A CRITICAL REVIEW OF THE EXPERIMENTAL DATA FOR
DEVELOPED FREE TURBULENT SHEAR LAYERS

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INTRODUCTION

In selecting test cases for use in this conference, both the Langley Conference Committee and the Data Selection Committee recognized the need to include shear layers among the selected test cases. However, because of the confusion and apparent contradictions which existed in the interpretation of the experimental data, three of the five shear-layer test cases were specified without reference to any particular data. The primary purpose of this paper is to review the relevant data and to present the results in a convenient form for comparison with the numerical predictions for these three test cases (test cases 1, 2, and 3). Since these flows were specified to be developed turbulent flows, this will be the primary concern of the present paper. Some mention will be made of transitional flow, but only to differentiate it from developed turbulent flow. This is not intended to be a detailed study of the transition process itself.

SYMBOLS

b width of shear layer

b^o spreading rate of shear layer

$c = \dfrac{b}{x}$

d, k constants

ℓ_1, ℓ_2 lengths

m density ratio, ρ_2/ρ_1

R Reynolds number

$R_x = \dfrac{u_1 x}{\nu}$, where x is the farthest downstream station surveyed

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r   velocity ratio, u_2/u_1

u   velocity

x,y   Cartesian coordinates

ϵ eddy viscosity

θ momentum thickness

$\lambda = \dfrac{u_1 - u_2}{u_1 + u_2}$

ν kinematic viscosity

ξ similarity parameter, $\sigma y/x$

ρ density

σ constant in similarity parameter ξ, also called spreading parameter

Subscripts:

o   value when $u_2 = 0$

1,2   conditions on high- and low-velocity side of shear layer, respectively

Primes denote fluctuating quantities.

ANALYTIC SOLUTIONS

The analytic solution of the boundary-layer equations generally used to compare with experimental data for the two-dimensional shear layer (fig. 1) was first derived by Görtler in 1942 (ref. 1). This solution is well known and, since details of its derivation are readily available (ref. 2, pp. 689-690), the results will only be briefly summarized here.

It is assumed that the effective kinematic viscosity is given by

$$\epsilon = kb\left(u_1 - u_2\right) \tag{1}$$

where k is a constant, b is the width of the shear layer, and u_1 and u_2 are the velocities on both sides of the shear layer. The adoption of a stream function and the assumption of similarity automatically satisfy the continuity equation and reduce the boundary-layer equations to a simple third-order differential equation

$$F'''(\xi) + 2\sigma^2 F(\xi)F''(\xi) = 0 \tag{2}$$

where $\xi = \dfrac{\sigma y}{x}$ with boundary conditions $F'(\xi) = 1 \pm \lambda$ at $\xi = \pm\infty$ where

$$\lambda = \frac{u_1 - u_2}{u_1 + u_2}$$

This leads to the general series solution in powers of λ

$$u = \frac{u_1 + u_2}{2}\Big[1 + \lambda\,\mathrm{erf}(\xi + d) + \ .\ .\ .\Big] \tag{3}$$

leaving two constants, σ and d, to be determined. The constant σ, which is a measure of the spreading rate of the shear layer, depends on the magnitude of the eddy viscosity and must be determined experimentally. The second constant d appears because only two of the three required boundary conditions have been specified. Note that this constant merely deflects the mixing region and has no direct influence on σ.

There has been considerable discussion in the literature on the correct third boundary condition (refs. 3, 4, and 5), but this relates primarily to the theoretical problem of semi-infinite streams. In practice the flow is always bounded. Experimentally and numerically the shear layer is generally approximated by the near field of a jet, and the requirement that $\partial u/\partial y$ be zero, on the line or plane of symmetry, is employed as the third boundary condition; thus the problem of indeterminancy is eliminated.

EFFECTS OF LOW REYNOLDS NUMBER

Before discussing the spreading rate in a developed turbulent shear layer, it is important to define what is meant by the term "developed." From an experimental point of view, it is seldom sufficient to require simply that the mean flow be self-similar, since it is often extremely difficult to establish when this has been achieved. This is especially true if only mean velocity measurements are available. The turbulence components are a much more sensitive indication of similarity, and it is strongly recommended that they be measured whenever possible. For present purposes the term "developed" will be used to refer to shear layers for which the data indicate that the turbulent and mean velocity components have achieved self-similarity and the maximum shear stress

has reached a constant value. However, it should be emphasized that this is no guarantee that the flow is truly developed. Some examples will be given of flows which appear to be self-similar, on the basis of mean and fluctuating velocity data, but which still do not seem to have reached their asymptotic spreading rate.

The variation of the center-line shear stress with downstream distance in a subsonic shear layer with zero velocity ratio is show schematically in figure 2. When the boundary layer is laminar at the point of separation, the near field can be divided into two more or less distinct regions. The first region ℓ_1 is shown here as the distance from the point of separation to the point of maximum shear stress. This region is sometimes defined as the distance to the point at which the fluctuating velocity component is a maximum (ref. 6) or as the distance to transition (ref. 7). The second region ℓ_2 is the subsequent distance to developed flow. The distance ℓ_1 has been reported to be virtually independent of Reynolds number (refs. 6 and 7), depending only on the initial boundary-layer thickness, but it is important to remember that ℓ_1 is a strong function of the initial disturbance level. This disturbance level will, in general, be different for different apparatuses and may also vary with Reynolds number for the same apparatus.

The second region ℓ_2 is much less sensitive to initial conditions, and at least for the range $500 < \dfrac{u_1 \theta}{\nu} < 1000$, Bradshaw (ref. 6) found that the Reynolds number based on ℓ_2 was nearly constant. Since for a typical laboratory experiment ℓ_2 is 5 or 10 times ℓ_1, it is generally sufficiently accurate to quote a combined length

$$\ell_1 + \ell_2 \approx 7.0 \times 10^5 \frac{\nu}{u_1} \tag{4}$$

as the length required for both the mean and the fluctuating velocity components to become similar, while the mean velocity profiles alone appear to be similar at about

$$\ell_1 + \ell_2 \approx 4.0 \times 10^5 \frac{\nu}{u_1} \tag{5}$$

Recent results by Spencer (ref. 8) indicate that if the fluctuating pressures are considered, true similarity may require a Reynolds number, based on shear layer length, of about 1.3×10^6. When the boundary layer is turbulent at the point of separation, the shear stress rises slowly and the maximum again becomes approximately constant at $x \approx 7.0 \times 10^5 \frac{\nu}{u_1}$. However, its value is higher than that produced by a laminar boundary layer (refs. 6, 9, and 10). This suggests that one or perhaps both of these flows are not truly fully developed and that a constant value of the peak shear stress, independent of initial conditions, is only achieved far downstream. At present there are no experimental results to confirm this conclusion. The available results all show that the shear stress

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in a shear layer depends on initial conditions, even in regions of the flow which otherwise appear to be developed. (See table 1, refs. 8 to 18.)

For velocity ratios u_2/u_1 greater than zero, the situation becomes very complex. The flow now develops from two boundary layers which will in general have different momentum thicknesses. Both boundary layers may be laminar or turbulent or one may be laminar and the other turbulent. It seems very unlikely that the developing region of such flows can be accurately characterized by any simple criteria, but, in general, the length required for the flow to become fully developed will increase with velocity ratio.

These results are based on a fairly limited range of conditions, but they do give a simple and useful guide for the design of experiments and provide an excellent first check on experimental data for which no turbulence measurements are available. To put it another way, failure to satisfy Bradshaw's criteria (eq. (4)) may not prove that a flow is transitional but it does place a burden on the experimenter to demonstrate that his flow is developed. Simply showing that the mean velocity data can be collapsed on a similarity plot is not good enough.

