Theory of One-Element Pumps for Propulsion[1]

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The development of conventional pumps has been aimed mainly at increasing the pressure of the pumped fluid. For propulsion applications, however, the desired output is in the form of kinetic energy rather than pressure. Propulsion pumps may therefore assume configurations considerably different from those of conventional pumps. They may, for instance, consist of one element only—i.e., an impeller without a stationary casing (as in the ordinary ship's propeller). The discharge from this type of pump contains a whirl component that cannot be used for propulsion and must therefore be considered a loss. This loss is balanced by the elimination of other losses, such as those due to disk friction, diffusion, seals, and leakage.

This paper develops a basic theory for one-element propulsion pumps, and does so without reference to internal details of a particular design. It establishes a limit to the attainable efficiency and indicates relationships among pump-performance parameters that result in the greatest efficiency. It shows that the peripheral velocity of the pump is directly related to ship speed and develops some relationships between performance parameters and pump geometry. An example of this family of pumps is described.

Development efforts for normal, conventional pumps have been aimed primarily at increasing the pressure of the pumped fluid in order to raise the fluid to a higher elevation, to force it into a pressure vessel, or to make up for pipeline losses. This objective requires that the impeller be enclosed in a stationary pump casing, which becomes part of a closed conduit system. The pump casing also serves to convert kinetic energy into pressure by diffusion and (in the case of centrifugal or mixed-flow pumps) to collect the flow from the entire periphery of the impeller and pass it

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into the discharge pipe. The casing and contained water constitute most of the bulk and weight of conventional pumps.

In seagoing propulsion applications, however, the desired output is in the form of kinetic energy (and not pressure), because water is taken from the ocean essentially at atmospheric pressure and is returned at the same pressure, but with a higher velocity. In addition, the propelling jet is discharged freely into the atmosphere, and it is therefore unnecessary to collect the outflow from the impeller. Peripheral discharge is equally acceptable.

Propulsion pumps may thus assume configurations that are very different from those of conventional pumps. They may, in fact, consist of one element only—i.e., an impeller without a stationary casing. The ordinary ship's propeller can be viewed as such a pump. If the flow enters a pump of this type, without prerotation, the discharge contains a whirl velocity component that is not useful for propulsion. The kinetic energy associated with that velocity must therefore be viewed as a loss chargeable to the pump. This loss, which does not occur in normal pumps, is counterbalanced by other factors that favor the one-element pump. These are

(1) Elimination of all the losses that occur in the stationary casing of normal pumps (e.g., disk friction, diffusion, leakage)

(2) Absence of the stationary casing, making this type simpler and more compact than normal pumps

(3) Operation at higher rotative speeds than conventional pumps, as a rule, thus further reducing the size required for a given task.

The losses in a pump of this type may be divided into (1) those due to viscosity (real-fluid effects), which may include boundary-layer friction, losses due to separation of the flow from a solid boundary, and energy-conversion losses (e.g., diffusion losses), and (2) the energy represented by the whirl component.

It is obviously desirable, for the sake of efficiency, to reduce losses of both types to the greatest degree possible. They cannot be eliminated altogether, however, because all hydraulic processes involve losses due to real-fluid effects and because a one-element pump accomplishes its function only by introducing whirl.

It thus becomes necessary, if one-element pumps are to have acceptable efficiency and be compact, to seek out all processes susceptible to optimization and to determine the conditions required for optimum performance. The designing of pumps generally is a complex process that leans heavily on empirical data, but such data are largely lacking for designs that differ substantially from conventional patterns. It therefore becomes necessary to inquire whether there are any simple, easily derived relationships that govern the performance of such pumps and are useful as guidelines for their design. Specifically, the following questions suggest themselves:

(1) Are there identifiable relationships between the relative magnitude of the whirl component and the efficiency of the pump?
(2) Is there an optimum relationship between the whirl-component loss and the friction loss?
(3) Is the ratio of peripheral speed of impeller to ship speed fixed, or is it open to choice?
(4) Are simple relationships available for use in calculating the blade angles of the impeller?

This paper covers an analytical investigation of the performance of one-element pumps for propulsion, insofar as this can be predicted from theory, and represents one aspect of a more general study reported in reference 1. It deals only with pumps that conform strictly to the limitation to one element; i.e., those that do not have guide vanes ahead of the impeller for the introduction of prerotation and do not have stationary casings surrounding or following the impeller. Propellers are not dealt with explicitly; however, as stated above, they may be considered as members of this family of pumps.

