac{\sqrt{(1+k^2)}\ \sin(\theta/2)}{\sqrt{[1+k^2\sin^2(\theta/2)]}}\right] \tag{7}$$

Previous work did not consider the case for $w < 0$. It may be shown [6] that for $-1 < (4Tw/p_o^2) < 0$,

$$\frac{x}{R} = 2\frac{T}{p_oR}\left[\left(1-\frac{2}{k^2}\right) F\left[\frac{\theta}{2},k\right] + \frac{2}{k^2} E\left[\frac{\theta}{2},k\right]\right] \tag{8}$$

$$\frac{s}{R} = 2\frac{T}{p_oR}\ F\left[\frac{\theta}{2},k\right] \tag{9}$$

For $(4Tw/p_o^2) < -1$,

$$\frac{x}{R} = 2\frac{T}{p_oR}\ \frac{1}{k}\ \left[2E\left[\beta,\frac{1}{k}\right] - F\left[\beta,\frac{1}{k}\right]\right] \tag{10}$$

$$\frac{s}{R} = 2\frac{T}{p_oR}\ \frac{1}{k}\ F\left[\beta,\frac{1}{k}\right] \tag{11}$$

where

$$\beta = \text{arc sin}\ [k\ \sin(\theta/2)] \tag{12}$$

For both of these cases,

$$\frac{y}{R} = \frac{p_o}{Rw}\ \{\sqrt{[1-k^2\sin^2(\theta/2)]} - 1\} \tag{13}$$

Boundary Conditions

Demiray and Levinson considered the case of the bag resting on a flat surface. In this work, we consider the membrane acted upon by loads uniformly distributed on a line which is perpendicular to the (x,y) plane. In the first case, we consider a central line load as shown in Fig 3. The perimeter of the membrane has a total length $2\pi R$ and the membrane is filled with a buoyant fluid. The load intensity is Q per unit length. Taking Q/R^2w as the dimensionless load per unit axial length of bag and setting this equal to the buoyancy force per unit length we obtain

$$\frac{Q}{R^2w} = \int_A \frac{dA}{R^2} \tag{14}$$

The equilibrium equation at the point of application of the force i.e. at $s/R = \pi$ is

$$\theta = \text{arc sin}(Q/2T) = \text{arc sin}\left[\frac{(Q/R^2w)/2}{(T/p_oR)(p_o/Rw)}\right] \tag{15}$$

in terms of the above dimensionless groups.

Equations (6), (7) and (15) may be solved simultaneously on a digital computer by an iterative process.

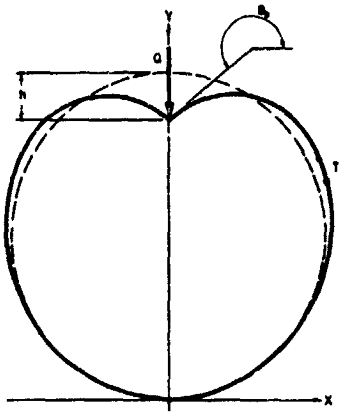

Fig 3 Central Suspension Case

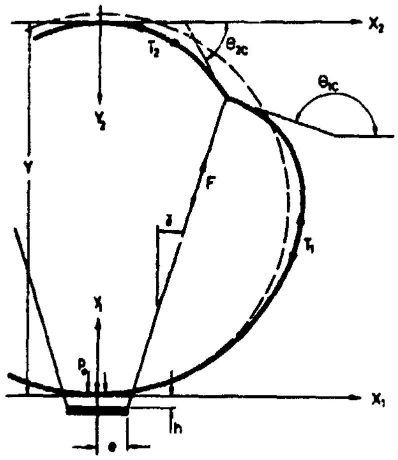

Fig 4 Twin Suspension Case

In the second case, we consider two equal line loads symmetrically disposed about the centreline as shown in Fig 4. The force intensity F arises physically from a set of inextensible cables which pass through sliding seals at the bottom of the membrane and anchor to a rigid frame of width 2e. For convenience it is assumed that if the membrane were circular, the lower anchor point would be in the plane $y = 0$; under equilibrium conditions with the membrane deformed, the anchor point has fallen a distance h. The angle of inclination of the suspension cables in the undeformed state is designated by γ. In Fig 4, variables with the subscript 1 refer to the lower portion of the membrane below the point of application of the load; variables with the subscript 2 refer to the upper section. For the upper section axis y_2 is directed downwards as shown and w is now negative. The differential pressure at $y_2 = 0$ is $p_o - wY$.

