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4.4 Asymptotic Analytical Methods in Fluid Mechanics Related to Drag Prediction

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Introduction

As a counterpart to the numerical prediction methods we've heard about, I thought it would be useful to describe some recent theoretical work of a purely analytical nature which promises to provide engineering predictions for the important drag-related phenomena of flow in the stall regime. This analytical work deals with rigorous asymptotic studies of the complete Navier-Stokes equations that govern the viscous flow around any aerodynamic body under conditions where boundary layer separation takes place from the body surface (see for example the summary given in a recent NATO-AGARD conference proceedings[1]).

Asymptotic Analysis Approach

In a nutshell, asymptotic analysis (sometimes also called the "method of matched asymptotic expansions" after Van Dyke[2] or the "multiple-deck" approach after Stewartson[3]) consists of a detailed local study of the Navier-Stokes ("N-S") equations behavior in the limit of large Reynolds number which delineates the basic layered substructure of the flow in the presence of adverse pressure gradients leading to separation, including viscous-inviscid interaction effects. As such, this approach is the logical next step in Prandtl's original notion of the boundary layer theory as an asymptotic approximation for large Reynolds numbers. The results establish how the flow leading to separation and beyond can be decomposed into a set of matched multiple decks, each of which are governed by equations appreciably simpler than the full N-S equations and hence far easier (and cheaper) to solve.

Although these mathematical studies are still rather esoteric compared to the needs of the practicing general aircraft aerodynamicist, they yield valuable insight to the "fine grain" structure of the flow and its basic scaling parameters (which is very useful in numerical work), and also show how to eliminate the notorious separation point singularity of classical boundary layer theory by inclusion of viscous-inviscid interaction.[4] Moreover, the results of asymptotic analysis can be used to devise simplified but accurate approximate engineering solutions which are applicable over a wide range of practical flow conditions. An example of this for

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laminar flow is given in Figure 1, where the approximate triple-deck viscous-inviscid flow model of separation proposed by Inger[5] is illustrated schematically. As outlined in Figures 2 – 4, the resulting analytical description of the flow is relatively simple and yields closed form results for the skin friction distribution along the surface including a very useful separation point – location criterion (Figure 4A). The turbulent flow counterpart of this layered approach can also be derived (assuming, of course, some appropriate turbulent eddy viscosity model), again resulting in a very simple engineering expression for the separation point (Figure 4B). It is gratifying indeed that these theoretical results have been found to be in a good agreement with experiment over a wide range of practical conditions.

The interaction effect on the pressure distribution due to the boundary layer displacement thickness growth $\delta^*(X)$ can also be accounted for in the context of this layered approach by means of a source-distribution simulation[5]; typical results showing the resultant smoothing out of the originally imposed non-interacting pressure are shown in Figure 5.

Concluding Remarks

With proper engineering adaptation, asymptotic analysis studies offer some valuable new methods for understanding and predicting the underlying fluid mechanics of drag (and lift) in the separation and stall regimes. Moreover, research is currently in progress on extending these methods to include suction or blowing through the surface (i.e., boundary layer control)[6] and the effects of Mach number (compressibility), including the presence of transonic shock – boundary layer interaction,[7] and finally to the case of three dimensional flows.