The overall performance of a one-element pump may be effectively investigated without reference to the internal details of the particular design—i.e., the impeller may be viewed as a black box. Impeller action on the water is then represented by velocity diagrams at the inlet and outlet, while internal losses are represented by a single loss coefficient. Existing pump theory is next applied to predict the overall performance and to answer some of the questions listed above. The results thus obtained are then used to establish guidelines for impeller design.

The efficiency of paramount importance in propulsion applications is the overall propulsive efficiency, η. It is defined as the ratio of thrust horsepower (THP) to shaft horsepower (SHP), where THP is the power required to move the ship through the water at a given speed (equivalent to towline power, if the ship were towed), while SHP is the power provided to the propulsion system by the prime mover. This efficiency is a function of several factors including pump efficiency. As shown in the next section, however, overall propulsive efficiency is directly proportional to pump efficiency. For the present study it is therefore sufficient to consider pump efficiency alone. The problem of matching the pump to the ship's requirements is discussed only with regard to its effect on the rotational speed of the pump.

RELATIONSHIP BETWEEN PUMP EFFICIENCY AND PROPULSIVE EFFICIENCY

The following expression, a modified version of equation (7) of reference 2 and derivable by combining equations (7) and (18*b*) of that reference, relates the pump efficiency and the overall propulsive efficiency of a waterjet system.

$$\frac{\eta}{\eta_\pi}=\frac{2k}{(1+k)^2(1+K_D)-1} \tag{1}$$

where

η overall propulsive efficiency (THP/SHP)
η_π pump efficiency
K_D duct-loss coefficient, including all internal losses, except those occurring in the pump
k jet-velocity ratio, $(V_j-V)/V=\Delta V/V$
V speed of advance of ship
V_j velocity of propelling jet relative to ship

The subscript π on η is used to distinguish the particular kind of pump efficiency considered here—i.e., one in which the energy of the whirl component is taken as a loss chargeable to the pump. The term K_D is defined by

$$H_D=K_D\frac{V_j^2}{2g}=K_D\frac{V^2}{2g}(1+k)^2 \tag{2}$$

where H_D is the head loss in ducting, elbows, and nozzles, and g is the acceleration of gravity.

Equation (1) shows that the propulsive efficiency is strongly affected by the choice of the jet-velocity ratio (k) and the value of the duct-loss coefficient (K_D). Given a pair of values for k and K_D, however, the overall propulsive efficiency is directly proportional to the pump efficiency.

ATTAINABLE EFFICIENCY

Basic Expression

The analysis that follows applies to all types of pumps (from axial to radial) in which the whirl component is not recovered, as well as to the ordinary ship's propeller, which may be viewed as a pump.

Because the type of pump under consideration does not require a stationary casing, the shaft torque (neglecting seal and bearing friction) is utilized fully to change the moment of momentum of the fluid passing through the impeller. The action of the impeller on the fluid is represented by velocity diagrams at the inlet and outlet, as shown in figure 1, for either an axial- or a radial-flow impeller. Equating the shaft torque (T_s) to the time rate of change of moment of momentum,

$$T_s = \rho Q (V_{w2} r_2 - V_{w1} r_1) \tag{3}$$

where

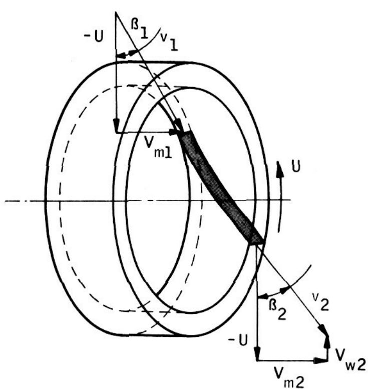
Q volume flow rate
V_w tangential velocity
r radial position
ρ fluid density

If the flow enters the impeller without prerotation,

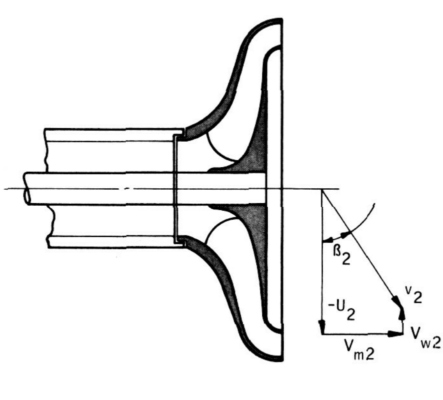
$$V_{w1} = 0$$

and

$$T_s = \rho Q V_{w2} r_2$$



a.—*Axial-flow impeller.*



b.—*Radial-flow impeller. (Velocity design corresponds to impeller discharge radius.)*

