

THE EFFECT OF CYCLIC FEATHERING MOTIONS  
ON  
DYNAMIC ROTOR LOADS

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Abstract

The dynamic loads of a helicopter rotor in forward flight are influenced significantly by the geometric pitch angles between the structural axes of the hub and blade sections and the plane of rotation.

The analytical study presented includes elastic coupling between inplane and out-of-plane deflections as a function of geometric pitch between the plane of rotation and the principal axes of inertia of each blade. In addition to a mean collective-pitch angle, the pitch of each blade is increased and decreased at a one-per-rev frequency to evaluate the dynamic coupling effects of cyclic feathering motions. The difference in pitch between opposed blades gives periodical coupling terms that vary at frequencies of one- and two-per-rev. Thus, an external aerodynamic force at  $n$ -per-rev gives forced responses at  $n$ ,  $n\pm 1$ , and  $n\pm 2$  per rev.

The numerical evaluation is based on a transient analysis using lumped masses and elastic substructure techniques. A comparison of cases with and without cyclic feathering motion shows the effect on computed dynamic rotor loads. The magnitude of the effect depends on the radial location of the pitch change bearings.

Introduction

For a stiff-in-plane rotor system, the blade chordwise stiffness may be 20 to 50 times greater than the blade beamwise stiffness. The elastic structure tends to bend in the direction of least stiffness, resulting in dynamic coupling between out-of-plane and inplane motions as a function of the geometric pitch angles due to collective pitch, built-in twist, forced cyclic feathering motions of the torsionally-rigid structure, and elastic deformation of the blade and control system in the torsional mode.

Typical cruise conditions for a modern helicopter require collective pitch angles of 14 to 16 degrees at the root, depending on the amount of built-in twist. Cyclic feathering motions of 6 to 7 degrees are required to balance the one-per-rev aerodynamic flapping moments. In current design practice, elastic torsional

deflections of the blade and control system of a stiff-in-plane rotor are generally less than one degree. The largest part of the angular motion in the blade-torsion degree of freedom, therefore, is the forced feathering motion due to cyclic pitch.

Periodic variations of the inplane/out-of-plane elastic coupling terms are caused when the geometric pitch angle of each blade is increased and decreased at a frequency of one cycle per rotor revolution. When one blade is at high pitch and the opposed blade is at low pitch, an asymmetrical physical condition exists with respect to a reference system oriented either to the mast axis or to the plane of rotation. One-half revolution later, the reference blade is at low pitch and the opposed blade is at high pitch. Thus, periodic dynamic coupling occurs at the principal frequency of one-per-rev with respect to a rotating coordinate system. The coupling terms are nonlinear functions of blade pitch; hence, these terms also have 2-per-rev content.

Both the steady and periodic coupling terms have been treated in an analytical study of the effects of one-per-rev cyclic feathering motions on dynamic rotor loads. Equations have been derived and programmed for a digital computer solution of the transient response of an elastic two-bladed rotor.

The rotor is modeled by elastic substructure elements and lumped masses, for which the accelerations and velocities are integrated over small time increments to determine time histories of deflections, inertia loads, bending moments, etc. The time-variant analysis includes the capability to calculate rotor instabilities. The present computer program has been tested for this capability, but further discussion of instabilities is beyond the intent of the paper.

Dynamic rotor loads have been calculated for a parametric series of rotors, where the coupled natural frequencies were tuned over the range of contemporary design practice for teetering rotors. A description of the analysis and a summary of computed results is presented.

## Objective

A primary consideration in the design of a helicopter rotor is to minimize oscillatory bending loads, or at least to reduce the loads to a level that will ensure satisfactory fatigue life. During early stages of design, the principal method of evaluating the dynamics of a proposed rotor is to calculate its coupled rotating natural frequencies. If required, design changes are made to achieve sufficient separation between the natural frequencies and harmonics of the rotor operating speed.

Current practice at Bell Helicopter Company is to require a separation of 0.3 per rev for all flight combinations of rotor speed and collective pitch. One purpose of the present analytical development is to determine whether the separation rule may be relaxed due to beneficial effects of cyclic feathering motions on rotor dynamic response.

### Collective and Cyclic Modes

The calculation of natural frequencies for semi-rigid rotors uses a coordinate system that is based on the plane of rotation. The orientation of the centrifugal force field, the angular motion allowed by the flapping hinge(s), and the constraints of opposing blades lead to the segregation of natural frequencies into collective modes, cyclic modes, and (for four-bladed rotors) scissor or reactionless modes. This procedure allows the use of continuous-beam theory for a single blade, where the centerline boundary constraints are imposed from conditions of symmetry or asymmetry to match deflections, slopes, shears, and moments for the other blades.