Data taken in low Reynolds number shear layers generally yield low values of σ, if the boundary layer was laminar before separation. This effect is believed to be similar to that described in references 19, 20, 21, and 22 for low Reynolds number boundary layers. The difference between the values of σ computed over the developing region of the flow and in the developed region is seldom more than 20 to 30 percent in subsonic flows with low velocity ratios. However, the peak shear stress can reach twice its developed value (ref. 6) and the difference does appear to increase with Mach number in supersonic flows.

VARIATION OF σ WITH u_2/u_1

So far discussion has been restricted mainly to shear layers with a velocity ratio $\dfrac{u_2}{u_1} = 0$. In 1962 Golik (ref. 23) proposed the relation

$$\frac{\sigma_0}{\sigma} = \frac{u_1 - u_2}{u_1} \tag{6}$$

based on data by Szablewski (ref. 24), for the variation of σ with u_2/u_1. The following year Sabin (refs. 25 and 26) and Abramovich (ref. 27, pp. 36-42) independently published the relation

$$\frac{\sigma_0}{\sigma} = \frac{u_1 - u_2}{u_1 + u_2} = \frac{1 - r}{1 + r} = \lambda \tag{7}$$

and supported their theoretical arguments with new experimental data. In 1968 Miles and Shih (ref. 28) proposed the relation

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$$\frac{\sigma_o}{\sigma} = \frac{1}{1 + 5r^2} \tag{8}$$

and in 1972 Yule (refs. 29 and 30) proposed

$$\frac{\sigma_o}{\sigma} = \frac{1 - r}{(1 + r)^{1/2}} \tag{9}$$

All these relations were supported by experimental data. These four relations are shown in figure 3, and obviously predict different variations of σ_o/σ with r. However, the situation is not as bad as it might seem at first sight. Golik's result can probably be discounted since it was based on very limited data which were not supported by later experimental results. Yule's relation appears to be based on only two new data points, and this does not seem to be sufficient to establish any relation between σ and r. In any case, these new data by Yule differ only slightly from the data obtained by Miles and Shih. However, equations (8) and (9) do differ significantly. In particular, note that equation (9) unlike equation (8) predicts that $\frac{1}{\sigma} \to 0$ as $r \to 1.0$, which seems to be required theoretically.

Before discussing the experimental data in detail, it is important to emphasize that the expression for eddy viscosity and the variation of σ and r are directly related and cannot be specified independently. Sabin characterized equation (7) as "a plausible functional relation between these two quantities." This statement is certainly true, but it has led to some confusion since it seems to imply that the result cannot be formally derived from Görtler's theory.

Consider Görtler's expression for ϵ

$$\epsilon = kcx\left(u_1 - u_2\right) \tag{10}$$

where c is a constant defined by

$$b = cx$$

and

$$\sigma = \frac{1}{2}(kc\lambda)^{-1/2} \tag{11}$$

or

$$\sigma^2 = \frac{1}{4kc\lambda}$$

If the width of the mixing layer is defined as

$$b = y_a - y_b = cx \tag{12}$$

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where y_a is the value of y where $\xi = 1$ and y_b is the value of y where $\xi = 0$, it follows since

$$\xi = \frac{\sigma y}{x}$$

or

$$\frac{\sigma b}{x} = 1$$

then

$$c = \frac{1}{\sigma}$$

In general, for any consistent definition of b, $c \propto \frac{1}{\sigma}$.

Inserting this into equation (11) gives

$$\sigma^2 = \frac{\sigma}{4k\lambda}$$

or

$$\frac{\sigma_0}{\sigma} = \lambda \tag{13}$$

Therefore, if Prandtl's hypothesis is used in the form

$$\epsilon = kb\left(u_1 - u_2\right)$$

where the viscosity is based on the actual width of the mixing region, as is usual in numerical solutions, it is not necessary to use equation (13) explicitly. If the resulting solution is self-similar, it will automatically satisfy equation (13).

The available experimental values of σ are shown in figure 3 as a function of the velocity ratio r. The data from reference 27 (pp. 36-42) were given in nondimensional form, and other values of σ (refs. 9, 12 to 14, 17, and 25 to 33) have been normalized by using a value of $\sigma_0 = 11$. Although this is common practice, it is an unsatisfactory method of illustrating the variation of σ with r for a number of reasons. First, it gives equal weight to all data points (some of which were measured in transitional flows). Second, the variation in σ_0 mentioned earlier tends to exaggerate the scatter in the data. (See tables 1 and 2.) It is not possible to establish the variation of σ with r by comparing values of σ taken in different apparatuses at different velocity ratios unless the corresponding values of σ_0 are known and in many experiments σ_0 was not measured. If the variation in σ is to be determined separately from each set of experiments, then it is suggested that σ be measured for at least four different values of r. Of the six sets of data which satisfy this requirement, one (ref. 31) must be discarded because of low

Reynolds number effects. Of the remaining five sets of data (fig. 4), three include determinations of σ_0, but in only one case (ref. 34)[*] is the numerical value of σ_0 given; in the other two cases, the data are given in nondimensional form (ref. 27, pp. 36-42). This leaves two sets of data for which no values of σ_0 are given. For these data a value of σ_0 was chosen which best collapsed the data. A value of $\sigma_0 = 11$ was used to normalize Sabin's data, which is the value originally used in references 25 and 26, while $\sigma_0 = 9.3$ was found to be best for the data published by Miles and Shih, compared with $\sigma_0 = 12$ which they used in reference 28. This last set of data was also modified slightly. As originally reported (ref. 35), mean velocity profiles were taken at five x stations for each value of r. The spreading parameter σ was then computed for each velocity profile, and the resulting five values of σ were averaged. The value of x used in the calculations was the actual distance from the end of the splitter plate rather than the distance from the virtual origin of mixing. This led to some error in the calculated values of σ which increased with velocity ratio r and with initial boundary-layer thickness but decreased with increasing x. Since two sets of data were given for different initial boundary-layer thicknesses, it was possible to partially compensate for these errors without repeating the calculations. First, the values of σ computed for the two stations closest to the origin were dropped, and a new average was calculated. Then, a linear extrapolation through these two sets of data was used to estimate the values of σ which would correspond to a flow with zero initial boundary-layer thickness. This resulted in no change in σ for velocity ratios less than 0.3, but the correction increased for higher velocity ratios, resulting in a maximum increase in σ of about 20 percent for a velocity ratio of 0.83. The resulting data were then normalized by using a value of $\sigma_0 = 9.3$. It is not clear why these data required a lower value of σ_0 to bring them into correspondence with the data from the other experiments, but it seems possible that the boundary layers may have been turbulent before separation. The agreement between these data and those reported in reference 29 seems to support this conclusion, since the boundary layers in the latter experiment were turbulent.

As can be seen in figure 4, the data from the five experiments, normalized as described previously, are all in good agreement with the prediction of equation (7). Note that the data points corresponding to the highest velocity ratio for each of the two experiments reported in reference 27 (pp. 36-42) have been omitted from figure 4. These data points deviate significantly from the rest of the data and obviously do not correspond to the mixing rates in fully developed shear layers. This deviation is discussed in reference 27 (pp. 36-42) and is attributed to the increased influence of free-stream turbulence on mixing rate at high velocity ratios.

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[*] The authors would like to thank B. G. Jones for pointing out that the σ_0 given in reference 34 was a measured value.

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It is perhaps worth emphasizing at this point that while the data in figure 4 seem to confirm the predictions of Görtler's simple theory, the result is, for many practical problems, rather academic. Görtler's theory is only valid for regions of the flow which are sufficiently far downstream that the effects of initial conditions can be ignored and the flow has become self-similar. This distance increases quickly with velocity ratio, and much of the available experimental data and many practical problems do not satisfy these criteria. For such flows the mixing rates will be affected by the initial conditions and may differ significantly from the prediction of equation (7).