FIGURE 1.—*Velocity diagrams.*

The power input (P_i) is

$$P_i = T_s\omega = \rho Q V_{w2}\omega r_2$$

where ω is the rotational speed of the impeller.
Because

$$\omega r_2 = U_2$$

it is possible to write

$$P_i = \rho Q V_{w2} U_2 \tag{4}$$

The input head, H_i (also called ideal head), or energy, may be written as

$$H_i = \frac{P_i}{\rho g Q} = \frac{V_{w2} U_2}{g} \tag{5}$$

We may define an input-head coefficient (ψ_i) based on peripheral velocity, by writing

$$H_i = \psi_i \frac{U_2^2}{2g} \tag{6}$$

Comparison of equations (5) and (6) shows that

$$\frac{V_{w2}}{U_2} = \frac{1}{2}\psi_i \tag{7}$$

The useful output head (H_π) is equal to the input head minus two losses: (1) that due to impeller friction (H_f) and (2) that due to non-recovery of the whirl component (H_w). These may be expressed, respectively, as

$$H_f = K_f \frac{v_2^2}{2g} \tag{8}$$

and

$$H_w = \frac{V_{w2}^2}{2g} \tag{9}$$

where K_f is a friction-loss coefficient based on v_2, the relative velocity. Thus, the output head is

$$H_\pi = \frac{2V_{w2}U_2 - K_f v_2^2 - V_{w2}^2}{2g} \tag{10}$$

The pump efficiency is the ratio of the output head to the input head:

$$\eta_\pi = \frac{2V_{w2}U_2 - K_f v_2^2 - V_{w2}^2}{2V_{w2}U_2} \tag{11}$$

Ideal-Fluid Efficiency

For an ideal, frictionless fluid ($K_f = 0$), equation (11) reduces to

$$(\eta_\pi)_i = 1 - \frac{1}{2}\frac{V_{w2}}{U_2} \tag{12}$$

and, using equation (7), this may be written

$$(\eta_\pi)_i = 1 - \tfrac{1}{4}\psi_i \tag{13}$$

It is thus evident that the efficiency in the frictionless case is a simple function of the input-head coefficient. When pumping a real fluid, the maximum attainable efficiency will be less than $(\eta_\pi)_i$. Therefore, one-element pumps must be designed to operate with low values of input-head coefficient if they are to have acceptable efficiency; in other words, as indicated by equation (7), they should operate at relatively high peripheral velocities.

Efficiency With a Real Fluid

The real-fluid efficiency is given by equation (11), which involves three velocity components. Before attempting optimization, a functional relationship must be established between the relative velocity (v_2) and the whirl component (V_{w2}). It is convenient also to reduce all velocity components to nondimensional ratios by dividing each by the peripheral velocity (U_2). The velocity diagram on the discharge side of the impeller (fig. 1) may now be labeled as shown in figure 2, recalling that $V_{w2}/U_2 = \frac{1}{2}\psi_i$. The functional relationships are

$$\tfrac{1}{2}\psi_i = 1 - k_v \cos\beta \tag{14}$$

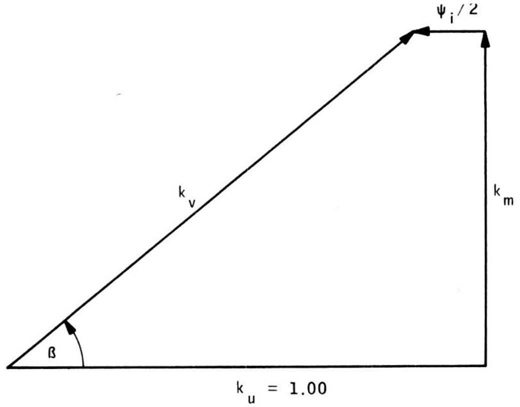


FIGURE 2.—*Dimensionless velocity diagram.*

and

$$k_m = k_v \sin \beta \tag{15}$$

where β is the relative flow angle measured from the tangential direction.

Equation (11) may now be written in terms of the velocity ratios as

$$\eta_\pi = \frac{\psi_i - K_f k_v^2 - \frac{1}{4}\psi_i^2}{\psi_i} \tag{16}$$

Each term, when multiplied by $U_2^2/2g$, represents a head (ideal input, friction loss, and whirl loss, respectively). Using equation (14) to substitute for ψ_i in equation (16), we obtain

$$\eta_\pi = \frac{2(1-k_v \cos \beta) - K_f k_v^2 - (1-k_v \cos \beta)^2}{2(1-k_v \cos \beta)} \tag{17}$$

which may be reduced to

$$\eta_\pi = \frac{1}{2}\left[\frac{1-k_v^2(K_f + \cos^2 \beta)}{1-k_v \cos \beta}\right] \tag{18}$$

Optimization with respect to β yields

$$K_f k_v^2 = \tfrac{1}{4}\psi_i^2 \tag{19}$$

which shows that the efficiency would be a maximum when the friction loss and the whirl-component loss have equal magnitudes.