The centerline boundary conditions for the collective mode (Figure 1) are:

- zero vertical (out-of-plane) slope change,
- vertical deflection constrained by mast tension/compression,
- inplane slope constrained by mast torsion, and
- zero inplane translation.

The centerline boundary conditions for the cyclic mode (Figure 2) are:

- vertical slope change unrestrained (except with flapping springs),
- zero vertical deflection,
- zero inplane slope change, and

- inplane deflection constrained by mast shear.

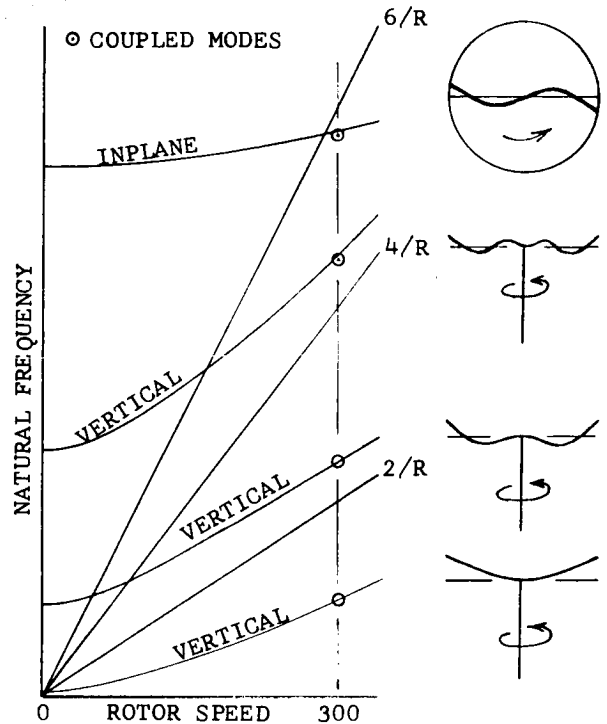


FIGURE 1. TYPICAL COLLECTIVE MODE FREQUENCIES AND MODE SHAPES.

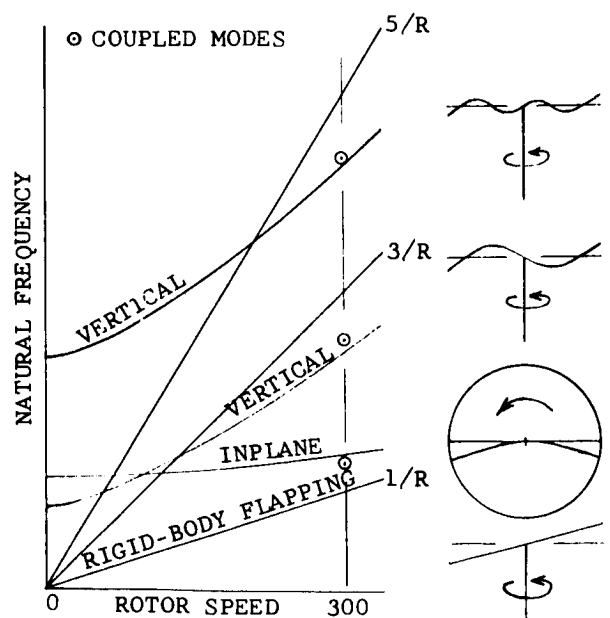


FIGURE 2. TYPICAL CYCLIC MODE FREQUENCIES AND MODE SHAPES.

For the reactionless modes, the centerline boundary conditions are:

- zero slope change and zero translation in both the inplane and vertical directions.

Uncoupled frequencies are determined by setting the geometric pitch angle of each elastic element to zero. The uncoupled frequencies are shown in Figures 1 and 2 by the labeled curves. Note that the frequencies of the vertical (out-of-plane) modes are highly dependent on rotor speed, and that the frequencies of the inplane modes are only slightly dependent on rotor speed.

Coupled natural frequencies are shown as small circles in the figures. Typical collective modes have very small frequency shifts as a function of collective pitch. However, the cyclic modes (Figure 2) couple significantly with collective pitch. Note that the inplane frequency decreases and the vertical frequencies increase with collective pitch. The method of determining these coupling effects is given in Reference 1.

By using only one blade plus appropriate boundary conditions, this method of calculating rotor natural frequencies is based on one explicit assumption, i.e., all other blades are at the same geometric pitch angle as the reference blade. If the blades are at different pitch angles, then the conditions of symmetry or asymmetry are not present. The inclusion of cyclic feathering motion, therefore, requires that the analysis treat separately each blade of the rotor and provide a means of matching the centerline slopes and deflections.

