THE COMPOSITE SCATTERING MODEL FOR RADAR SEA RETURN

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In this paper a composite scattering model, suitable for explaining the behavior of measured scattering cross sections of the ocean surface, will be presented. Furthermore, utilizing this scattering model, the spectrums of the small gravity, gravity-capillary, waves will be predicted for NASA/MSC, 13.3 GHz Scatterometer data.

INTRODUCTION

From the viewpoint of radar scattering at high frequencies, the ocean scene is best described by the composite surface function $\xi(x,y)$ given by

$$\xi = \xi_s + \xi_1 + \xi_2 + \xi_3 + \cdots\cdots , \qquad (1)$$

where ξ_s is the swell, ξ_1 is the sea, ξ_2 is the gravity-capillary structure, and so on. The solution of the electromagnetic scattering from the sea requires the probability density function of the height and the correlation function of each component of ξ, as well as the joint probability densities of the components of ξ. The interaction of the surface wind with each component of ξ must be formulated to study the wind dependence of the scattering cross section. However, a mathematical model of $\xi(x,y)$ as a function of surface wind velocity is not available.

Most of the ocean studies in the past have been devoted to developing models for ocean surface wave forecasts. Toward this goal low frequency gravity wave spectrums for fully developed seas for various windspeeds and fetches have been studied (ref. 1). The dependence of significant wave height, $H_{1/3}$, on surface windspeed has also been reported (ref. 2). The measurement of high frequency gravity-capillary waves has been reported in only a few instances, with the most recent investigation reported by Dobson (ref. 3).

Due to the lack of complete mathematical description of $\xi(x,y)$, the composite model for sea surface scattering will be studied using the NASA/MSC scattering cross section (σ_0) data.

SCATTERING THEORIES AND COMPARISONS

Numerous approaches have been advanced to explain scattering from rough surfaces. The three theories which have received attention and show promise of efficient interpretation of experimental data are as follows:

- The Kirchhoff method
- The small perturbation theory for slightly rough surfaces
- The composite scattering theory

In the Kirchhoff method the field scattered by rough surface is formulated according to Huygen's principle and is given by the Stratton-Chu integral. The total field and its normal derivatives are determined by tangent plane approximation on the surface. These requirements generally restrict this method to locally flat surfaces.

The small perturbation method is valid for large values of the angles of incidence and exhibits meaningful polarization dependence. It is useful in the low frequency limit and can therefore be applied to a class of slightly rough surfaces when very low frequencies are used. A comparison of theoretical and experimental results over slightly rough water surface has been given by Wright (ref. 4). As pointed out in his paper, the measured average backscattering cross sections are in good agreement with the calculated values. The depolarized return from slightly rough ocean surface has been obtained by Valenzuela (ref. 5) by using Rice's small perturbation method. A slightly different approach using the small perturbation theory has also been used for slightly rough surfaces by Bass et al. (refs. 6 and 7).

It is interesting to examine closely what parameters are needed in the formulation of the electromagnetic problem. If only the large-scale structure is considered, the tangent plane approximation can be used. With this theory σ_0 can be expressed as a function of tan β_0, the root-mean-square slope of the surface, in the high frequency limit. Only a few measurements of the sea slope distributions have been reported. The most widely used of these are the optical measurements of Schooley (ref. 8) and Cox and Munk (ref. 9). Figure 1 shows the value of β_0 as a function of windspeed. The curves C(1), C(2) have been calculated using the spectrums of Kitaigorodskii and Pierson and Moskowitz (ref. 10). The lower value of C(1) and C(2), as compared to other curves in figure 1, is attributed to the fact that these curves are not direction dependent (upwind, downwind, etc.) but involve all facets of the sea surface.