Optimization with respect to k_v results in

$$K_f k_v^2 = \frac{1-\psi_i}{1+\psi_i}\,(\tfrac{1}{4}\psi_i^2) \tag{20}$$

which shows that the efficiency would be a maximum when the friction loss is somewhat less than the whirl-component loss.

The preceding two equations are satisfied simultaneously only when $\psi_i = 0$ (i.e., when the pump is doing no work and when the friction loss is zero), in which case the efficiency would be unity. Because useful pumps operate with positive values of ψ_i, the efficiency of a one-element pump has only one theoretical limit, which is the one derived for the ideal-fluid case as expressed in equations (12) and (13). The real-fluid efficiency will be less than that, and its magnitude will depend on the minimum friction loss that can be achieved with a given pump configuration.

It is interesting to examine the maximum efficiency that results when either of the two foregoing equations is used separately. Equation (14) may be used in combination with either one to make substitutions in equation (17) so as to express the efficiency in terms of ψ_i only. When equation (19) is used, the result is

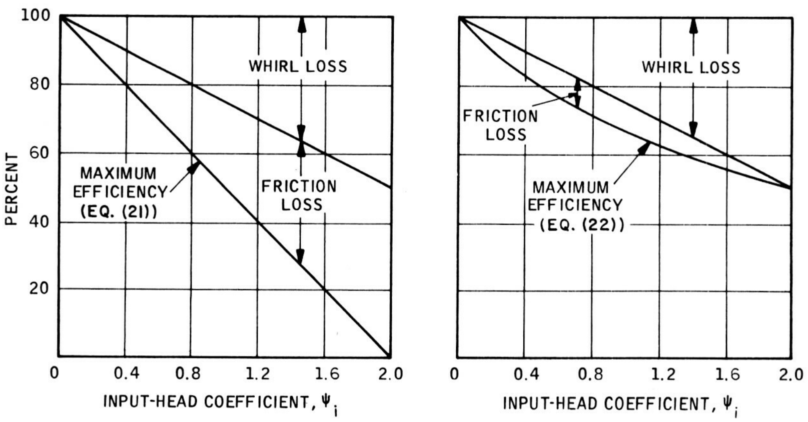
$$(\eta_\pi)_{\max,1} = 1 - \tfrac{1}{2}\psi_i \tag{21}$$

while equation (20) yields

$$(\eta_\pi)_{\max,2} = \frac{1}{1+\frac{1}{2}\psi_i} \tag{22}$$

When ψ_i is small in comparison with unity, these equations give substantially the same result. However, when ψ_i is not very small, the two efficiencies are quite different. The maximum efficiencies, and the corresponding loss distributions, of equations (19) and (21) are plotted in figure 3*a*, while those of equations (20) and (22) appear in figure 3*b*. The efficiency is greater in plot *b*, but its attainment requires that the friction loss be smaller than shown in plot *a*. That smaller friction may be difficult to achieve in practice, especially when ψ_i approaches either 0 or 2. On the other hand, when ψ_i is not small, it should be possible to achieve friction losses below those shown in figure 3*a*.

It may therefore be said that practical designs should be aimed for efficiencies that fall between the values shown in plots *a* and *b*. These two lines of (so-called) maximum efficiency are replotted in figure 4. For propulsion we require pumps of high efficiency (e.g., at least 80 percent). The region of practical interest is thus the shaded portion of this figure, preferably in the upper end of that portion.