#### Elastic Substructured Rotor Analysis

A digital computer program has been developed to study the effects of cyclic feathering motions on dynamic rotor loads. The Bell Helicopter computer program is identified by the mnemonic ESRA for Elastic Substructured Rotor Analysis.

The analysis is a transient solution of elastic rotor blade motions, where the coupling terms for each blade are treated separately and the necessary constraints are imposed on each blade to insure slope and deflection continuity at the rotor centerline. Each blade is considered as being divided into a discrete number of segments, with uniform weight and stiffness properties over the length of a segment. The geometric pitch angle of each segment is a function of rotor azimuth position and input values of fore/aft and lateral cyclic pitch. To represent the hub structure that is inboard of the pitch-

change bearings, the inboard elastic element may be specified as an uncoupled element (geometric pitch equals zero).

All forces are applied at the ends of the elastic elements. Slope and deflection changes over the length of a segment are based on linear moment distributions versus span. For compatibility, shear over the segment length must be constant, which requires concentrated forces for both internal and external forces. In its simplest form, the analysis follows a lumped-mass approach. All of the important rotor dynamic characteristics may be retained with this method, however, by using the concepts of equivalent structural segments.

The dynamic response equations are solved by a step-by-step iterative method in order to include transient conditions. If the initial deflections and velocities are specified (spanwise distributions for each blade), then the internal bending moment distributions are found with respect to the rotating reference system. Internal shear distributions are obtained from the moment distributions, and summed with applied airload forces and inertial components of the centrifugal force field to determine spanwise distributions of accelerations.

The first estimates of deflection and velocity changes are calculated for constant acceleration during the integration time step. Then bending moments, shears, and accelerations are calculated for the end of the time step. Subsequent deflection and velocity estimates are based on accelerations changing linearly with time, and the iterations continue until a prescribed error limit is satisfied for the entire set of accelerations, or until a limit is reached on the number of iterations.

In recognition of the problems inherent with this type of numerical integration, the initial development of the ESRA computer program has been limited to a qualitative study of cyclic feathering effects. The current program represents each blade with four elements, each with beamwise and chordwise bending elasticity. Only the forced rigid-body motion is allowed in the blade-pitch degree of freedom, i.e., elastic blade torsion is not considered.

The current computer program is limited to two-bladed rotors, and the torsional impedance of the drive system is assumed to be zero. In practice, the rotor senses a two-per-rev torque from the mast that is proportional to the drive-system impedance times the Hooke's-joint angular oscillation, which is a function of rotor flapping. The rotor is the predominant inertia component of the drive system, and a good approximation for two-bladed rotors is to assume

that the true axis of rotation remains perpendicular to the tip-path plane even when the tip path plane is not perpendicular to the mast. Thus, Coriolis accelerations equal to the product of coning times flapping are not appropriate in a two-bladed rotor analysis.

Bending deflections of the elastic elements are linearized; therefore, Coriolis accelerations from radial foreshortening are excluded also. Vertical and in-plane translational motions of the rotor center are not included in the current version of the program.

Referring to the description above, the formulation of the analysis allows the removal of these limiting assumptions. For instance, nonlinear bending deflections and Coriolis accelerations may be included by a direct addition to the inertial forces acting on each mass. Translation of the rotor centerline, additional blades, control system flexibility, elastic blade torsion, and nonlinear hub and control kinematics also may be added within the existing computational method.

With the limitation of four elastic elements for each blade, plus provisions for slope and deflection continuity at the rotor centerline, the current ESRA program allows 15 distinct vibration modes for the rotor:

- 3 rigid-body modes (flapping, mast torsion, blade pitch)
- 6 coupled elastic collective modes (3 vertical, 3 inplane)
- 6 coupled elastic cyclic modes (3 vertical, 3 inplane)

In attempts to predict rotor loads for two-bladed rotors, emphasis is placed on response components at least up to the third harmonic of rotor speed. Three-per-rev airloads excite the cyclic mode that derives from the first elastic asymmetric mode in the out-of-plane direction. Four elastic elements for each blade should provide a very satisfactory dynamic representation for this frequency range. At a frequency of five-per-rev, the second elastic mode would be excited and computed loads may be marginally valid. Current design practice is to minimize higher frequency loads by proper tuning of the rotor natural frequencies, as discussed earlier.

#### Numerical Evaluation

A parametric computer study was accomplished to resolve a basic question:

With respect to the natural frequency of the first coupled vertical

elastic cyclic mode at or near 3 per rev, how much does cyclic feathering motion affect 3-per-rev dynamic rotor loads?