For comparison, a value of $\tan \beta_0 = 0.27$ was taken corresponding to a 20.5-knot upwind speed as given in figure 1. The value of σ_{vv} was calculated for the Gaussian height probability function. The calculated value of the scattering cross section (using the Kirchhoff method) given by

$$(\sigma_{vv})_L = \frac{\sec^4 \theta}{\tan^2 \beta_0} |R_{11}(0)|^2 \exp\left(-\frac{\tan^2 \theta}{\tan^2 \beta_0}\right) \tag{2}$$

for $\varepsilon = 55 + j30.25$ is plotted in figure 2. In equation (2), θ is the incidence angle, and $R_{11}(0)$ is the Fresnel reflection coefficient for normal incidence. Also shown in this figure are the NASA/MSC, 13.3-GHz, F4L8R1, 21-knot, forebeam data. The evident disagreement is attributed to the fact that equation (2) is a limiting solution, and only one component of the composite surface is considered.

It has often been suggested that, near the normal direction for backscattering cross sections, scattering of the optics type (Kirchhoff method) predominates. In other directions, however, the slight roughness on top of the large-scale roughness constitutes the major source of scattering. The scattering cross sections are calculated from the scattered component of the field. In view of this, Wright (ref. 11) and Guinard and Daley (ref. 12) ignore the effect of large structure to account for the scattering at higher backscattering angles. The procedure by Wright, Guinard and Daley parallels that of Rice (ref. 13), Barrick and Peake (ref. 14), and Valenzuela. For Rice's method the backscattering cross sections for a slightly rough surface using first order terms are given in reference 14.

$$(\sigma_{\gamma\delta})_s = 4\pi k_0^4 \cos^4 \theta |\alpha_{\gamma\delta}|^2 W(p,q) \; . \qquad (3)$$

In equation (3)

k_0 = wave number of the incident radar energy

θ = the incident angle

$W(p,q)$ = the roughness spectral density of the surface, and p,q are radian wave numbers

$$\alpha_{HH} = \frac{\varepsilon - 1}{\left[\cos\theta + \sqrt{\varepsilon - \sin^2\theta}\right]^2}$$

$$\alpha_{VV} = \frac{(\varepsilon - 1)[(\varepsilon - 1)\sin^2\theta + \varepsilon]}{\left[\varepsilon\cos\theta + \sqrt{\varepsilon - \sin^2\theta}\right]^2}$$

ε = the complex dielectric constant of the surface.

The scattering cross sections can be computed from equation (3) for the exponential and Gaussian surface height correlation function. For the Gaussian correlated surface, the result is

$$(\sigma_{VV})_s = 4k_0^4 h^2 \ell^2 \cos^4\theta |\alpha_{VV}|^2 \exp(-k_0^2 \ell^2 \sin^2\theta) \qquad (4)$$

where ℓ is the surface height correlation distance, and h^2 is the surface-mean-square height.

Several ocean wave spectrums have been proposed over the last few years. However, no experimental spectrums of the fine structure (the capillary waves) have been reported for different wind velocities. In the equilibrium range the isotropic spectrum is of the form

$$W(r) = kr^{-4} \qquad r = \sqrt{p^2 + q^2} \tag{5}$$

There is some uncertainty for the value of k , but Phillips (ref. 15) gives the following estimates:

6×10^{-3}	for the equilibrium range spectrum for gravity waves
1.5×10^{-2}	for the capillary wave spectrum

Evaluating at wave numbers satisfying the Bragg scattering condition (ref. 11) equation (3) yields the limiting form of the cross sections as

$$(\sigma_{\gamma\delta})_s = 1.5\pi \times 10^{-3} |\alpha_{\gamma\delta}|^2 \cot^4 \theta \tag{6}$$

In the case of the ocean, it is thought that the sea waves, the small gravity waves, and the gravity-capillary structure combined produce the scattering. The swells are assumed absent. To the first order of approximation, the composite scattering cross sections were calculated by adding the average incoherent scattering cross sections from the very rough surface (Kirchhoff method) to that of the slightly rough surface. The mathematical proof of this is given in papers of Semenov (ref. 16) and Fuks (ref. 17). The comparison of composite scattering cross section and NASA/MSC, 13.3-GHz, F4L8R1, forebeam vv data is shown in figure 3. The theoretical curve is the summation of $(\sigma_{vv})_L$ and $(\sigma_{vv})_s$ as given in equations (2) and (4) respectively. The value of the dielectric constant ε is taken as 55+j30.25 and, furthermore, $\tan \beta_0 = 0.27$. The result is encouraging. Comparisons such as shown in figure 3 made it obvious that the scattering by small gravity-capillary structure plays a significant role at higher angles.