Stephen J. Kline of Stanford University recently suggested a new method of plotting the data shown in figure 4. If σ_0/σ is plotted against λ, the result is a straight line passing through the origin and the point (1,1). This method of plotting the data seems to offer some advantages over other methods: It simplifies the selection of a σ_0 to optimize the fit between theory and experiment, and it may also be very useful for studying flows in which the velocity ratio r is varying. The data plotted in this form are shown in figure 5.

VARIATION OF σ WITH MACH NUMBER

Figure 6 shows experimental values of σ for zero-velocity-ratio shear layers as a function of Mach number. This includes a determination of σ at a Mach number of 5 recently obtained at the NASA Langley Research Center (LaRC). The total temperature in each of these flows was approximately constant. Again some selection was involved in the presentation of these data in that not all the available data listed in table 3 (refs. 16, 36 to 44) are shown in figure 6. To present all the available results would lead to a scatter of data, matched only by the attempts to correlate them (refs. 27 (pp. 293-302), 45 to 53), and would tend to mask any real experimental trend which may be indicated. It seems reasonable to discount Johannesen's early results (ref. 39) since the author himself suggested that the mixing rate was affected by shock waves in the nozzle, and he later repeated the experiment with a new nozzle (ref. 40). The discrepancy between Cary's data (ref. 38) and those obtained by other workers over the same Mach number range is not as easy to explain. It is pointed out in reference 16 that the data, which were obtained by using an interferometer, would have limited accuracy especially for the lower Mach number flows. Although this is true, the discrepancy appears to be larger than can be explained as a simple experimental error and probably indicates some difference in the actual flows. While the present authors can offer no clear explanation for the difference, it does seem that the preponderance of data, which indicate lower values of σ for this Mach number range, is more representative of the mixing in a developed shear layer. Therefore, to illustrate the general experimental trend more clearly, Cary's data have been omitted from figure 6.

The remaining data seem to follow two more or less distinct curves. For clarity, these two sets of data have been marked by open and solid symbols in the supersonic region. One set of data, marked by open symbols, indicates a sharp increase in σ with Mach number as the flow becomes supersonic with a tendency to level out again for hypersonic velocities. The second set of data, marked by solid symbols, shows little or no variation with Mach number. At a Mach number of 5 the two sets of data differ by about a factor of 3, and this difference appears to be increasing with further increases in Mach number. The only obvious significant difference between these two sets of data is in the Reynolds number, the data marked by solid symbols generally having a lower value of R_x. Note that the Reynolds number for the Mach 4 and the Mach 8 data (ref. 41) is considerably lower than the R_x of at least 4×10^6 required to achieve fully developed mean profiles for the Langley Mach 5 shear layer. The data from reference 44 refer specifically to the developing region of a shear layer and extend only about 10 to 15 initial boundary-layer thicknesses downstream from the separation point. There is no reason to believe that the mixing rates calculated for these flows will equal the mixing rates in developed turbulent flows. It is therefore suggested that on the basis of the data available at present, the faired line shown in figure 6 best represents the variation of σ with Mach number for developed shear layers. However, because of the limited data available at present and because effects of initial conditions are still poorly understood, some uncertainty must exist as to the absolute accuracy of these data, and further high Reynolds number data for supersonic and hypersonic shear layers are very desirable.

Interest in the effects of Reynolds number on the turbulence levels in supersonic and hypersonic free shear flows has been heightened by a recent paper by Finson (ref. 54). In this paper Finson draws attention to the significant differences between the turbulence levels in high and low Reynolds number hypersonic wakes. While the flow in a hypersonic wake is considerably more complex than the mixing in a simple shear layer, the Reynolds number effects described in reference 54 seem to be at least qualitatively similar to those discussed previously in this paper.

VARIATION OF σ WITH DENSITY RATIO

Although a number of correlations of the variation of σ with Mach number have been proposed, the authors know of no published eddy viscosity model, applicable to a wide range of free turbulent flows, which includes a specific Mach number effect. Most models assume that the change in σ is due to the associated change in density ratio across the shear layer and that for a given velocity and density distribution, the mixing rate is independent of whether the density distribution is due to changes in temperature, composition, or Mach number. This conclusion has recently been challenged by Brown

and Roshko (ref. 32), who claim that density differences in subsonic flows have relatively small effect on the turbulent mixing rate.

The spreading rate in a developed turbulent shear layer appears to be linear with x for both homogeneous and heterogeneous flows at all velocity ratios, and conditions on both sides of the mixing layer do not change with distance downstream. This would seem to make the shear layer an ideal flow in which to study the effects of density ratio on the turbulent mixing rate. Although this is probably true, the design of a suitable apparatus poses a number of serious problems. It is no coincidence that most of the detailed studies of the mixing in homogeneous shear layers employed rather large apparatuses (refs. 8, 9, and 14). This may have been due in part to a desire to generate a shear layer which was large enough to allow detailed measurements to be made, but more important, it was necessary to employ a large apparatus or a high unit Reynolds number (ref. 6) to insure that the flow would be developed. To achieve suitably high Reynolds numbers in a heterogeneous experiment can be quite difficult. The experimenter finds that in selecting a suitable gas combination which will give the required large density difference, he often must season his selection with considerations of expense and danger. As a result of these difficulties there are little data available for the mixing in heterogeneous shear layers, and in none of these experiments is the flow clearly developed.

Of the four available experimental studies, three list values of σ (table 4). Values of σ are not available for the fourth (ref. 55), but the variation in spreading rate with density ratio m is given. These experiments cover approximately the same density ratio and Reynolds number range as those described in reference 32. Although there are insufficient data at any one velocity ratio for a meaningful plot of the variation of σ with m, it is obvious that there is considerable disagreement between the four sets of experimental results. There is good agreement between the measured values of σ in references 18 and 33 for a density ratio of 4.0 and a velocity ratio of about 0.25, but the results differ by nearly a factor of 2 at a velocity ratio of approximately 0.5. At a velocity ratio of 0.377 Brown and Roshko found that σ varied by only about 60 percent when the density ratio was changed by a factor of 49; Abramovich et al. found that the spreading rate changed by nearly a factor of 4 over the same range of density ratios. In all these experiments R_x was less than that generally required to give developed flow in a homogeneous shear layer. It is difficult to draw any definite conclusion from these results except that most of the data are probably influenced by low Reynolds number effects. A comparison of the measured values of σ in the homogeneous mixing experiments from references 18, 30, and 33 with those from references 8 and 25, tends to support this conclusion. (See fig. 3.)

The turbulent mixing in heterogeneous shear flows has been studied for more than years, and it is surprising and disappointing to find that one of the most fundamental

questions still remains unanswered: Does density ratio have any significant effect on the mixing rate? This question is important and must be answered before we can claim any real understanding of the turbulent mixing in supersonic or variable-density free shear flows.

Although the absolute values of σ from these heterogeneous mixing experiments are of questionable value for determining the effect of density ratio on mixing rate, the reported variation of σ with velocity ratio is still of some interest. This is due to the widely different predictions of available eddy viscosity models. These data plotted as a function of velocity ratio r are shown in figure 7. Except for the experiments from reference 55, where the spreading rates at $r = 0$ are given, the data have been normalized by using a value of σ_0, which fits the points corresponding to the lowest velocity ratios to the curve $\frac{\sigma_0}{\sigma} = \lambda$. Note that in spite of the expected scatter most of the data fall close to the curve. This suggests that the variation of σ_0/σ with r is not strongly dependent on the density ratio across the shear layer. In contrast to this, many eddy viscosity models show a strong dependence on density ratio. For example, a simple mass flow difference model of the form

$$\epsilon = kb \left| (\rho u)_1 - (\rho u)_2 \right| \tag{14}$$

predicts a variation given by

$$\frac{\sigma_0}{\sigma} = \left| \frac{1 - mr}{1 + r} \right| \tag{15}$$

Equation (15) is plotted in figure 8, and it can be seen that it predicts a strong dependence on m. For values of $m > 5.0$ and $r > 0.2$, σ_0/σ increased with r, and in some cases, values of σ_0/σ were nearly an order of magnitude greater than those found experimentally. It should also be noted that equation (14), unlike Prandtl's constant exchange hypothesis, is not invariant with respect to a Galilean transformation.