#### Selection of Rotor Dynamic Characteristics

Corresponding to a Huey main rotor, the computer study was based on a 48-foot diameter 2-bladed semi-rigid rotor, operating at 300 RPM. Two basic design approaches were selected as end points for the evaluation.

1. A constant blade weight distribution of 1.20 lb/in. with no dynamic tuning weights was picked to simulate the early production Huey rotors. Uniform beamwise and chordwise stiffness values were determined to locate the two lowest coupled cyclic mode frequencies at 1.40/rev (inplane) and 2.60/rev (vertical) for a collective pitch of 14.75 degrees. The fan plot of cyclic mode natural frequencies for this rotor is shown in Figure 3.

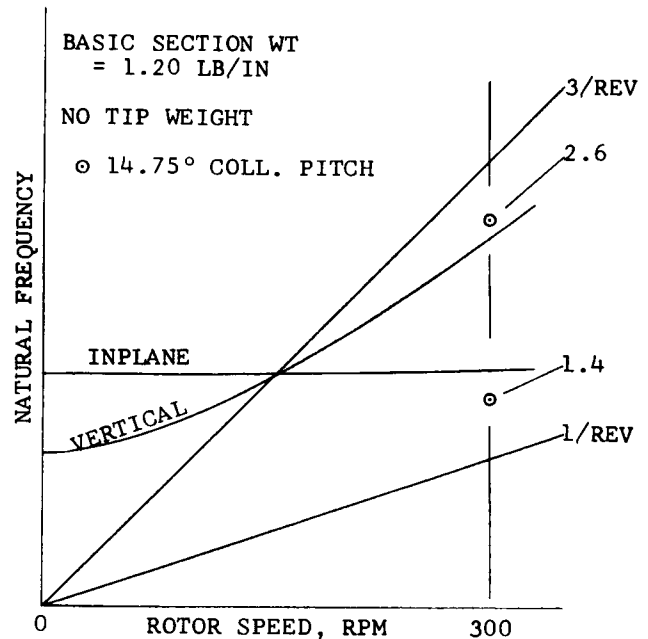


FIGURE 3. CYCLIC MODE, TUNED BELOW 3/REV.

2. A very recent rotor development at Bell (the Model 645 rotor) was simulated by a configuration with a constant blade weight distribution of 1.00 lb/in plus a dynamic tuning weight of 100 pounds located at the blade tip. Uniform beamwise and chordwise stiffness values were determined to locate the coupled cyclic mode frequencies at 1.40/rev (inplane) and 3.40/rev (vertical), as shown in Figure 4, again for 14.75 degrees of collective pitch.

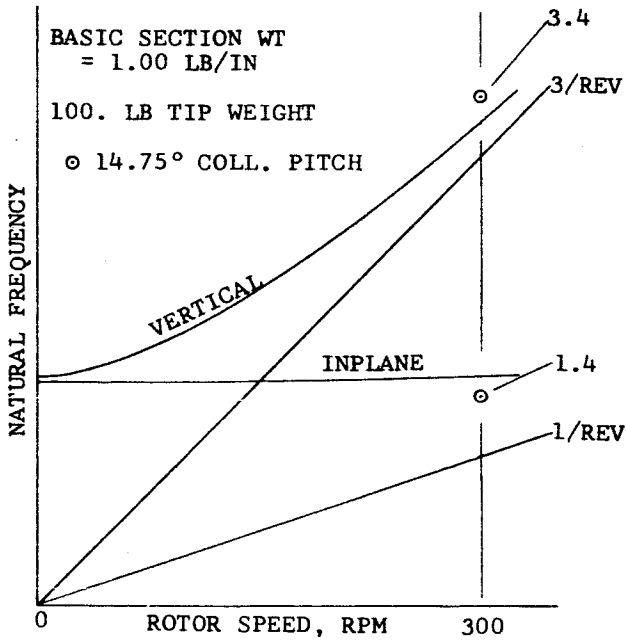


FIGURE 4. CYCLIC MODE, TUNED ABOVE 3/REV.

Between the two basic configurations, a series of intermediate rotor parameters was established by stepping the uniform blade weight from 1.20 down to 1.00 by increments of 0.025 lb/in., while increasing the tip weight from 0. to 100. by increments of 12.5 pounds. Beamwise and chordwise stiffnesses were varied to hold the coupled inplane frequency at 1.40/rev while tuning the coupled vertical frequencies from 2.60/rev to 3.40/rev in increments of 0.10/rev. Thus, to compensate for the program restriction of no hub motion, proper placement of the coupled inplane frequency was maintained by the selection of rotor stiffness. This approach affects the spanwise distribution of inplane bending moments, but is entirely adequate for a qualitative evaluation.