a.—*Equations (19) and (21).*

b.—*Equations (20) and (22).*

FIGURE 3.—*Maximum efficiencies and loss distributions.*

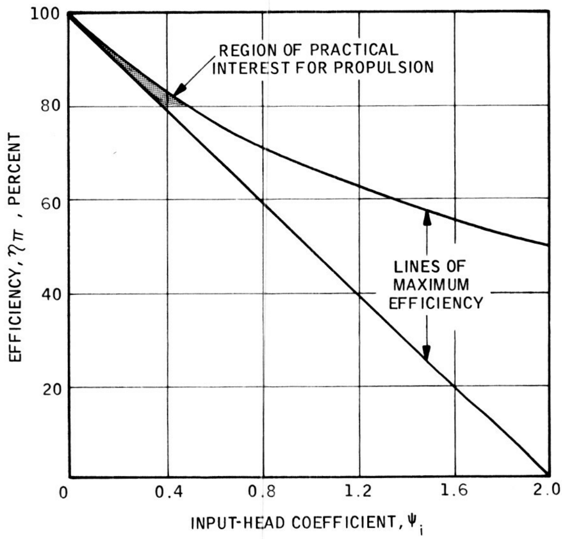


FIGURE 4.—*Efficiency range for practical design.*

RELATIONSHIPS BETWEEN EFFICIENCY AND IMPELLER GEOMETRY

Certain guidelines for the design of impellers for one-element pumps may be derived from the foregoing expressions for maximum efficiency and from the geometrical relationships represented by velocity diagrams (fig. 1). The results that follow pertain only to the terminal stations. Subscript 1 refers to the inlet to the impeller vanes, while subscript 2 refers to the station where the propelling jet is discharged freely from the impeller. The pump designer must provide transitions between these stations. The efficiency to be used hereafter is the lower value of maximum efficiency, that of equation (21).

The conditions prescribed here for the impeller outlet apply to the station at which the flow is discharged into the atmosphere in the form of free, thrust-producing jets. The angle β_2 gives the relative-flow direction, which differs somewhat from the geometrical blade angle in some cases. In some arrangements, especially with centrifugal or mixed-flow impellers, the radial position of the jet discharge may be different from that of the outlet end of the working blades (as in the Hydrocket® propulsion unit described in a succeeding section as an example of a one-element pump). In these cases, the product UV_w remains constant in the flow between the blades and the nozzles that form the jets, although all the velocity components may change magnitude.

Centrifugal and Mixed-Flow Impellers

Combining equations (21) and (7),

$$(\eta_\pi)_{\max}=1-\frac{V_{w2}}{U_2} \tag{23}$$

Substituting

$$V_{w2}=U_2-v_2\cos\beta_2$$

and

$$U_2=U_1\frac{r_2}{r_1}=v_1\cos\beta_1\frac{r_2}{r_1}$$

in equation (23) yields

$$(\eta_\pi)_{\max}=\frac{r_1v_2\cos\beta_2}{r_2v_1\cos\beta_1} \tag{24}$$

The relative-velocity ratio (v_2/v_1) in this equation may be replaced with the flow-area ratio (r_1b_1/r_2b_2) by making the following substitutions:[2]

$$v_1\cos\beta_1=\frac{V_{m1}}{\tan\beta_1}$$

and

$$v_2\cos\beta_2=\frac{V_{m2}}{\tan\beta_2}$$

From continuity,

$$V_{m1}=\frac{Q}{2\pi r_1b_1}$$

and

$$V_{m2}=\frac{Q}{2\pi r_2b_2}$$

where Q is the volume flow rate and b is the width of the flow passage at each station. Equation (24) may now be rewritten as

$$(\eta_\pi)_{\max}=\frac{r_1^2b_1\tan\beta_1}{r_2^2b_2\tan\beta_2} \tag{25}$$

Axial-Flow Impellers

For the usual type of axial-flow impeller (i.e., cylindrical hub and casing, noncavitating), $r_2=r_1$, $b_2=b_1$, and $V_{m2}=V_{m1}$. Equation (25) may

[2] Editor's Remarks: The author does not distinguish between the mass averaged radii used hitherto and the arithmetic averaged radii used in the next four equations. The error involved in equating these two averages is of the order of 5 percent for a hub/tip radius ratio of 0.5. In view of the death of the author, the editors felt impelled not to modify the analysis.

therefore be simplified to read

$$(\eta_\pi)_{\max}=\frac{\tan \beta_1}{\tan \beta_2} \tag{26}$$

or, from Equation (24),

$$(\eta_\pi)_{\max}=\frac{v_2 \cos \beta_2}{v_1 \cos \beta_1} \tag{27}$$

In some special types of axial pumps (e.g., supercavitating or superventilated), the meridional velocities (V_{m1} and V_{m2}) are not equal. However, the relative velocities (v_1 and v_2) are very nearly equal, and we may write

$$v_2=k_b v_1$$

where k_b is nearly unity. Equation (27) then becomes

$$(\eta_\pi)_{\max}=\frac{k_b \cos \beta_2}{\cos \beta_1} \tag{28}$$

RELATIONSHIP BETWEEN PERIPHERAL SPEED AND SHIP SPEED

It can be shown that the peripheral speed of a one-element pump has a definite relationship to the design speed of a ship that is to be propelled by a pump of this type. This is done by equating the useful head developed by the pump to the head required for propelling the ship.