CONCLUSIONS

One of the most important conclusions of this study must be that many, if not most, of the apparent inconsistencies which exist in the interpretation of the experimental data for free shear layers result from confusing data taken in developed turbulent flows with those taken in transitional or developing flows. Only a small fraction of the flows studied thus far appear to be developed, and many workers are apparently unaware that the effects of the initial conditions can persist far downstream. The present authors are not suggesting that experimental studies of developing flows are unimportant. On the contrary

many flows of practical importance are not developed, and one of the major advantages of the better turbulence models is their potential ability to predict such flows. However, as the authors have attempted to show in this review, experimental studies in developing flows can lead to erroneous conclusions if the mixing rates in the corresponding developed flows are not known.

The conclusions of this study as they relate to the first three conference test cases are as follows:

1. The variation of σ_0/σ with r in a developed subsonic homogeneous shear layer is best represented by

$$\frac{\sigma_0}{\sigma} = \frac{1 - r}{1 + r}$$

where $r = \dfrac{u_2}{u_1}$, σ is the spreading parameter, σ_0 is the spreading parameter at $u_2 = 0$, and u_1 and u_2 are velocities on high- and low-velocity side of shear layer, respectively. Although some of the data do not support this relation, the discrepancies appear to be satisfactorily explained as low Reynolds number effects or the effects of initial conditions.

2. The effects of Mach number are more uncertain primarily because of limited data and the absence of any turbulence measurements for supersonic shear layers. On the basis of the data available at present, the faired line shown in figure 6 seems to best represent the variation of the spreading parameter with Mach number for a developed supersonic shear layer.

3. The data available for heterogeneous shear layers are not sufficient to clearly establish the effect of density ratio on mixing rate. Although there is little experimental evidence to suggest that variations in the density ratio across a shear layer will greatly change its mixing rate, it appears to the present authors to be more appropriate at this time to emphasize the need for better data at high Reynolds number than to speculate on the absolute accuracy of the available data.

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TABLE 1.- EXPERIMENTAL DETERMINATIONS OF σ_o IN SUBSONIC SHEAR LAYERS

Reference	Investigator	Date	Nozzle shape and dimensions (a)	Initial conditions (a)	Theory used	R, per meter	$^b R_x$	σ_o	Comments
12	Tollmien	1926	A; 224 cm		Tollmien	20.5×10^5	2.3×10^6	11.8	Only one profile used to compute σ_o
11	Cordes	1937	R; 25 by 65 cm		Tollmien	10.2×10^5	6.0×10^5	12.0	
13	Reichardt	1942	R; 32 by 32 cm		Görtler	2×10^5	2.8×10^6	13.5	Very high value of σ_o
14	Liepmann and Laufer	1947	R; 152 by 19 cm (60 by 7.5 in.)	S.P. (laminar)	Görtler and Tollmien	12.1×10^5	1.1×10^6	11.0	Note change in σ_o
						12.1×10^5	1.1×10^6	12.0	
15	Crane	1957	A; 8.9 cm (3.5 in.)	Nozzle	Crane	14.4×10^6	3.3×10^6	12.7	Based on rather limited data by Lawrence
16	Maydew and Reed	1963	A; 7.6 cm (3.0 in.)	Nozzle	Crane	15.7×10^6	4.6×10^6	10.5	No information given about boundary layer before separation (probably turbulent)
			A; 7.6 cm (3.0 in.)		Crane	19.0×10^6	5.6×10^6	10.8	
			A; 7.6 cm (3.0 in.)		Crane	21.3×10^6	6.3×10^6	11.0	
17	Mills	1968	R; 20 by 5 cm (8 by 2 in.)	S.P.	Numerical solution	11.2×10^5	2.6×10^5	10.0	No information given about boundary layer before separation (probably turbulent)
						8.5×10^5	1.75×10^5	10.4	
9	Wygnanski and Fiedler	1970	R; 50.8 by 17.8 cm (20 by 7 in.)	St., T.W.	Comparison with reference 14	7.9×10^5	4.7×10^5	9.0	T.W. just upstream of separation
8	Spencer	1970	R; 38 by 17.8 cm (15 by 7 in.)	S.P. (laminar)	Korst	20.7×10^5	2.6×10^6	11.0	Appears to be best data available at present
10	Batt, Kubota, and Laufer	1970	R; 63.5 by 12.7 cm (25 by 5 in.)	S.P. (laminar) St. (laminar) St., T.W.	Profiles were compared with those in references 14 and 9	10.2×10^5	6.6×10^5	12.0	Turbulence measurements tend to confirm effect of trip wire on σ_o
						10.2×10^5	6.6×10^5	12.0	
						10.2×10^5	6.6×10^5	9.0	
18	Johnson	1971	R; 2.5 by 2.5 cm (1.0 by 1.0 in.)	S.P.	Görtler	12.5×10^5	1.3×10^5	9.0	Very low R_x

[a] Abbreviations used are

A	Axisymmetric
R	Rectangular
S.P.	Splitter plate
St.	Backward-facing step
T.W.	Trip wire

[b] $R_x = \dfrac{u_1 x}{\nu}$ where x is the farthest downstream station surveyed.

TABLE 2.- EXPERIMENTAL VALUES OF σ FOR VARIOUS VALUES OF u_2/u_1 IN SUBSONIC SHEAR LAYERS

Reference	Investigator	Date	u_2/u_1	σ	σ_o/σ	R_x
25, 26	Sabin	1963	0.36	25		2.3×10^5
			.46	32		
			.62	47		
			.66	54		
27 (pp. 36-42)	Yakovlevskiy	1963	0		1.0	
			.1		.83	
			.12		.76	
			.235		.615	
			.40		.52	
			.58		.51	
27 (pp. 36-42)	Zhestkov	1963	0		1.0	
			.17		.67	
			.22		.585	
			.28		.52	
			.465		.34	
			.52		.32	
			.70		.29	
31	Seban and Back	1962	.2	14.7		
			.36	16.4		
			.6	27.9		9×10^4
17	Mills	1968	0	10.4		1.75×10^5
			.3	18.3		
			.6	25.0		2.6×10^5
			0	10.0		
			.3	16.4		
			.6	26.0		
33	Baker and Weinstein	1968	.111	12.1		8×10^4
			.25	13.75		
			.43	16.01		
			.666	37.5		
28	Miles and Shih	1968	.15	12.6		1.9×10^6
			.225	15.0		
			.36	21.6		
			.475	28.2		
			.55	34.8		
			.65	43.2		
			.755	54.6		
			.83	66.6		
8, 34	Spencer and Jones	1971	0	11		2.6×10^6
			.15	13.3		
			.22	15.7		
			.305	20.4		
			.54	41.4		
			.61	50.4		
			.76	77.0		
32	Brown and Roshko	1972	.143	12.8		5×10^5
			.377	18.5		
18	Johnson	1971	0	9		1.3×10^5
			.3	14.85		
			.6	25.2		
29, 30	Yule	1972	.3	19		8×10^5
			.6	36		

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TABLE 3.- EXPERIMENTAL DETERMINATIONS OF σ_o IN SUPERSONIC SHEAR LAYERS