All of the above frequencies were tuned with the first segment uncoupled (hub structure to .25 radius), which maximized the coupling of the vertical mode near 3/rev and minimized the coupling of the inplane mode near 1/rev.

Additional input data was taken directly from the Bell Helicopter Rotorcraft Flight Simulation, program C81-68 (References 2, 3), for a Model 309 King-Cobra flying at 150 knots. Data used in the present computer evaluation included a collective pitch setting at the root of 14.75 degrees, a total cyclic pitch of 6.30 degrees, and the spanwise distributed airloads up to and including the third harmonic components.

The study results presented below are based, therefore, on full-scale parameters that are realistic with regard to current helicopter design practice. Although direct correlation with measured loads is not possible because of the simplifying assumptions, it may be noted that the magnitude of calculated bending moments is well within the expected range.

#### Computed Results

The forced response was computed for the series of nine parametric rotor configurations, where the inplane coupled frequency was held at 1.40/rev and the vertical coupled frequency was varied from 2.60/rev to 3.40/rev. The dynamic rotor loads for each configuration were calculated twice, once with cyclic feathering and once without cyclic feathering.

Figure 5 shows the 2/rev vertical bending moment at the rotor centerline as a function of natural frequency of the vertical elastic cyclic mode. The 1/rev variation in structural coupling due to cyclic pitch, and the 3/rev applied airloads produce a 2/rev component of bending moment. This additional component peaks and changes sign as the vertical mode is tuned through 3/rev. For the two-bladed rotor, 1/rev and 3/rev vertical bending moments at the centerline are negligible.

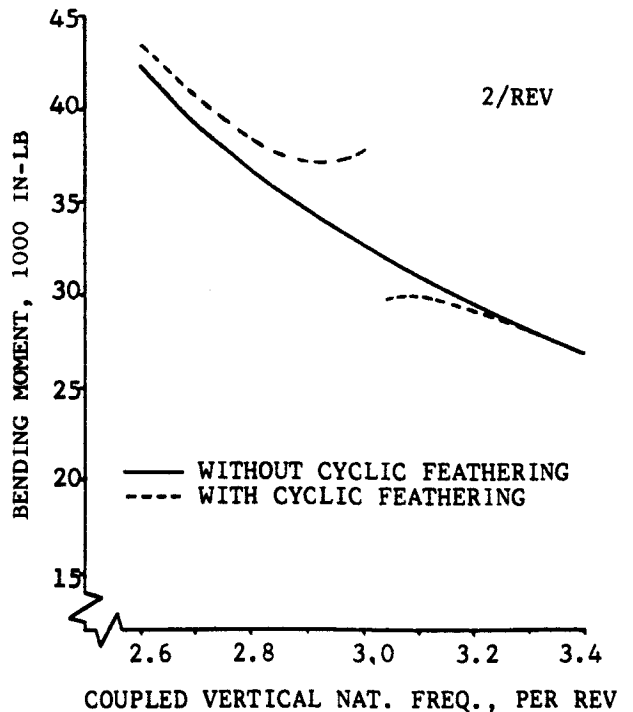


FIGURE 5. VERTICAL MOMENT AT CENTERLINE

Inplane bending moments at the rotor centerline are shown in Figure 6. The large peak in the overall oscillatory moment occurs as the coupled vertical mode is tuned through resonance at 3/rev. Note that the coupling associated with cyclic feathering increases the 1/rev response by about 5 percent for the vertical frequency tuned to 2.6/rev. In other respects, the effect of cyclic feathering appears to be minimal.

Beamwise moments and chordwise moments at midspan are shown in Figures 7 and 8, respectively. Two-per-rev moment