The head developed by any pump may be expressed in terms of the peripheral velocity (U_2), the input-head coefficient (ψ_i), and the pump efficiency (η_π), as follows:

$$H_\pi=\frac{U_2^2}{2g}\psi_i\eta_\pi$$

It may be assumed that, at the design ship speed, the pump will be operating at its best efficiency, and that (for one-element pumps) the best efficiency and the input-head coefficient are related. We may therefore use equation (21) to replace ψ_i by its equivalent, $2(1-\eta_\pi)$. The pump-output head then becomes

$$H_\pi=\frac{U_2^2}{g}(\eta_\pi-\eta_\pi^2)$$

The pumping head required to propel a ship at a speed V, with a jet-velocity ratio of $k=\Delta V/V$, and having a duct-loss coefficient K_D, can be shown (see, for instance, eq. (12) of ref. 2) to be

$$H_\pi = \frac{V^2}{2g}\left[(1+K_D)(1+k)^2 - 1\right]$$

Equating the two expressions for H_π and rearranging,

$$\frac{U_2}{V} = \left[\frac{(1+K_D)(1+k)^2-1}{2(\eta_\pi - \eta_\pi{}^2)}\right]^{1/2}$$

This expression is plotted in figure 5 for several values of pump efficiency and duct-loss coefficient.

DESCRIPTION OF A ONE-ELEMENT PUMP

One arrangement of a one-element pump with a centrifugal impeller is shown in figure 6. It was conceived by Mr. C. A. Gongwer (formerly of

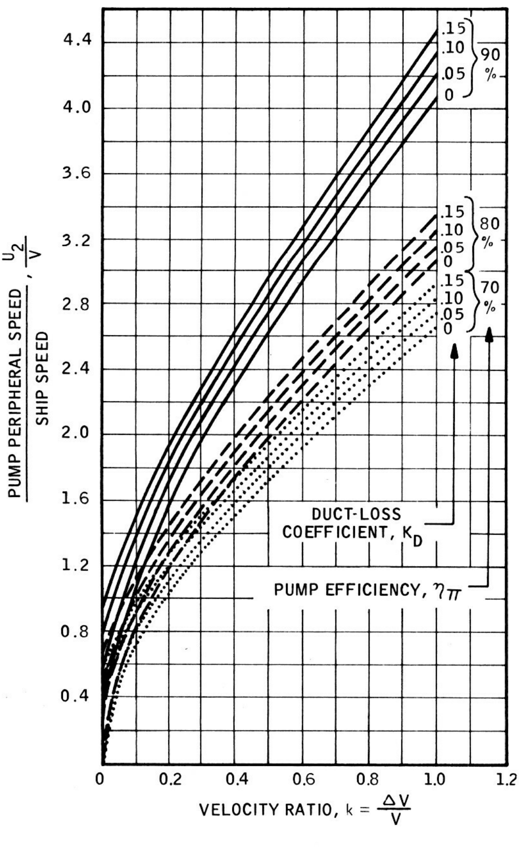


FIGURE 5.—*Peripheral-speed ratio versus jet-velocity ratio.*

Aerojet-General Corporation) for the propulsion of pleasure craft—as well as large ships—and was named the Hydrocket® propulsion unit.

The Hydrocket unit consists of an impeller (either radial-flow or mixed-flow), similar to a conventional pump impeller except for modifications at the periphery. The conventional impeller is open at the periphery, and its flow discharges radially into the pump casing. The Hydrocket impeller terminates in a gallery that is closed, except for a ring of nozzles, and turns the flow from the radial to the axial direction before discharging it through the nozzles. The gallery and nozzles are integral parts of the impeller and rotate with it. The nozzles are canted in a direction opposite to the peripheral velocity, making the angle β_2 with the impeller face, so that the absolute velocity of the jet is nearly in the meridional, or thrust, direction but does have a small whirl-velocity component (V_{w2}).

Several boats, including an 18-ft hydrofoil, were equipped with Hydrocket propulsion systems and were operated successfully for several years.