Reference	Investigator	Date	Mach number	R, per meter	R_x	σ_o
36	Gooderum, Wood, and Brevoort	1950	1.6	6.2×10^7	1.2×10^7	15.0
37	Bershader and Pai	1950	1.7	6.9×10^7	4.8×10^5	17.0
38	Cary	1954	1.0	28.2×10^6	9.5×10^5	16.0
			1.33	4.3×10^7	8.9×10^5	17.0
			1.59	6.2×10^7	1.7×10^6	33.0
39	Johannesen	1957	1.4	4.9×10^7	1.3×10^6	21.9
					3.1×10^6	11.9
40	Johannesen	1959	1.4	4.9×10^7	3.1×10^6	13.2 & 13.7
16	Maydew and Reed	1963	1.49	4.3×10^7	1.1×10^7	15.0
			1.96	7.5×10^7	2.2×10^7	20.0
41	Rhudy and Magnan	1966	3.99	7.5×10^6	7.67×10^5	12.4
			7.53	8.5×10^6	1.03×10^6	14.0
42	Sirieix and Solignac	1966	3.0			29.0
			4.0			34.0
43	Eggers	1966	2.22	10.8×10^7	1.6×10^7	22.0
44	Hill and Page	1969	2.1	4.9×10^7	1.1×10^7	16.0
			2.5	4.6×10^7	1.0×10^7	16.0
			3.2	4.2×10^7	6.4×10^6	16.0
			3.5	4.2×10^7	6.4×10^6	16.0
			3.7	4.2×10^7	4.8×10^6	16.0
[a] Unpublished	Morrisette and Birch	1972	5.0	31.5×10^6	8×10^6	37.0

[a] Data obtained at NASA Langley Research Center by E. Leon Morrisette.

TABLE 4.- EXPERIMENTAL VALUES OF σ IN HETEROGENEOUS SHEAR LAYERS

Reference	Investigator	Date	ρ_2/ρ_1	u_2/u_1	σ	$a\,\sigma_o/\sigma$	R_x
33	Baker and Weinstein	1968	4.0	0.5	36.9		$\approx 6 \times 10^4$
				.25	15.4		
				.111	14.6		
				.143	10.5		
55	Abramovich et al.	1969	.27	0		0.22	10^5 to 10^6
			$\longrightarrow$	.5		.1	
			.27	0		.32	
			1.3	.22		.20	
			$\longrightarrow$	.5		.12	
			$\longrightarrow$	.75		.08	
			7.0	0		.9	10^4 to 10^5
			$\longrightarrow$	.2		.27	
			$\longrightarrow$	.7		.115	
32	Brown and Roshko	1972	.143	.377	23		7×10^4 to 5×10^5
			7.0	.143	9		
			7.0	.377	15		
18	Johnson	1971	4.0	.28	15.9		$\approx 6 \times 10^4$
				.45	19.8		

a The spreading rate b^o of the shear layer is not defined in reference 55, but it is assumed that $\dfrac{b^o}{b_o^o} \approx \dfrac{\sigma_o}{\sigma}$.

32
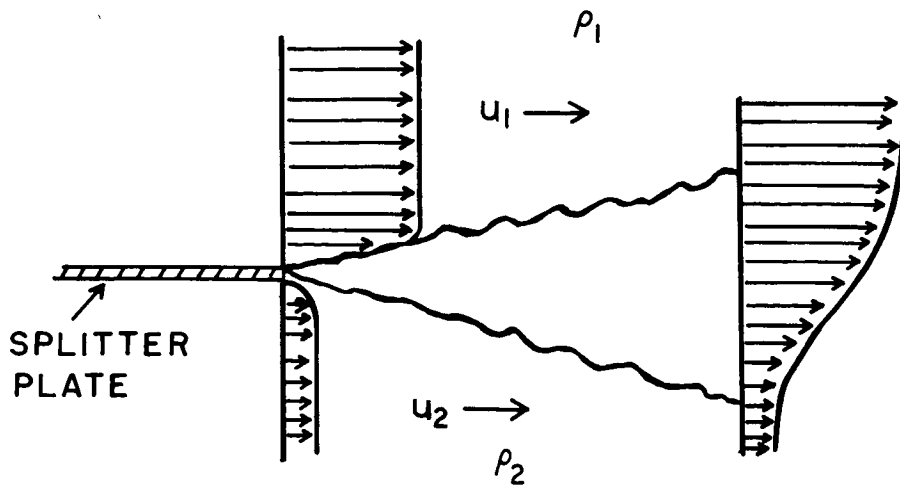
Figure 1.- Free shear layer.
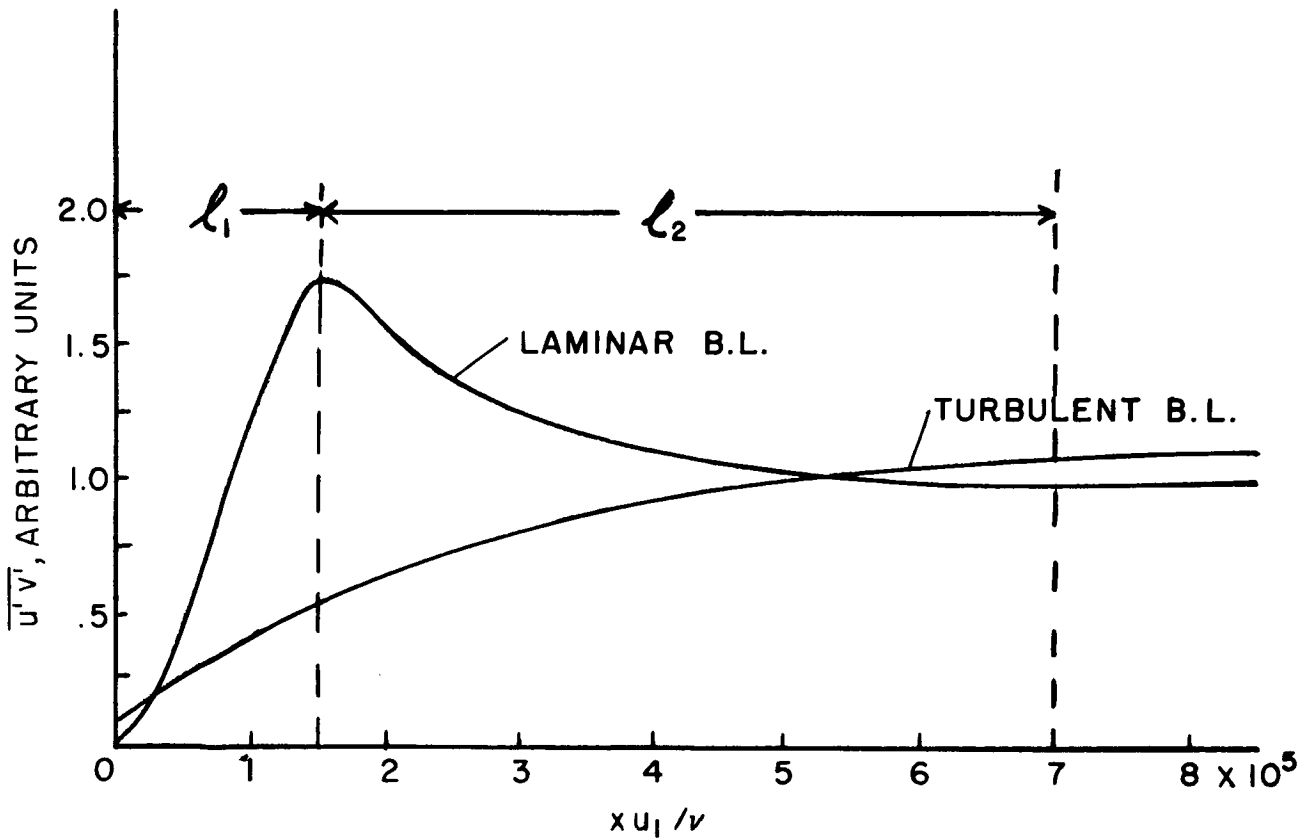
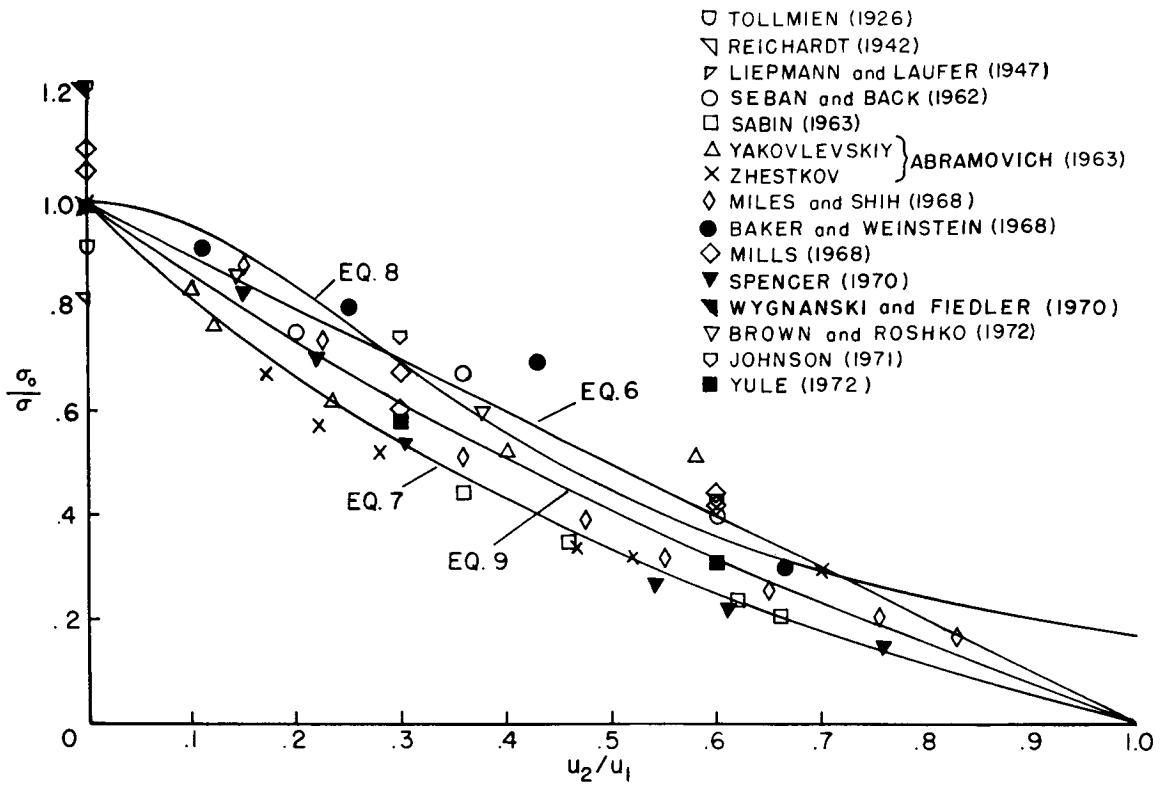Figure 2.- Variation of shear stress $\overline{u'v'}$ in near field of a shear layer.