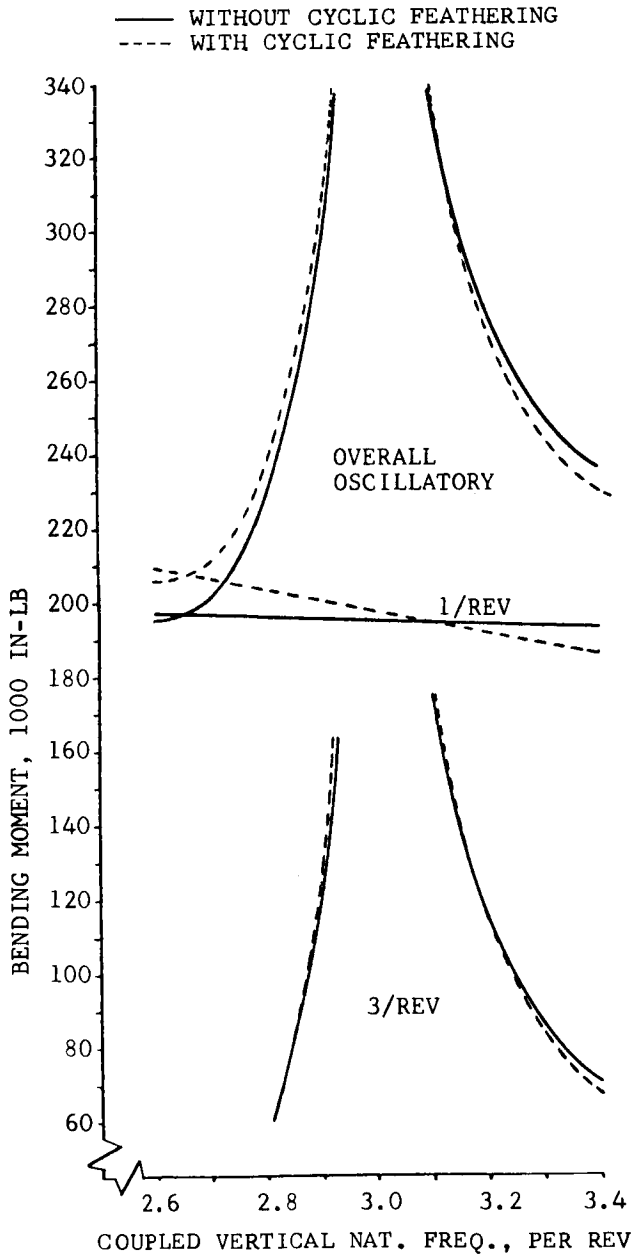


FIGURE 6. INPLANE MOMENT AT CENTERLINE

components are not shown in the figures because of their small magnitudes. The significance of the cyclic feathering effects at midspan is consistent with that indicated in earlier figures for the rotor centerline.

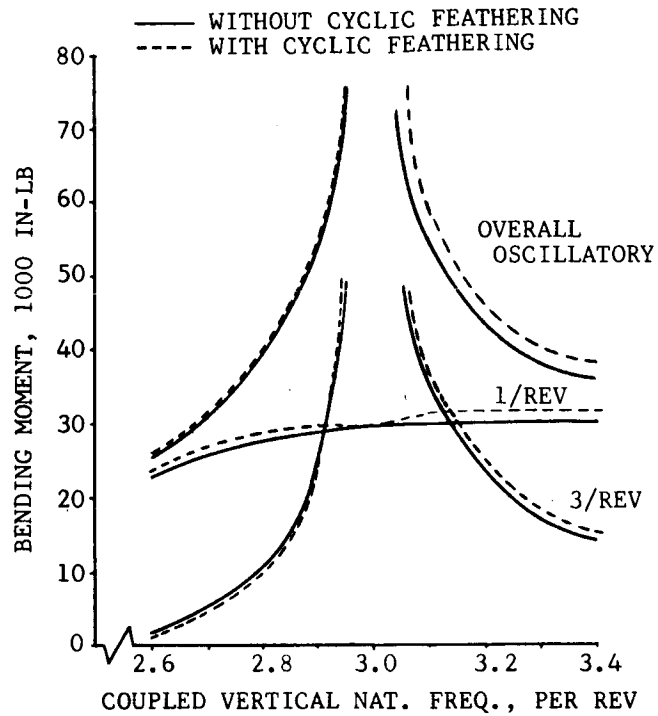


FIGURE 7. BEAMWISE MOMENT AT MID SPAN

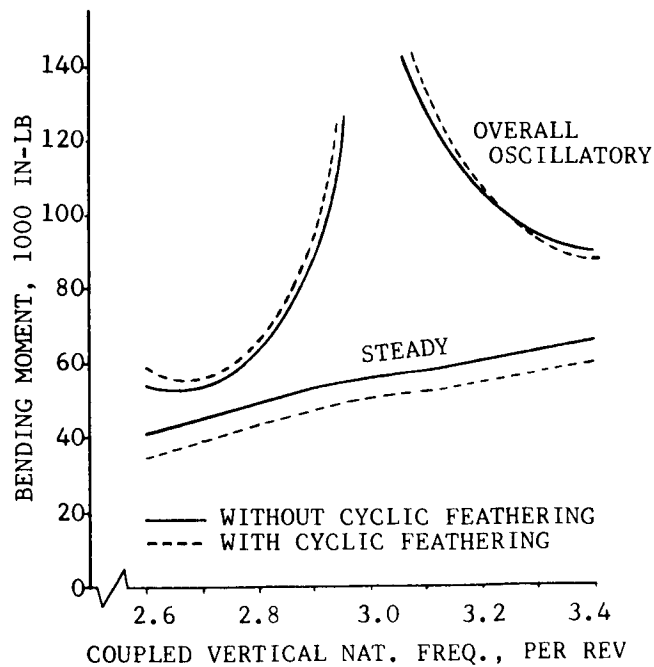


FIGURE 8. CHORDWISE MOMENT AT MID SPAN

As discussed in a previous section, the aeroelastic effect of blade bending velocity was excluded from this study by basing the response calculations on a prescribed set of airloads. No inference is intended regarding the magnitude of aerodynamic damping that may be associated with elastic bending velocities. Conversely, the procedure was selected so that the time-variant structural couplings could be studied in an analytical environment that does not include other sources of damping.

