# CALCULATION OF TURBULENT FREE MIXING

## STATUS AND PROBLEMS

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## INTRODUCTION

The first impulse when compiling an introductory paper to a conference such as the present one is to provide a review or overview of the research in the subject area. However, since several fairly recent reviews are already available (refs. 1 to 4), some background on the motivation in organizing this conference and one view of the status and problems of turbulent free mixing calculations will be presented. The other attendees at the conference have their own views of the present status in this area, and indeed, the major purpose of this conference is to ascertain the present capability to predict several of the simpler turbulent free mixing flows. It is hoped that each attendee will air his views in the papers to be presented and in the discussions which follow – both formal and informal.

The motivation behind much of the turbulent shear layer research is one or more of the large number of possible applications. Some of these applications are given in the following list, which is obviously biased toward aerospace:

Propulsion
Shock interference heating
Noise
Tangential slot injection (film cooling)
Pollution
Wakes
Augmenters and ejectors
Separated flows
Nuclear rockets
V/STOL high-lift devices

At the Langley Research Center (LaRC) free mixing in practically all the areas shown has been an important concern, with the most effort involved in slot injection, V/STOL, noise, and propulsion. In other industrial fields there are many more applications of turbulent free mixing research which could be listed. Therefore, even in the present climate of applied technology over basic research there is still a strong mandate to develop accurate calculation schemes for free mixing.

In the present conference only the basic mixing problems of free shear layers, jets, and wakes are considered. However, within these basic flows there is included a considerable range of conditions including nonsimilarity, compressibility, and secondary and heterogeneous flows. A brief outline of the various combinations included in the present conference is given in the following list:

Free shear layers:

- Similar
- Nonsimilar

- Incompressible
- Compressible

Jets:

- Similar
- Nonsimilar

- Incompressible
- Compressible

- Single
- Coaxial

- Axisymmetric
- Two dimensional

- Homogeneous
- Heterogeneous

Wakes:

- Similar
- Nonsimilar

- Incompressible
- Compressible

- Axisymmetric
- Two dimensional

Paper no. 2 by Stanley F. Birch and James M. Eggers will discuss further details concerning the data chosen as test cases. This is a formidable set of conditions with which to confront a calculation method. Most of the published procedures were generally applie to only a few of the test cases to be considered by the predictors at this conference.

This fairly complete confrontation of turbulence closure method with basic data should give a clearer picture of which method works where and which approaches deserve further development. By "further development" is meant application to some of the important "real life" effects which are not specifically included in the basic data considered for this conference. Several of these effects will be briefly discussed.

## FREE MIXING PHENOMENA NOT CONSIDERED IN PRESENT CONFERENCE

### Transverse Pressure Gradients

Several authors (e.g., refs. 5 and 6) have indicated that the common assumption of constant static pressure across a free mixing layer is not borne out by the available data. This assumption becomes increasingly suspect as Mach number increases (ref. 5), and the fact that the static pressure is variable may have a profound effect on turbulence spreading rates. The probable cause of this static pressure variation is the $\bar{\rho}\overline{V'^2}$ $\left(\bar{\rho}\right.$ is density, $\overline{V'^2}$ is mean-square transverse turbulence velocity$\left.\right)$ term in the normal momentum equation (ref. 7). There are also more complicated flows where a static-pressure variation occurs because of outside influences (such as shock interactions). In paper no. 4 David H. Rudy and Dennis M. Bushnell discuss transverse pressure gradients in free turbulent flows in more detail.

### Longitudinal Pressure Gradients

Longitudinal pressure gradients can occur quite often, especially in combustors, interactions between shock waves and shear layers, and separated flows. Very little is presently known, either experimentally or theoretically, concerning the influence of longitudinal pressure gradients on turbulent free mixing. Ferri (ref. 8) indicates a large effect on wake mixing due to the passage of a weak shock. Detailed data are necessary in this case before the turbulence closure models can be adequately tested for application to combustor design and so forth.