The boundary conditions arising from continuity of the membrane are

$$x_{1_C}/R = x_{2_C}/R\,, \quad y_{1_C}/R + y_{2_C}/R = Y/R$$
$$s_{1_C}/R + s_{2_C}/R = \pi \qquad (16)$$

The equilibrium equation at the point C yields the further condition

$$\frac{(x_{1_C}/R)-(e/R)}{(y_{1_C}/R)+(h/R)} = \frac{-(T_2/p_oR)\cos\theta_{1_C} - (T_1/p_oR)\cos\theta_{1_C}}{(T_2/p_oR)\sin\theta_{2_C} - (T_1/p_oR)\sin\theta_{1_C}} \qquad (17)$$

where e/R is the dimensionless cab frame width and h/R is the dimensionless suspension deflection.

NUMERICAL RESULTS

Central Support Case

Membrane shapes and stress resultants were obtained for typical conditions which might apply to a small non-rigid airship, i.e. diameter ≈ 40 ft, inflation pressure in the range 5 to 15 lbf/ft^2 and w = 0.0696 lbf/ft^3 which is the lift of pure helium at 0°C (this is a conservative value, 0.0625 lbf/ft^3 often being taken for airship calculations [5]). The results are presented with dimensions for convenience. Fig 5 shows the deflected shapes and Table I the stress resultants and deflections of the suspension point from the position for a circular membrane.

Table I

$p_o(lbf/ft^2)$	$T(lbf/ft^2)$	h(ft)	Q(lbf/ft)
5	112.3	5.233	85.6
10	212.9	2.727	86.6
15	313.3	1.831	87.0

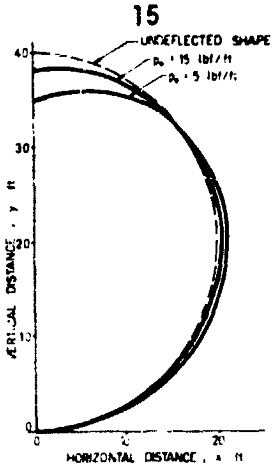

Fig 5 Cross-sectional Shapes for various inflation pressures for the central suspension case

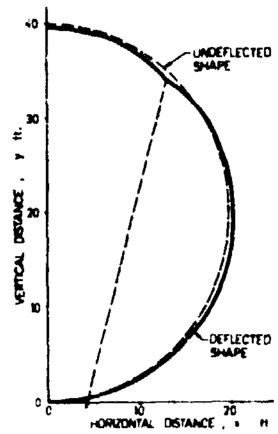

Fig 6 Cross-sectional Shape for an inflation pressure of 10 lbf/ft^2 for twin suspension

Two Support Case

This case, shown in Fig 4, is the more usual situation in airships and is considered further. Results were obtained for the particular geometry γ = 15° and e/R = 0.2. Fig 6 shows the deflected shape for a membrane of nominal diameter 40 ft and an inflation pressure p_o = 10 lbf/ft^2. Fig 7 shows the non-dimensional deflection h/R versus the non-dimensional pressure parameter p_o/Rw. Fig 8 shows the non-dimensional membrane stresses T/p_oR versus p_o/Rw.

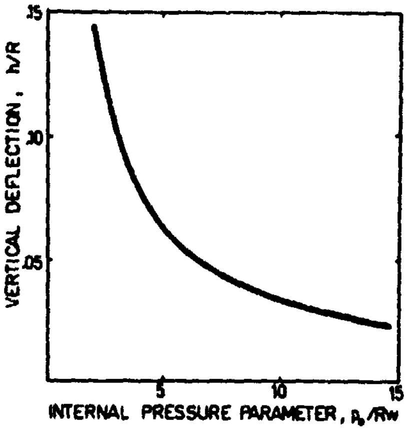

Fig 7 Suspension Deflection, h/R, versus p_o/Rw for the case shown in Fig 4; e/R=0.2, γ=15°

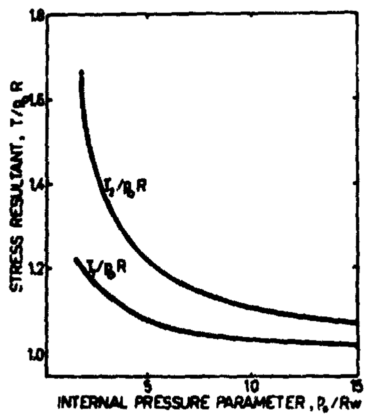

Fig 8 Stress Resulatants, T/p_oR versus p_o/Rw for the case shown in Fig 4; e/R=0.2, γ=15°