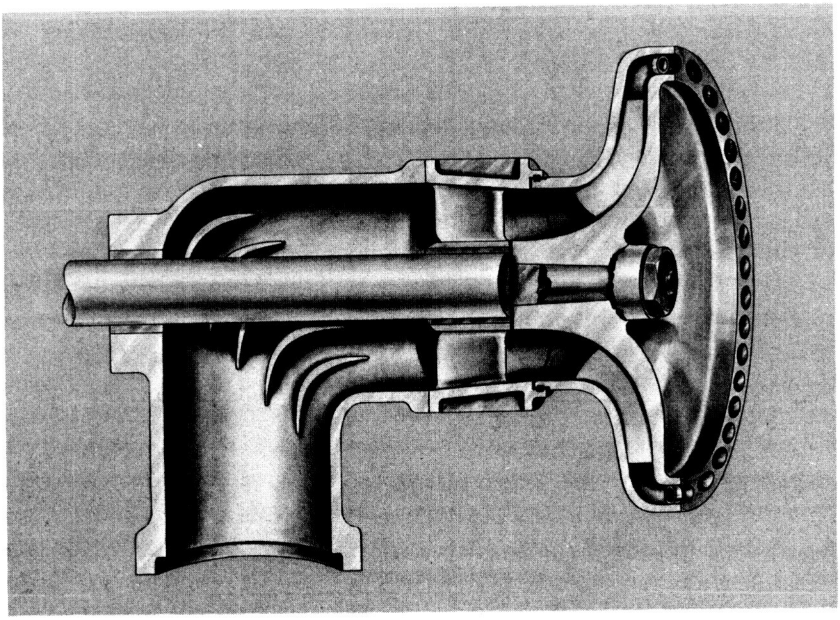


FIGURE 6.—*Hydrocket® propulsion unit (artist's drawing).*

CONCLUDING REMARKS

The relationships developed in this paper provide preliminary design guidelines for a wide range of one-element pumps for propulsion, although they prescribe only the conditions at the pump inlet and outlet. Blade shapes and shrouds may be sketched in, to provide smooth transitions between the prescribed terminal conditions, and internal-loss coefficients may then be estimated. Pumps of this general type can have acceptable efficiency and are more compact and simpler than conventional pumps.

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2. LEVY, J., The Design of Water-Jet Propulsion Systems for Hydrofoil Craft. *Marine Technology,* Vol. 2, No. 1, January 1965.

DISCUSSION

G. F. WISLICENUS (Tucson, Arizona): The paper discloses the competence in the hydrodynamic propulsion field for which its author is well known. Therefore, the writer of this discussion will limit his remarks to a few questions and only one general comment.

It is not clear to this discussor why a one-element propulsion pump can run under the same operating conditions at a higher speed of rotation than a conventional pump. This statement is made in the third paragraph of the introduction, item (3). It would seem that generally a single-element pump would be limited to the same suction specific speed as a conventional pump. Is it that the author feels single-element pumps can be operated with fully developed cavitation or ventilation, and the conventional pump cannot? This contention would have to be justified.

Equations (20) and (22) and figure 3*b* are not clear to this discussor as to their *physical* origin and meaning. Some explanatory comments by the author as to *why* this must be so would be helpful.

This discussor has never understood why the inventor of the Hydrocket (Calvin Gongwer) separated the discharge nozzle ring of his ingenious device from its pump vane system. Either a reason for this arrangement, or a statement that this separation is not necessary, perhaps not even desirable, would be very instructive.

This brings this discussor to his general comment: The Hydrocket (or any one-element pump) can be reasonably efficient only with low ψ_i values; i.e., low ratios of the circumferential discharge velocity component to the peripheral velocity of the runner. Standard centrifugal pumps are reasonably efficient only if ψ_i is in the vicinity of 1 or this velocity ratio in the vicinity of $\frac{1}{2}$. For a one-element propulsion pump such values would lead, according to equations (12) and (13) of the paper to *ideal* pump efficiencies in the neighborhood of 0.75, which is unacceptable. The reason why one-element pumps of the type of the Hydrocket can be operated with much lower ψ_i values than conventional pumps (and therefore with acceptable efficiencies) is the fact that the exterior of the Hydrocket rotor is exposed to air, not to water. This reduces the so-called disc-friction losses radically, thus removing the reason why ψ_i values cannot be much lower than unity. It seems that the recognition of this difference between the Hydrocket and a conventional centrifugal pump was essential for making the Hydrocket a practical possibility.

This discussor believes that further consideration of fluid friction losses in rotating vane systems will be necessary for arriving at optimum designs for the Hydrocket.

F. GILMAN (Worthington): As I looked at this paper, I was struck with the idea of a single element and the need for a jet, and to me a single element and a jet is a Pelton wheel. Now if one could collect the fluid from the sea with relatively low velocity and direct it into a nicely formed rearward jet by some means, using a very low-specific-speed machine such as a Pelton wheel, I think that we would have very low friction within the unit, and we would have no loss due to a whirl component. I've been quite interested in what a nice jet can be formed, for instance, with a spoon under the kitchen tap, and I think that something the reverse of the Pelton wheel might be constructive.

LEVY (author): The author wishes to thank Dr. Wislicenus for having reviewed the paper and for raising some penetrating questions. There are five separate questions (although some are interrelated) as follows:

(1) Why can a one-element pump run at a higher speed of rotation than a conventional pump operating under the same conditions? Perhaps the term speed of rotation was used too loosely in the paper. Most one-element pumps must operate at higher peripheral speeds than their conventional counterparts, as shown by equations (5) and (23). This implies a higher speed of rotation in those cases where impeller diameters are approximately equal. The problem of suction specific speed does apply to one-element pumps, although it was not treated in the paper.