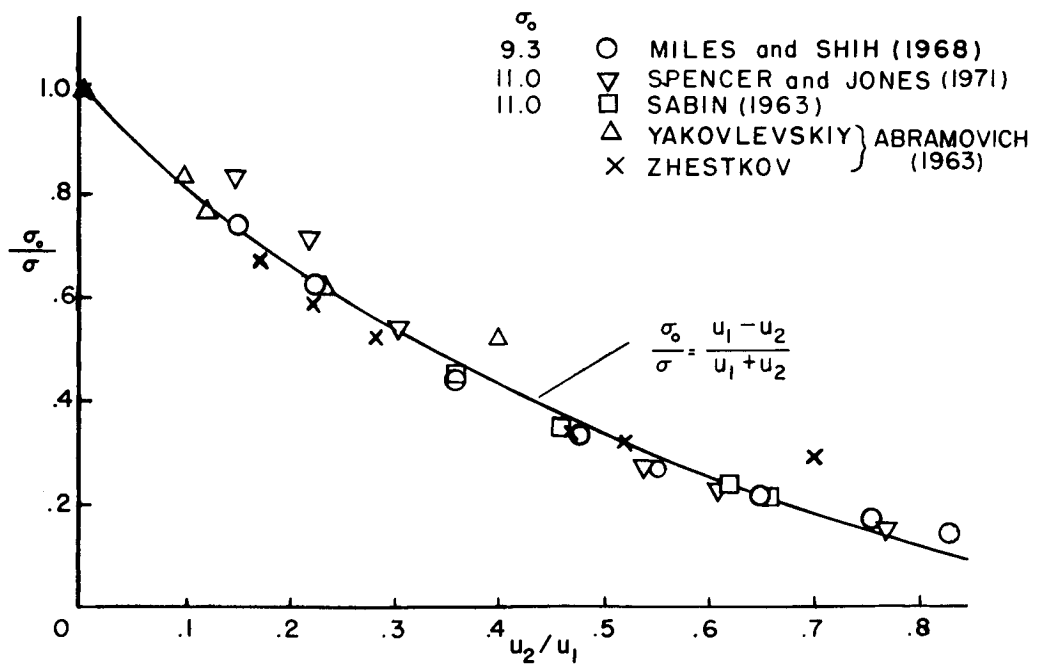Figure 3.- Variation of spreading parameter σ_0/σ with velocity ratio u_2/u_1.