A comparison of equation (6) with 13.3GHz, NASA/MSC data in the range of angles $20° \leq \theta \leq 50°$ showed that the angular variation of the data was approximately the same as that given in the equation. It was therefore concluded that the directional spectrums of the small gravity and gravity-capillary structure of the sea could be expressed as

$$W(r) = kr^{-k_3} \qquad (7)$$

The values of k and k_3 are wind dependent. After substitution of equation (7), and toward the goal of studying the change of spectrum as a function of wind velocity, an expression of the following form was used.

$$\sigma_0(\theta) = k_1 W_1^{k_2} |\alpha_{vv}|^2 (\cos\theta)^4 (\operatorname{cosec}\theta)^{k_3} \qquad (8)$$

In equation (8), W_1 is the wind velocity reduced to a 19.5-meter anemometer height. After using equation (8) it was found that the value of $(\cos\theta)^{4-k_3}$ (in the range of angles 20° ≤ θ ≤ 50°) could be taken as 1 for most data. Consequently, the following simplified form of equation (8) was also used:

$$\sigma_0(\theta) = k_1 W_1^{k_2} |\alpha_{vv}|^2 (\cot\theta)^{k_3} \tag{9}$$

By using algorithm 178 "direct search" from ACM communication (ref. 18), a FORTRAN program was used to find the values of k_1, k_2, and k_3. The program then searches for a minimum value. The value of dielectric constant was taken as $\varepsilon = 55+j30.25$.

Two typical sets of data will be analyzed here. One set consists of NASA/MSC Mission 119, 13.3 GHz data for vertical-transmit vertical-receive polarization combination. The upwind forebeam data, for F9L1R19 (flight 9, line 1 and run 19), F2L1R1 and F3L1R1, the corresponding surface wind velocities are 6 knots, 22.5 knots and 33 knots respectively, were processed. The values for the constants using equation (8) for this set were as follows:

$$k_1 = 0.026$$

$$k_2 = 1.324$$

$$k_3 = 5.47$$

In figure 4, the experimental and calculated data using preceding value of constants is presented. Similar results were obtained for other sets of Mission 119 data. The same set of data using equation (9) gave the following values:

$$k_1 = 0.043$$

$$k_2 = 1.33$$

$$k_3 = 5.00$$

The second set of data was chosen from NASA/MSC Mission 156 data. The data is the aft beam 13.3 GHz (vertical-transmit vertical-receive) data for upwind conditions. The F2L8R1, F5L4R5 and F6L4R2 data corresponding to 33 knots, 15 knots and 3 knots of average wind speed respectively were processed. The values for constants using equation (8) were:

$$k_1 = 0.0207$$

$$k_2 = 1.1$$

$$k_3 = 6.6$$

Figure 5 shows the comparison of the experimental and calculated data using the preceding values of the constants. It should be pointed out here that the flight 6 data was for very calm conditions with extremely low surface wind velocity.

CONCLUSION

It can be concluded that a theoretical composite model can explain the dependence of the scattering cross section on the angle of incidence θ . The spectrum of the small scale structure is found wind dependent. In general, as the wind velocity increases, the magnitude of the spectrum for high spectrum frequencies increases. The value of the constant k_3 is also a function of the radar incident wave length. Two sets of data (F7L3R1, forebeam Mission 119 data) gathered under identical conditions for a surface wind of 15.5 knots using equation (9) gave the following values for k_3

$$k_3 = 7.3 \text{ for } 0.4 \text{ GHz , and}$$

$$k_3 = 4.7 \text{ for } 13.3 \text{ GHz data .}$$

This dependence is expected since 0.4 GHz radar would be relatively insensitive to small gravity-capillary waves.