THE DYNAMIC CASE

We consider the case in which the bag and the air surrounding it are subject to a vertical acceleration ng upwards. If we assume that the differential pressure at y = 0 is p_o as before, it may easily be shown that for the dynamic case, the differential pressure at any other point is

$$p' = p_o + (n+1)wy \tag{18}$$

The buoyancy force per unit length will be

$$Q' = w(n+1) \int_A dA \tag{19}$$

We consider that the suspension is attached to a mass which under static conditions gives rise to a vertical force per unit length $Q=w\int_A dA$ (from equation 14). Under dynamic conditions the vertical component of the suspension force will now be Q(1+n) and clearly this is equal to Q'. Thus the dynamic case may be obtained from the preceeding results by replacing the relative specific weight w by w' where

$$w' = (1+n)w \tag{20}$$

DISCUSSION

The analysis provides membrane shapes and stress resultants for the two-dimensional problem of the fluid filled bag and these can be applied to both the static and dynamic cases.

In a non-rigid airship, the inflation pressure p_o must be sufficient to maintain the shape of the structure under both static and dynamic loads. As an example, we consider an airship designed for a 75 mph maximum speed. Allowing for a frontal gust of 15 mph, the maximum stagnation pressure at the nose would be 22 lbf/ft^2. Usually the nose contains a stiffening structure, which permits lower inflation pressures of, for example, 60% of the stagnation pressure or 13 lbf/ft^2 in this case. For a helium filled bag of 40 ft diameter, the non-dimensional pressure parameter would have the value of 9.3. From Figs 7 and 8, we obtain values for the vertical deflection of 8.4 inches and for the stress resultants of $T_1 = 270$ lbf/ft and $T_2 = 292$ lbf/ft. The stress resultant due to pressure only, i.e. p_oR is 260 lbf/ft so the effect of the suspension on the envelope stresses is quite small.

In the design of airships, it is customary to consider the effect of a transverse gust of about 30 ft/sec [7]. A vertical gust of this magnitude could give rise to accelerations in excess of 1g. In the example chosen, the parameter p_o/Rw would be halved for an upward acceleration of 1g and the deflection would be 16.6 inches i.e. approximately doubled.

There are important differences, however, between the problem formulated and the real case of an airship. These are:-

1. The analysis is for a two dimensional system. Airships will have a fineness ratio (overall length to maximum diameter) of between 3 and 5, thus curvature in the axial plane will significantly diminish the stress resultants and probably increase the overall stiffness.
2. It is not possible to arrange the suspension system in such a way that the suspension force F in Fig 4 exactly balances the buoyancy at that section. Consequently bending moments arise in the axial plane and the associated shear forces are transmitted through the membrane. These will give rise to deformations of the section which differ from those in the two-dimensional case.
3. It is customary to have a secondary suspension system in the form of a skirt or fairing between the cab and the envelope as shown in the schematic diagram in Fig 9. This will be considerably stiffer than the upper suspension system so that under dynamic loading the additional loads will be transferred to the envelope by

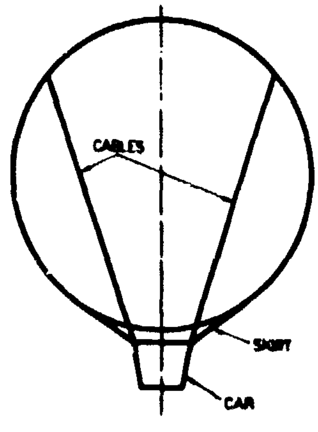

Fig 9 Schematic Illustration showing the skirt location

the skirt rather than by the cables.

4. Below the ceiling altitude of the airship, not all of the envelope is filled with helium. Up to 10% of the internal volume can be taken up by air in ballonets as shown in Fig 1. These serve to maintain the inflation pressure and allow for expansion of helium at higher altitudes i.e. on ascending air is bled off from the ballonets and this prevents loss of helium. These ballonets may have a significant effect on the deformed shape at a section.

5. The dynamic case assumes that the surrounding fluid has the same acceleration as the bag. This is not truly representative of the situation in a vertical gust where there will be an aerodynamic pressure distribution on the section due to the relative transverse velocity of the surrounding air. This problem, as well as the effect of the pressure distribution due to forward velocity are outside the scope of this work.

Other factors give rise to stress distributions and deformations in airship envelopes which have not been considered here. These include instantaneous and creep strains in the fabric, improper rigging and the effects of the empennage. It is considered, however, that the analysis and numerical results presented will assist the designer in the preliminary investigation of envelope and suspension performance in non-rigid airships.

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