The computed responses appear as undamped resonances centered at 3/rev, from which it follows that cyclic feathering motions do not provide any significant amount of equivalent damping to suppress 3/rev dynamic loads. Regarding the vertical cyclic mode near 3/rev, in particular, the effect of cyclic feathering motion does not provide relief for the design rule that requires 0.3/rev separation of coupled frequencies from excitation harmonics of rotor speed.

The results presented above are all based on rotor structural simulations with the inboard 25 percent radius treated as non-feathering hub structure. This option of the program was selected to maximize the coupling (as a function of collective pitch) of the vertical cyclic mode near 3 per rev. As noted, the largest change in rotor loads due to the inclusion of cyclic feathering motions was a 5 percent increase in inplane bending moments at the rotor centerline.

The pitch-change or feathering bearings of production two-bladed main rotors are located typically at about 10 percent radius. In this respect at least, the above results are based on a dynamic model that is not representative of actual design practice.

To evaluate the importance of the radial location of the bearings, another set of rotor loads was computed for a case where the entire radius is in the feathering system.

A constant blade weight distribution of 1.20 lb/in with no dynamic tuning weights was selected, as before, to simulate the early production Huey rotors. The structural properties of the rotor were modified to maintain a 1.40/rev natural frequency for the coupled inplane cyclic mode. For the modified parameters, the natural frequency of the coupled vertical cyclic mode is 2.87/rev.

The computed results are shown in Figures 9 through 12 for the case in which the feathering bearings are located at zero percent radius. The bar graphs show first, second, and third harmonics plus

overall levels of oscillatory bending moments. The open bars are for the condition of no cyclic pitch, i.e., the geometric pitch of the elastic elements held fixed at the specified value of collective pitch. The closed bars are for the condition that the geometric pitch of the elastic structure is a function of both collective pitch and cyclic pitch.

Vertical and inplane oscillatory bending moments at the rotor centerline are shown in Figures 9 and 10, respectively. The vertical moments are not changed

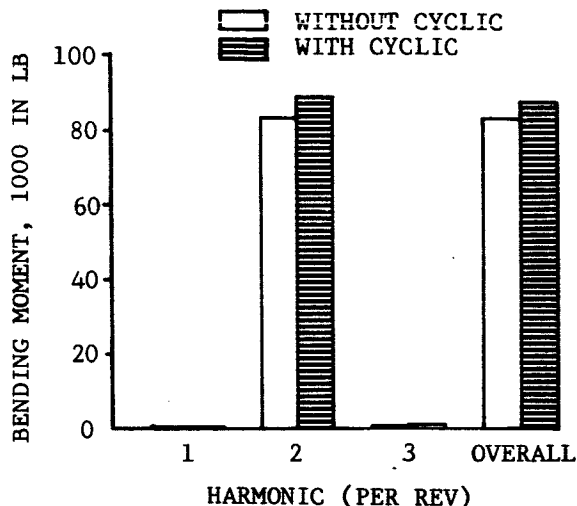


FIGURE 9. VERTICAL MOMENT AT CENTERLINE

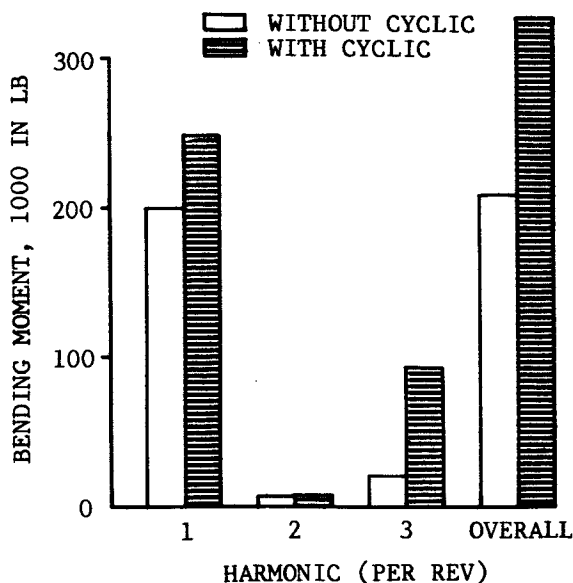


FIGURE 10. INPLANE MOMENT AT CENTERLINE

significantly by the inclusion of cyclic pitch. However, the inplane centerline moments increase by 57 percent, with both the first and third harmonics contributing to the increase.