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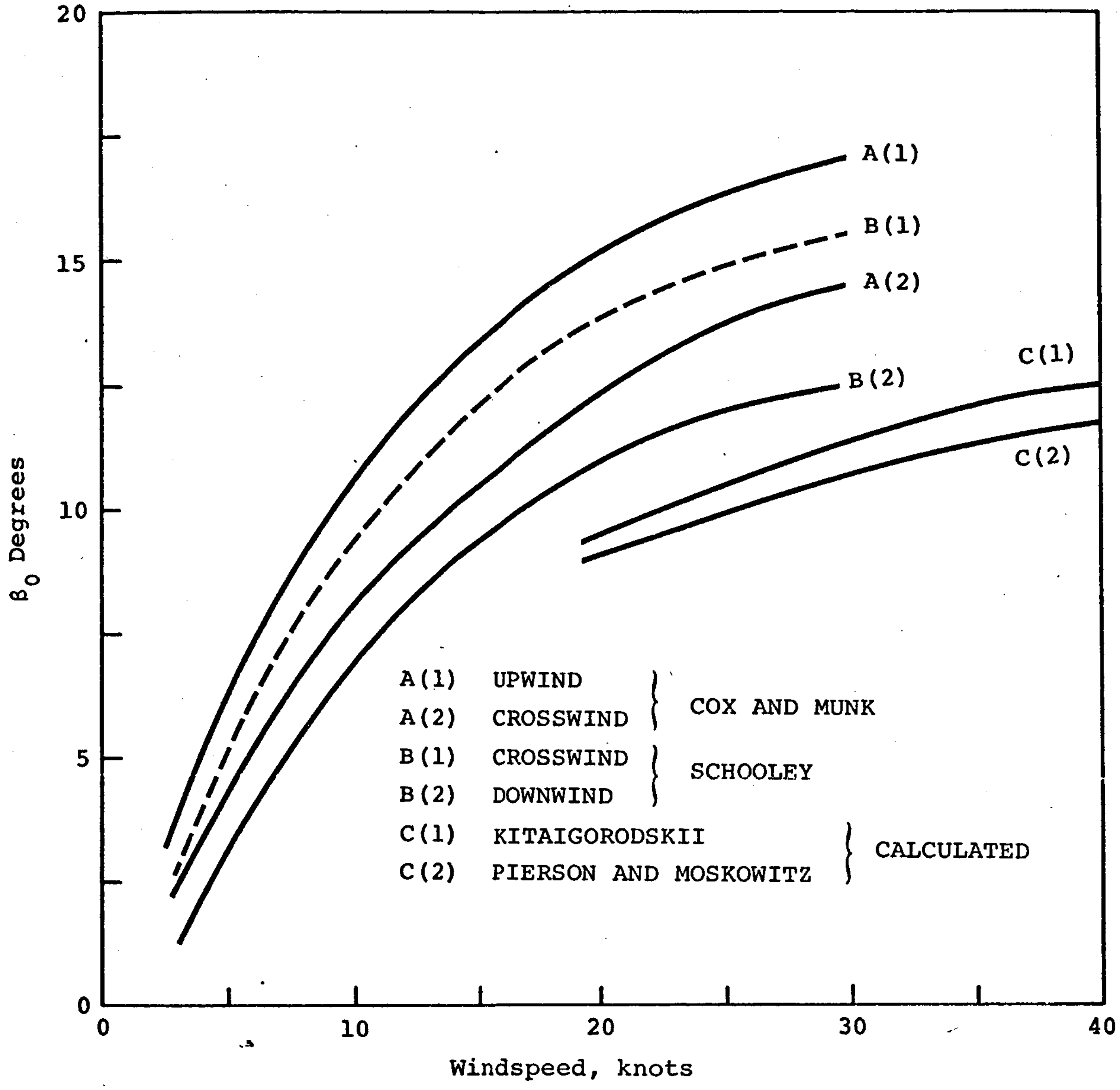


Figure 1. – Values of β_0 as a function of windspeed.