### Transitional and Low Reynolds Number Turbulence

Transitional and low Reynolds number turbulence can be quite important in many applications, especially at low unit Reynolds numbers. Recent experience in calculations of turbulent boundary layer (refs. 9 to 12) indicates a pronounced increase in turbulent shear stress near the end of transition and beginning of turbulent flow. This high shear condition is aggravated at high Mach number and can occur for quite large Reynolds numbers $\left(R_{e,\theta} \rightarrow 10^4\right.$ at high Mach number $\left(R_{e,\theta}\right.$ is momentum-thickness Reynolds number$\left.\left.\right)\right)$. This low Reynolds number effect may also occur in free shear flows (ref. 13) and indeed may account for some of the anomalies present in the available data for these flows. In any event, a viable calculation method should have the capability of computing through transition. Again, further detailed experimental data are necessary to calibrate the calculation methods for this effect.

## Longitudinal Curvature

It is well known in boundary-layer flows that concave longitudinal curvature can significantly increase the level of turbulent shear stress (e.g., ref. 14). In addition, concave curvature can trigger embedded longitudinal Görtler vortices, even in nominally two-dimensional flows (refs. 15 and 16). Free mixing flows with longitudinal curvature occur in actual applications (e.g., Coanda effect, shear layer near attachment), and several investigators have considered this problem (e.g., refs. 17, 18, and 19), but further detailed data are needed, especially in the compressible case, before the turbulence closure models can be seriously confronted with the influences of longitudinal curvature.

## Chemical Reactions

The possibility of an increase in turbulent shear due to density and pressure fluctuation terms associated with chemical reactions has been postulated by several authors (e.g., refs. 20 and 21) and is currently under investigation theoretically (ref. 22). However, for a low speed combusting boundary-layer flow (ref. 23), calculations using a "conventional" eddy viscosity model were found to provide good prediction of the data when the temperature dependence of the mean properties was taken into account. The possibility of using "conventional" turbulence closure techniques in combusting flows must be investigated further.

## Nonparallel Flows and Confined Mixing

For the general case of nonparallel flows or confined mixing, the problem is no longer parabolic and the formation of regions of separated and secondary flow is certainl a possibility. Efficient numerical methods are becoming available to handle these cases (using the two-dimensional Navier-Stokes equations, refs. 24 to 27). The problem is one of developing adequate turbulence models to handle the three-dimensional nature of the shear flow (refs. 28, 29, and 30). Again, further detailed data are necessary.

There is also the possible effect of acoustical feedback; the paper of Glass (ref. 31) is a very chastising experience and is a warning of the sensitivity of free turbulence spreading rate not only to outside influences but to self-induced effects as well.

# MOTIVATION FOR PRESENT CONFERENCE

The motivation for holding the present conference stems from recent work at LaRC on free shear layers and jets by members of the LaRC conference committee. In the course of their research several anomalies appeared, which are included in the following list:

1. Nonunique variation of $\sigma$ (spreading rate) with Mach number for shear layers
2. Question of density difference versus density level viscosity models
3. Appearance in the literature of different predictions from the same turbulence model
4. Apparent nonuniversality of many of the available methods
5. Available turbulent boundary-layer expertise (nonsimilar scale adjustments, low Reynolds number effects) should be applied to the free-mixing problem
6. Comprehensive review and comparisons with data needed for latest models – particularly to indicate the efficacy of turbulent kinetic energy approaches

The $\sigma$ (spreading rate) variations and the problems associated therewith are discussed in paper no. 2 and also in paper no. 4. The question of density difference versus density level models is an old problem in that a model which relies on an eddy viscosity proportional to the mass flow difference across the mixing zone does not predict the correct behavior when the velocity and density are not equal across the layer, but the mass flow is (ref. 32). Therefore, the use of this model is questionable when compared with the extrapolation of boundary-layer viscosity models which are proportional to the local density.