References

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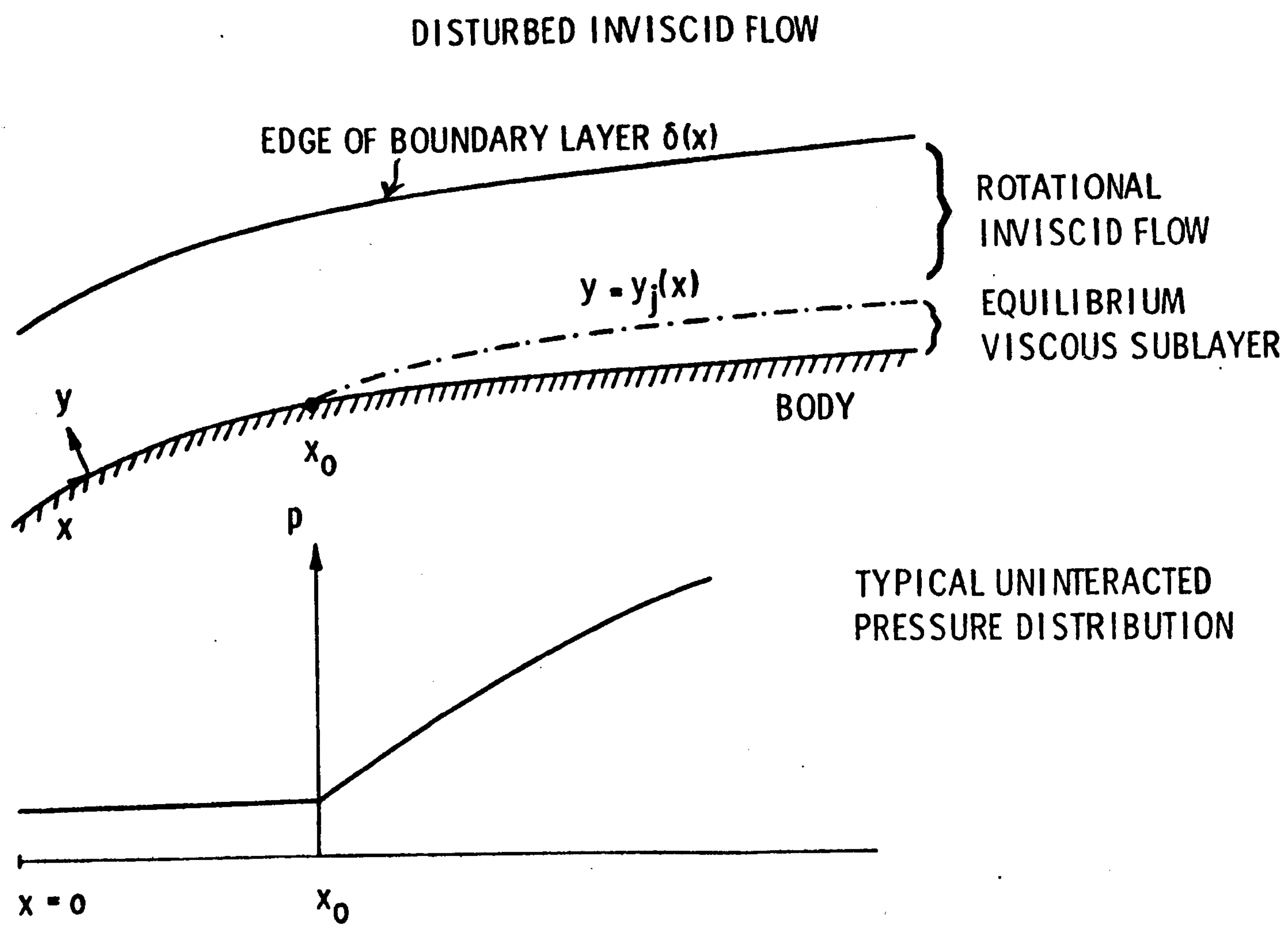

Figure 1. Schematic of Three Layered Flow Model

OUTER $(y_i \le y \le \delta)$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{1}{\rho}\frac{dp}{dx} \simeq \nu \left(\frac{\partial^2 u}{\partial y^2}\right)_{REF}$$

GIVING

$$u^2(x,\psi) \simeq \underbrace{U_{REF}^2(x,\psi)}_{\text{KNOWN APPROX. FOR TOTAL HEAD LOSS}} - \frac{p - p_0}{\rho}$$

INNER $(0 \le y \le y_i)$

$$\frac{dp}{dx} \simeq \frac{\partial \tau}{\partial y} = \mu \frac{\partial^2 u}{\partial y^2}$$

GIVING

$$\mu u \simeq \tau_w y + \frac{1}{2}\frac{dp}{dx} y^2 + a(x) y^N$$

WITH $a(x)$ RELATED TO y_i

N GIVEN $(\gtrsim 3)$

Figure 2. Analytical Relations

RESULTS OF MATCHING

ψ, u, $\partial u/\partial y$ & $\partial^2 u/\partial y^2$ MATCHING AT $y = y_j$ YIELDS

$$\left(X \frac{dC_P}{dX}\right)^2 C_P = K_1(N)(1-T)^3\left[1 + K_2(N) \cdot T\right]$$

$$y_j = \left(\frac{N-1}{N-2}\right)(1-T)\frac{C_{f_{REF}}}{dC_P/dX}$$

$$a(x) = -(dp/dX)/N(N-1)\,y_j^{N-2}$$

WHERE

$$T = \tau_w/\tau_{w,REF} = \text{NON-DIMENSIONAL SKIN FRICTION}$$

$$K_1(N) = \left[\left(X\; dC_P/dX\right)^2 C_P\right]_{SEP} = \frac{.0041(N-1)^3(N^2+3)}{N^2(N+1)(N-2)^2}$$

$$K_2(N) = 3(N^2-1)/(N^2+3)$$

Figure 3. Results of Matching

a) LAMINAR FLOW

$$C_p \left(X \frac{dC_p}{dX} \right)^2_{sep} \simeq .0104$$

b) TURBULENT $(u \sim y^{1/N})$

$$C_p^{\frac{N-2}{4}} \sqrt{X \frac{dC_p}{dX}}_{sep} \simeq 1.43 \left(\frac{N-2}{N+1} \right)^{\frac{N-2}{4}} \left(\frac{3}{N+1} \right)^{\frac{1}{4}} \frac{Re_{x,sep}^{1/10}}{\sqrt{(N+1)(N+2)}}$$