Beamwise and chordwise oscillatory bending moments at 50 percent radius are shown in Figures 11 and 12. Most of the increase in the overall oscillatory moments at mid-span is due to an increase in 3/rev response.

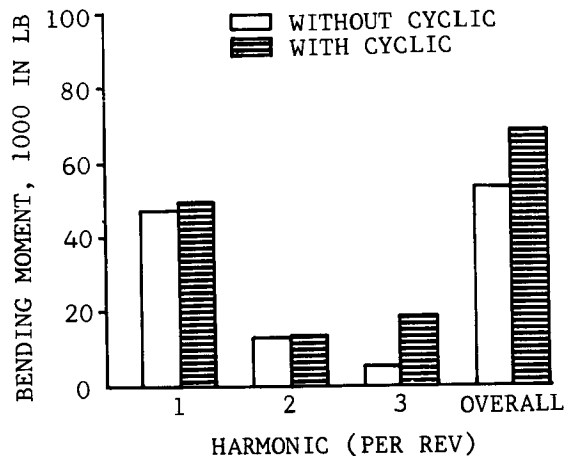


FIGURE 11. BEAMWISE MOMENT AT MID-SPAN

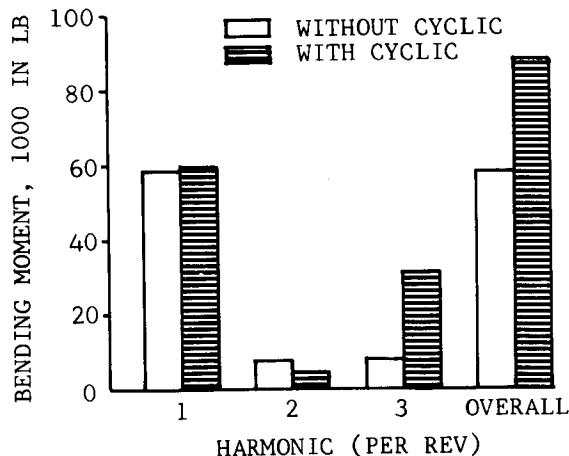


FIGURE 12. CHORDWISE MOMENT AT MID-SPAN

Due to cyclic feathering motions, a significant increase (57 percent) in rotor loads is indicated with the feathering bearings at zero percent radius; a minimal increase (5 percent) is indicated with the feathering bearings at 25 percent radius. This suggests that the radial location of the feathering bearings may have a controlling influence on the magnitude of the cyclic-feathering effect. Further study of this relationship is in progress.

### Conclusions

1. The cyclic feathering motions of a helicopter rotor cause time-dependent elastic coupling due to asymmetrical pitch on opposed blades. The effect of these motions on dynamic loads may be calculated by modeling the rotor with elastic sub-structure elements, by providing individual treatment of each blade, and by matching slopes and moments at the rotor centerline.

2. Cyclic feathering motion of the elastic blade structure does not cause any significant damping effect on the 3-per-rev dynamic loads of a two-bladed semi-rigid rotor. The design rule requiring 0.3/rev separation between coupled natural frequencies and aerodynamic excitation frequencies should not be relaxed on the expectation of beneficial effects from cyclic feathering.

3. The inplane one-per-rev rotor loads of a stiff-in-plane rotor are affected significantly by cyclic feathering of the elastic structure. The magnitude of the effect is decreased as the feathering bearings are moved radially away from the rotor centerline.

### References

1. Blankenship, B. L. and Harvey, K. W., A DIGITAL ANALYSIS FOR HELICOPTER PERFORMANCE AND ROTOR BLADE BENDING MOMENTS, *Journal of the American Helicopter Society*, Vol. 7, No. 4, October 1962, pp 55-69.
2. Duhon, J. M., Harvey, K. W., and Blankenship, B. L., COMPUTER FLIGHT TESTING OF ROTORCRAFT, *Journal of the American Helicopter Society*, Vol. 10, No. 4, October 1965, pp 36-48.
3. USAAVLABS Technical Report 69-1, ANALYTICAL STUDY OF HELICOPTER GUST RESPONSE AT HIGH FORWARD SPEEDS, Harvey, K. W., Blankenship, B. L., and Drees, J. M., U.S. Army Aviation Materiel Laboratories, Ft. Eustis, Virginia, September 1969.