Another problem was the appearance of different predictions in the literature where supposedly the same viscosity model was employed. Was this due to a difference in numerical techniques? If so, which result was correct? Also, several of the models (and numerical methods) were developed for a particular class of flows (e.g., coaxial jets, wakes), and the range of application of several of the available closure assumptions was therefore in some doubt.

It was felt that a confrontation of as many of the methods as possible with a broad data base would tend to resolve some of these questions. This approach is similar to that employed by Harsha (ref. 4).

## HEIRARCHY OF CLOSURE SCHEMES

A brief sketch of the various computational (closure) techniques is given below, along with their approximate representation at the present conference:

| Method | Approximate representation | Prime usage |
|---|---|---|
| Integral | 1 | Equilibrium flows |
| Differential-mean field | 6 | Near-equilibrium, nonequilibrium flows |
| Differential-mean turbulence field without length-scale equation | 6 | |
| Differential-mean turbulence field with length-scale equation | 1 | Nonequilibrium flows, especially mixing of flows with differing scales |
| Statistical fluid mechanical approaches | | To be determined; should be more accurate for a wider range of turbulent shear flows |

At the Stanford Conference (ref. 33), the integral methods outnumbered the differential approaches more than 2 to 1, whereas in the present conference practically all the methods are numerical (differential). This is probably due in part to a bias on the part of the conference committee, but it also indicates a continuing shift in the last 4 years toward more fundamental closure techniques. This was, of course, made possible by large digital computers and numerical solutions to highly nonlinear partial differential equations (ref. 34).

In the near future, for nonequilibrium and general turbulent flows the most promising methods (based on the author's experience) are probably those which include some length-scale equation in a so-called mean turbulent field approach. Several investigators are developing this type of method (refs. 35 to 39). Of these approaches, reference 39 presents the most complete study of free-mixing applications.

The last category in the heirarchy of closure methods is the possibility of applying statistical fluid mechanical approaches to the calculation of "practical" shear flows. J. R. Herring comments on this in paper no. 3, but mention should be made of recent unpublished work by Dr. A. B. Huang at Georgia Institute of Technology. His distribution function approach is essentially that of Lundgren (ref. 40) and seems to hold some promise of significantly reducing the empirical input generally involved in the development of a closure model. In this regard a quote from Kraichnan (ref. 41) seems appropriate: ". . . the variables in the direct-interaction equations are statistical averages. They can be expected to vary smoothly with their arguments and therefore can be adequately represented by relatively few numbers. At turbulent Reynolds numbers, the individual velocity fields vary jaggedly and unpredictably with distance and time and require relatively many numbers for a good description."

For compressible applications eddy-viscosity models may be the only viable methods until several questions are settled. Some of these are indicated in the following list:

1. Additional input (kinetic energy or Reynolds stress) is required (ref. 4).

2. Highly nonequilibrium flows require a length-scale equation; approximately six "constants" must be evaluated and optimized (functions of Reynolds number ?).

3. Rigorous application to compressible cases is difficult because of lack of detailed turbulence data, especially for $p'$ (pressure fluctuation) terms (which could be large).

4. Compressible application also entails additional equations for second-order correlations involving temperature fluctuations with more constants.

The most serious problem is probably the question of just what influence do the $p'$ terms (which can be quite large) have on the shear stress in compressible flows. Recent work (ref. 9) indicates that for boundary-layer flows with quite large density ratios (up to 140) the Reynolds stress can be modeled with low speed eddy-viscosity approaches once the low Reynolds number effect is recognized. Does this mean that the $p'$ terms are small even though $p'$ itself is quite large $\left(\sqrt{\overline{p_w'^2}}/\bar{p}\right.$ up to 20 percent $\left(\sqrt{\overline{p_w'^2}}\right.$ is root-mean-square wall pressure fluctuation, $\bar{p}$ is mean static pressure$\left.\left.\right)\right)$? This question requires considerable research including very difficult $p'$ correlation measurements before compressible flows can be confidently modeled by using mean turbulent field approaches.

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