Figure 4. Separation Criteria

$$C_p \simeq C_{p_{BASIC}}(x) - \frac{2}{\pi}\sqrt{1 - C_{p_{BASIC}}} \int_0^\infty \frac{(d\delta^*/dx)\, d\xi}{x - \xi}$$

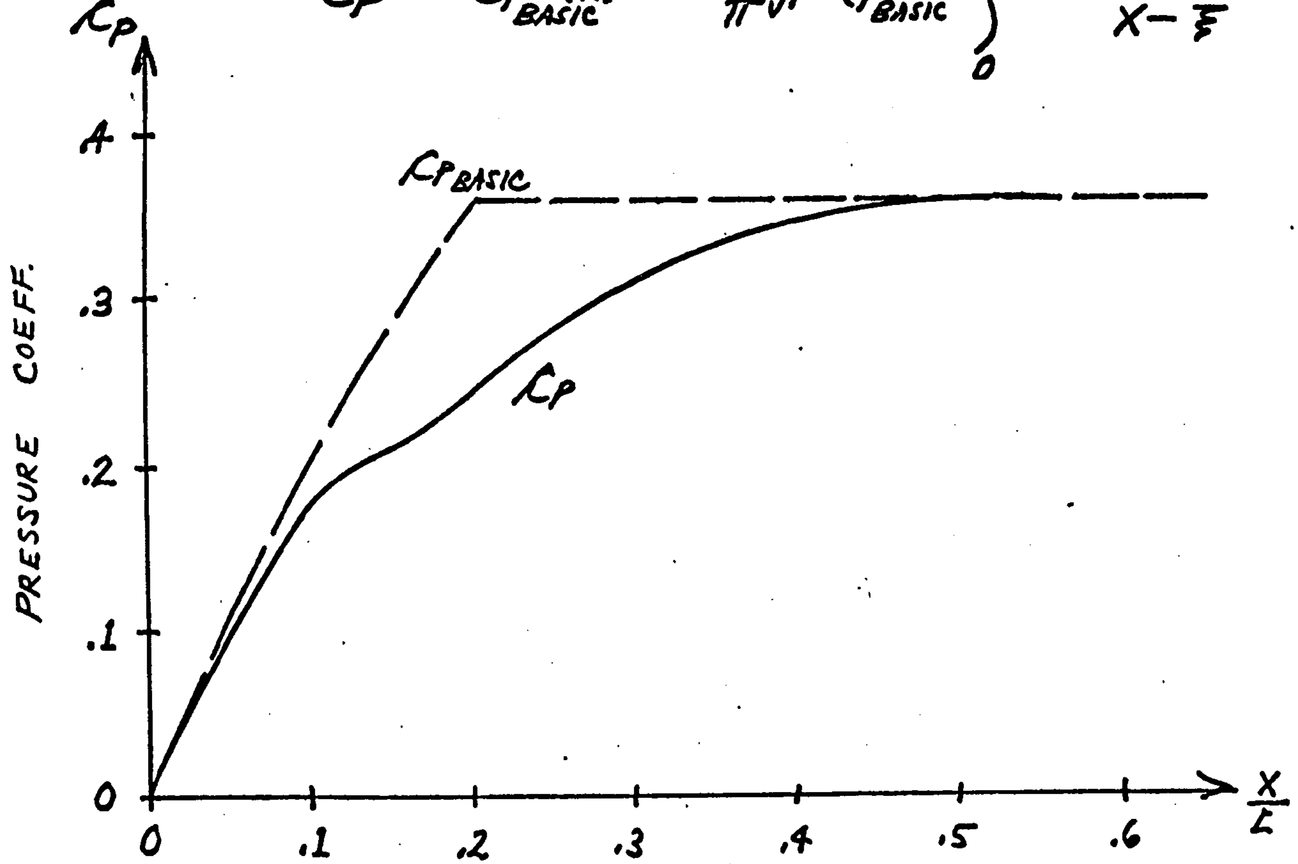

Figure 5. Typical Viscous-Inviscid Interaction