(2) Does the author feel that one-element pumps can operate with fully developed cavitation or ventilation, whereas conventional pumps cannot? Both types of pumps may operate in this manner, and, in this respect, the Hydrocket is subject to the same considerations as normal pumps. However, other types of one-element pumps may discharge into the atmosphere without throttling the flow. Therefore, cavitation or ventilation voids do not have to close or collapse within the impeller. This eliminates the problems of cavitation erosion and of reentrant jets and the associated turbulence that results from the collapse of voids in liquid flows.

(3) What is the *physical* origin and meaning of equations (20) and (22) and of figure 3*b*? It should be noted that the same question could be asked with regard to equations (19) and (21) and figure 3*a*. The results obtained (in both cases) are due to the restrictions imposed by the mathematical process of partial differentiation, in which it is assumed that we can change one independent variable without affecting any of the others. This may not be achievable in the physical world. Equations

(19) and (21) (and fig. 3*a*) may appear to be more credible (as pointed out by Dr. Wislicenus in the oral discussion following presentation of the paper) because the result is similar to that obtained in other cases, wherein efficiency is a maximum when two types of losses (in this case whirl component and friction) are of equal magnitudes. Nevertheless, this result should also be questioned, as may be seen from the following example. Let us assume that ψ_i has the value 1.00; the whirl component loss ($\frac{1}{4}\psi_i^2$) would then have the value 0.25. In the absence of friction (i.e., $K_f=0$), the pump efficiency would be 0.75, as given by equation (13). Equation (16) states that, in the presence of friction, the efficiency would be lower than 0.75 by the amount $K_f k_v^2$. Thus, for instance, the efficiency would be 0.65, if $K_f k_v^2$ had the value 0.10. However, equations (19) and (21) and figure 3*a* state that, for this example, the maximum attainable efficiency is 0.50 and that this occurs when $K_f k_v^2=0.25$. Now, there is no physical reason for accepting this limitation. It may be said, in conclusion, that the limitation expressed by equation (13) is a hard ceiling; that is, the real efficiency will always be lower than that. How much lower will depend on the magnitude of the smallest friction loss that can be achieved in the real case.

(4) Why is the Hydrocket (or any one-element pump) efficient only if ψ_i has low values, while a normal centrifugal pump is efficient when ψ_i is in the vicinity of unity? It should be recalled that the theory presented in the paper is based on analysis of the velocity diagram at final discharge into the atmosphere. Velocity diagrams inside the impeller may be quite different. Let us examine the case of a normal centrifugal impeller that is converted into a one-element pump, as was done for the Hydrocket shown in figure 6. In a normal pump, the impeller shrouds would end where the vanes end in this figure; the discharge from the impeller would be radial, and it would flow against the back pressure imposed by the pump casing and the discharge pipe; the relative velocity (v) and the meridional velocity (V_m) would be small by comparison with the peripheral velocity (U); the energy added by the impeller would be partly pressure and partly kinetic energy. Conversion to a one-element pump is achieved (in this case) by extending the shrouds, first turning the flow toward the axial direction and then discharging it through nozzles that make some angle (β) with the face of the impeller. The nozzles perform a number of different functions. Some of these could, perhaps, be eliminated or achieved by other means. These functions are

(*a*) To throttle the flow so that velocities inside the impeller are kept within reason, thereby reducing internal losses.

(*b*) To convert pressure energy into kinetic energy.

(*c*) To guide the flow into the desired direction.

(*d*) To return energy to the impeller.

The conversion of energy and the guidance result in substantial increases in the relative velocity (v) and the meridional velocity (V_m) and in a reduction in the magnitude of the whirl component (V_w). It should be recalled that, at final discharge, V_m must be greater than ship speed (V) in order to achieve a positive thrust; also that V_w must be small in order to achieve good pump efficiency.

The nozzle ring, in the arrangement shown in figure 6, acts as a turbine; i.e., some of the energy added by the impeller vanes helps to drive the impeller. The energy (per unit time or per pound of water) input by the shaft is less than the energy added by the vanes. This input is equal to the sum of the kinetic energies of the two components of the absolute velocity at final discharge, and result in acceptably small internal loss. Many arrangements are possible; however, the author was not at liberty to describe them in this open publication.

In reply to Mr. Gilman's question, if we stipulate that there be no loss due to a whirl component, we imply that this component should be oriented in the direction of the thrust vector. This is precisely the mode of action of the paddle wheel. If such a wheel could be designed to produce "a nicely formed rearward jet," it would indeed be an efficient propulsor, although it might be very large and complex. Since (in this case) the whirl component is not lost, such devices are not in the class of turbomachines discussed in the present paper.