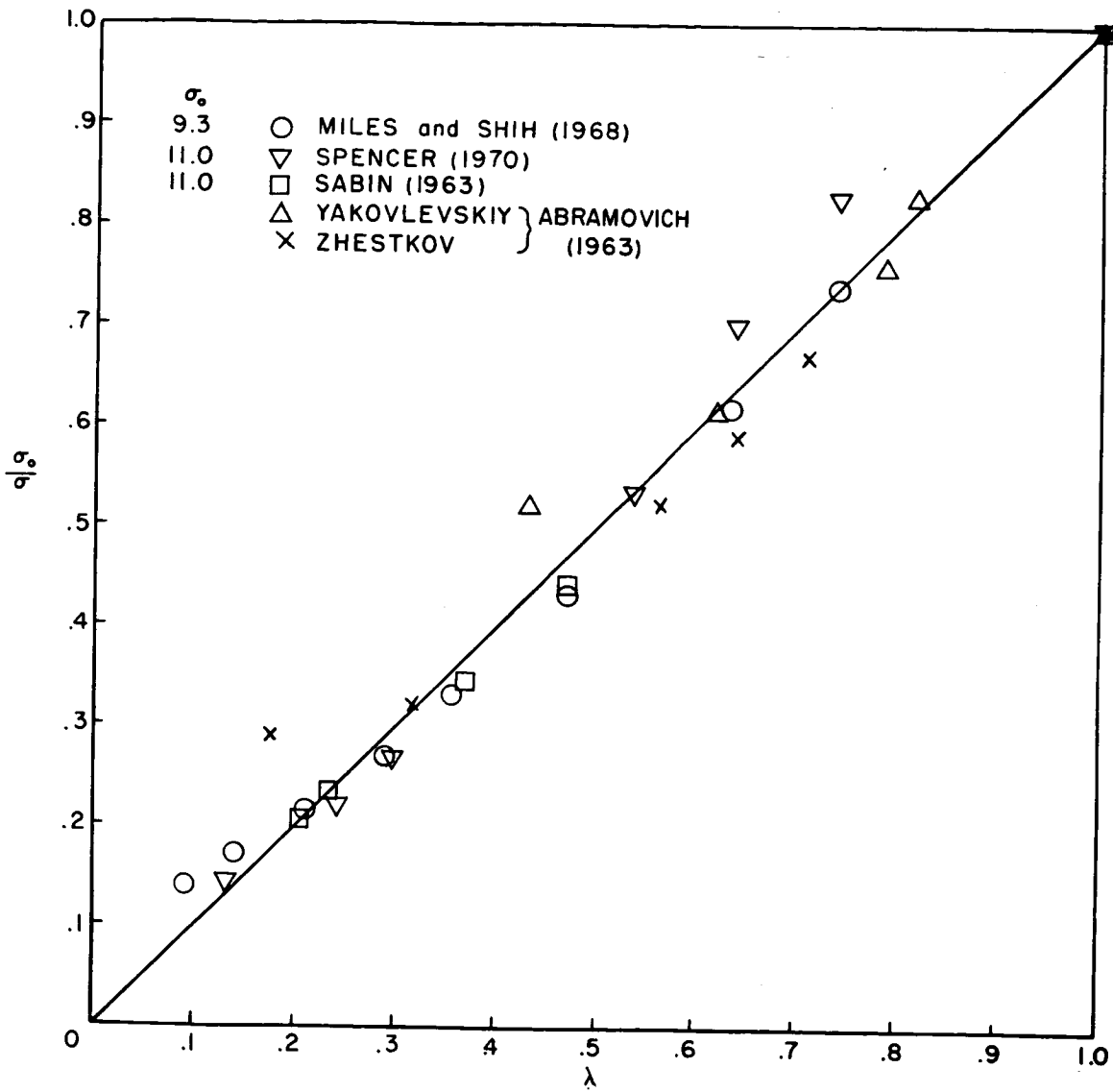Figure 4.- Variation of σ_0/σ with velocity ratio u_2/u_1.
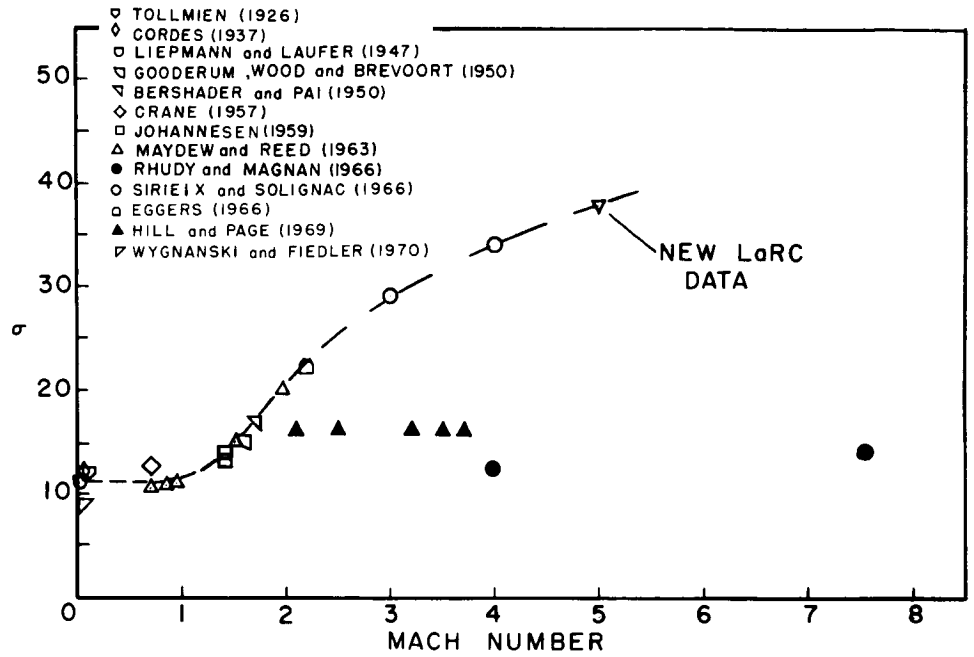
Figure 5.- Variation of σ_0/σ with λ.
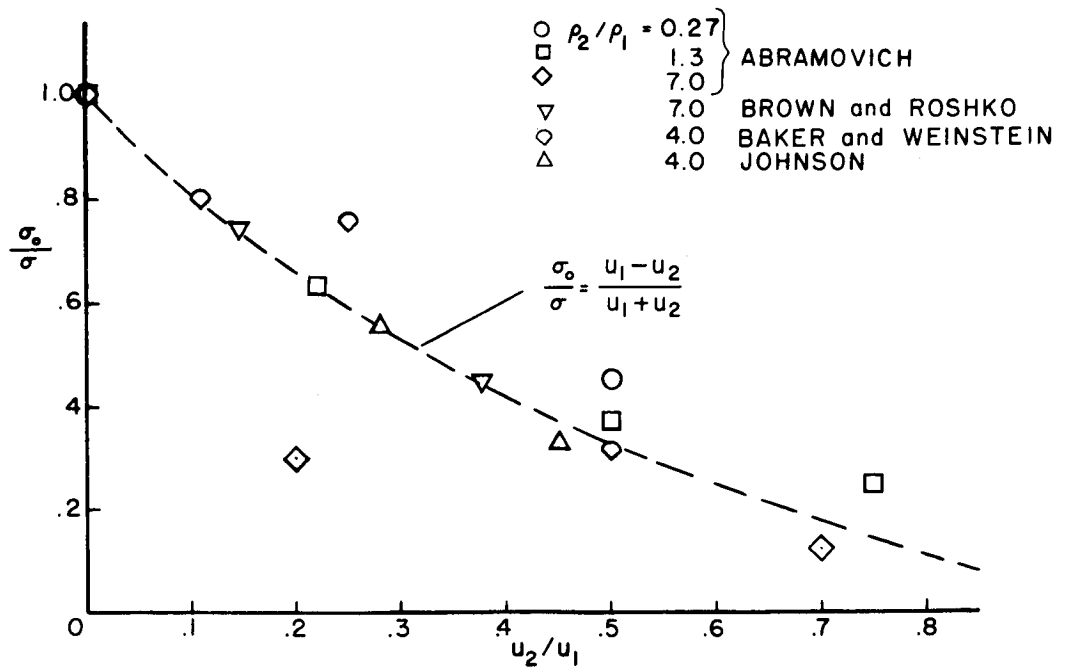
Figure 6.- Variation of σ with Mach number.

Figure 7.- Variation of σ_0/σ with velocity ratio u_2/u_1 for heterogeneous shear layers.
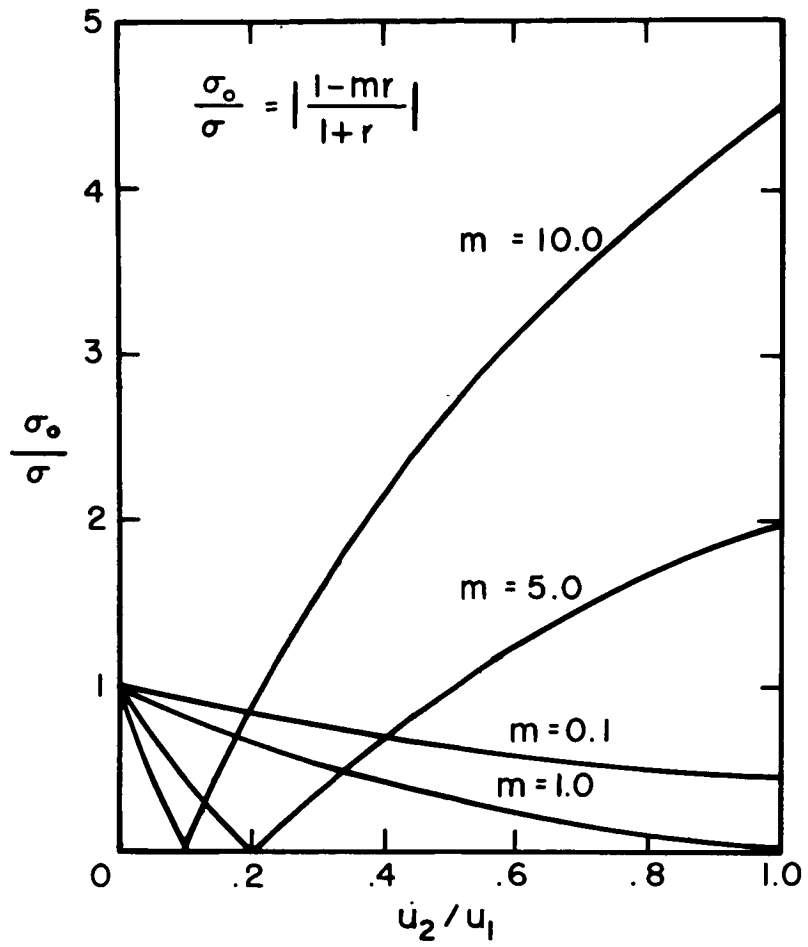
Figure 8.- Variation of spreading parameter σ_o/σ with velocity ratio for different values of density ratio $m = \dfrac{\rho_2}{\rho_1}$.

DISCUSSION

S. J. Kline: I merely want to comment in respect to Sabin's theory that the curve of σ_0/σ versus velocity ratio was not fitted. It was derived before there were any data, and he derived it in a different way than you suggested — he derived it on the basis of the assumption that the transverse velocity component is a function of velocity ratio only. It does not require anything but that, and the answer pops out and then the data were subsequently plotted on, so the theory was never fitted to the data.

S. F. Birch: No, the point I was trying to make was that in his paper he was somewhat modest on this point. He characterized it as a plausible functional relation between the two quantities. I believe this has been confusing to a number of people, in the sense that he did not emphasize the fact that this was derived directly from Prandtl's constant exchange hypothesis. It does follow directly. One method has been indicated by Professor Kline, and there is a second way of doing it, described in my paper. We have also checked the result numerically.

M. V. Morkovin: What flows do you feel are good touchstones for the theories that we are going to hear, and will the audience know "what flow is what number" when they are discussed?

S. F. Birch: Everyone was sent a list of the test cases with the letter of invitation. Is this what you are referring to?

M. V. Morkovin: Everyone in the audience or just the predictors?

S. F. Birch: No, everyone who received an invitation got a list of the test cases. If they have not brought them with them, we can supply some extra copies.

M. V. Morkovin: The question was also which are good touchstones?

S. F. Birch: Well, we believe that the shear layers (or near field region of jets) are more sensitive than some of the coaxial downstream mixing regions to such things as density ratio and Mach number ratio. This was the reason they were included in the flow test cases. To a certain extent it will be up to the committees, in general, to decide whether or not this is correct.

A. Roshko: I have a comment on a remark made toward the end of your talk. I am not sure I understood you correctly — did you say, or imply, that you would want Galilean invariance for an eddy-viscosity model?

S. F. Birch: I think it is possible to model some flows with a model which is not invariant. However, if it is not invariant, you do restrict yourself in its application or range of application.

A. Roshko: But, if you require Galilean invariance, then you are immediately requiring that the effect of ρ_2/ρ_1 be the same as that of ρ_1/ρ_2.

S. F. Birch: The ratio ρ_1/ρ_2 is not changed by Galilean transformation.

A. Roshko: Well, that is what I mean. For example, let us say you have the low density on the high-speed side, that will have one effect on the spreading rate, but if you require that the model be Galilean invariant, then you could reverse the velocities without reversing the densities. Then you ought to expect the same spreading rate on the requirement that it be Galilean invariant. I do not think that for the steady flows you can require them to be Galilean invariant, or equivalent to a nonsteady flow, say an infinite sheet spreading simply in a transverse direction.

S. F. Birch: I am not sure that I really follow your remarks. It seems to me that the shear stress must be invariant with respect to Galilean transformation in the full equations. Perhaps, I do not understand your point.

A. Roshko: Possibly we could take it up separately.

S. F. Birch: Yes, certainly.

I. E. Alber: I do not know if you have considered the parameter which describes the effect of the initial turbulent boundary layer or whether you have reached a similarity state or not, but some earlier calculations show that you have to go a distance of at least about 100 initial momentum thicknesses before you have essentially washed out the character of the initial boundary layer, at least in terms of its scale and this increase significantly with Mach number as well, so that, in fact, many of these cases at higher Mach numbers may not really be similar.

S. F. Birch: Yes, this is a point which perhaps I did not emphasize enough in my talk. In the experiments which were run here at Langley at Mach 5, we found that the Reynolds number required to get what looked like fully developed flow was 5 to 10 times higher than would have been required for a subsonic flow. Therefore, apparently this distance does increase with Mach number. I am not absolutely certain that our Reynolds numbers were high enough, in spite of the fact that we did get good similarity behavior for the mean profiles. The data reported here were for the highest Reynolds number test run so far, so presumably it best represents the fully developed spreading rate.

The following comments were submitted in writing before the conference and are included here because of their relevance to the discussion:

P. Bradshaw: It seems just possible that Brown and Roshko's [1] "two-dimensional large eddies" are the same as the posttransitional disturbances that delayed the onset of self-preservation in my experiments. [2] However, I am not confident about the use of my sim-

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[1] Brown, Garry; and Roshko, Anatol: The Effect of Density Difference on the Turbulent Mixing Layer. Turbulent Shear Flows, AGARD-CP-93, Jan. 1972, pp. 23-1 – 23-12.

[2] Bradshaw, P.: The Effect of Initial Conditions on the Development of a Free Shear Layer. J. Fluid Mech., vol. 26, pt. 2, Oct. 1966, pp. 225-236.

ple Reynolds number criterion for self-preservation $\left(u_1 \frac{x}{\nu} > 7 \times 10^5\right)$ at exit boundary-layer Reynolds numbers very different from the range I used. Certainly my results indicated a Reynolds number criterion rather than an x/θ_0 criterion but it is not obvious physically why the viscosity should matter to the posttransitional decay. As usual we need more data. It would be very helpful to have measurements of velocity or density fluctuations in Brown and Roshko's rig to see if rms values (ρ' or u') are self-preserving.

I don't think the uncertainties about self-preservation are strong enough to invalidate the conclusion that density ratio has little effect on spreading rate. We can take Brown and Roshko's order-of-magnitude arguments a little further and examine the Mach number fluctuation, which Morkovin's hypothesis requires to be small. For simplicity look at $M_T = \frac{\sqrt{|\overline{uv}|}}{a}$: in a mixing layer at $M = 1$ (about the Mach number at which the spreading rate starts to change significantly) we have $M_{T,max} \approx 0.1$. In a boundary layer at $M_1 = 4$ with $c_f = 0.001$, we again have $M_{T,max} = M_1\sqrt{\frac{c_f}{2}} \approx 0.1$. Therefore, if the Morkovin limit is roughly $M_1 = 4$ in a boundary layer, it is plausible that it should be only $M_1 \approx 1$ in a jet. Of course, the insensitivity to density ratio implies that Morkovin's hypothesis breaks down because of the effect of <u>pressure</u> fluctuations on the turbulence $\left(\text{if rms } p \propto \rho\overline{uv}, \text{ rms } \frac{p}{p_{abs}} \propto \gamma M_T{}^2\right)$. This deserves further thought, but will cheer the people who are interested in large density ratios at low speeds.