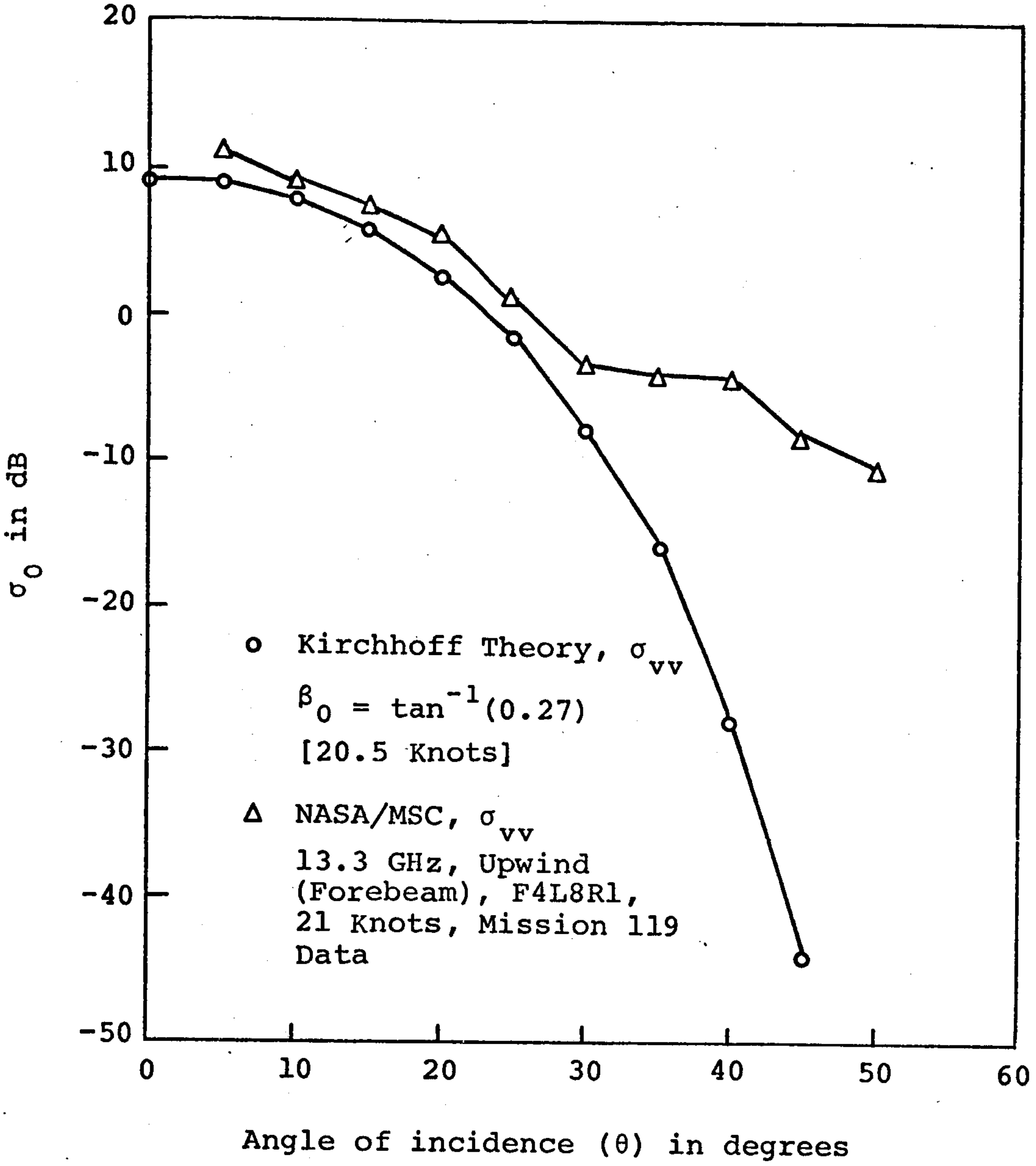
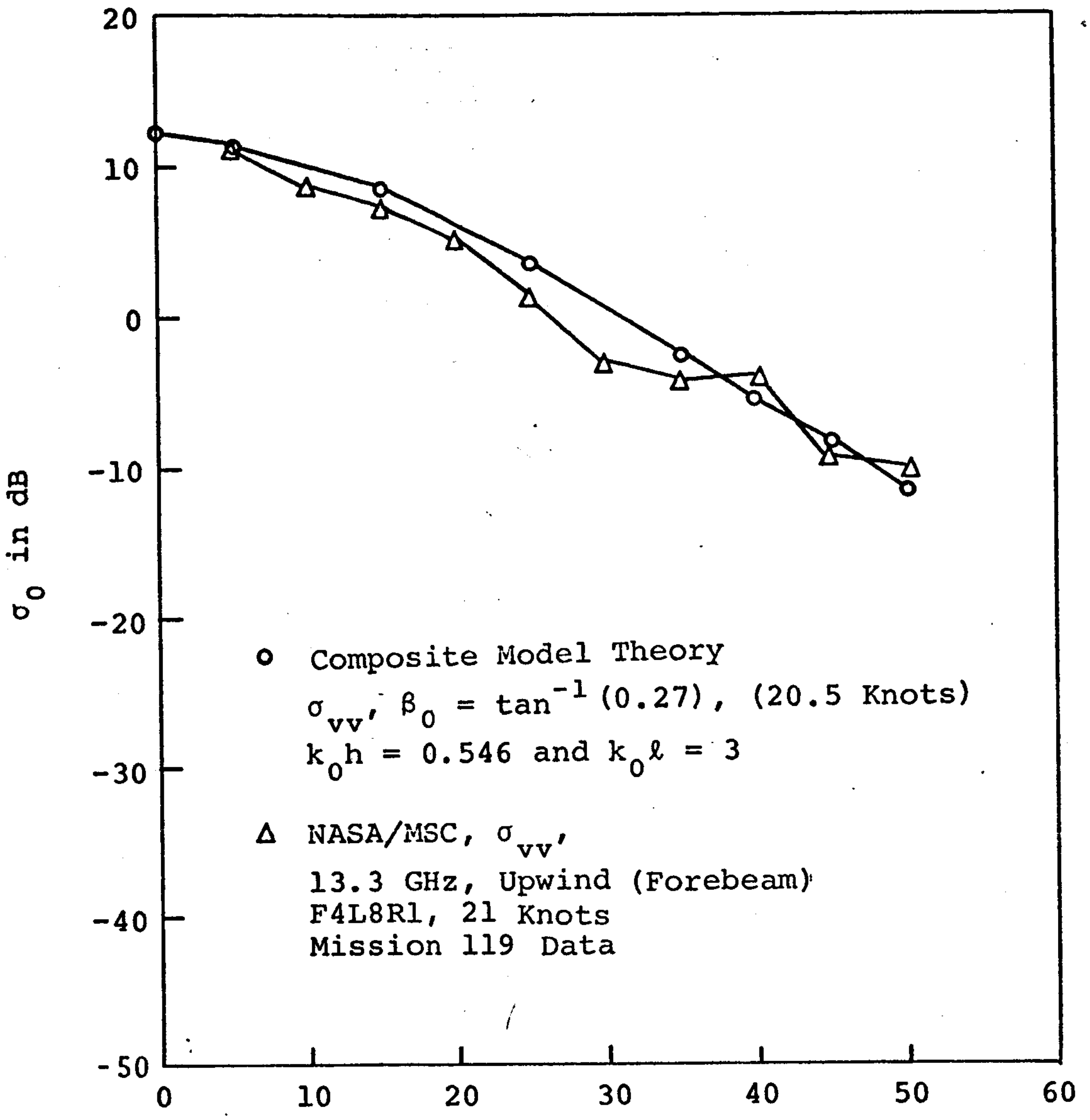


Figure 2. — Comparison of NASA/MSC, 13.3-GHz data with the Kirchhoff theory.



Angle of incidence (θ) in degrees

Figure 3. – Comparison of NASA/MSC, 13.3-GHz data with model theory.

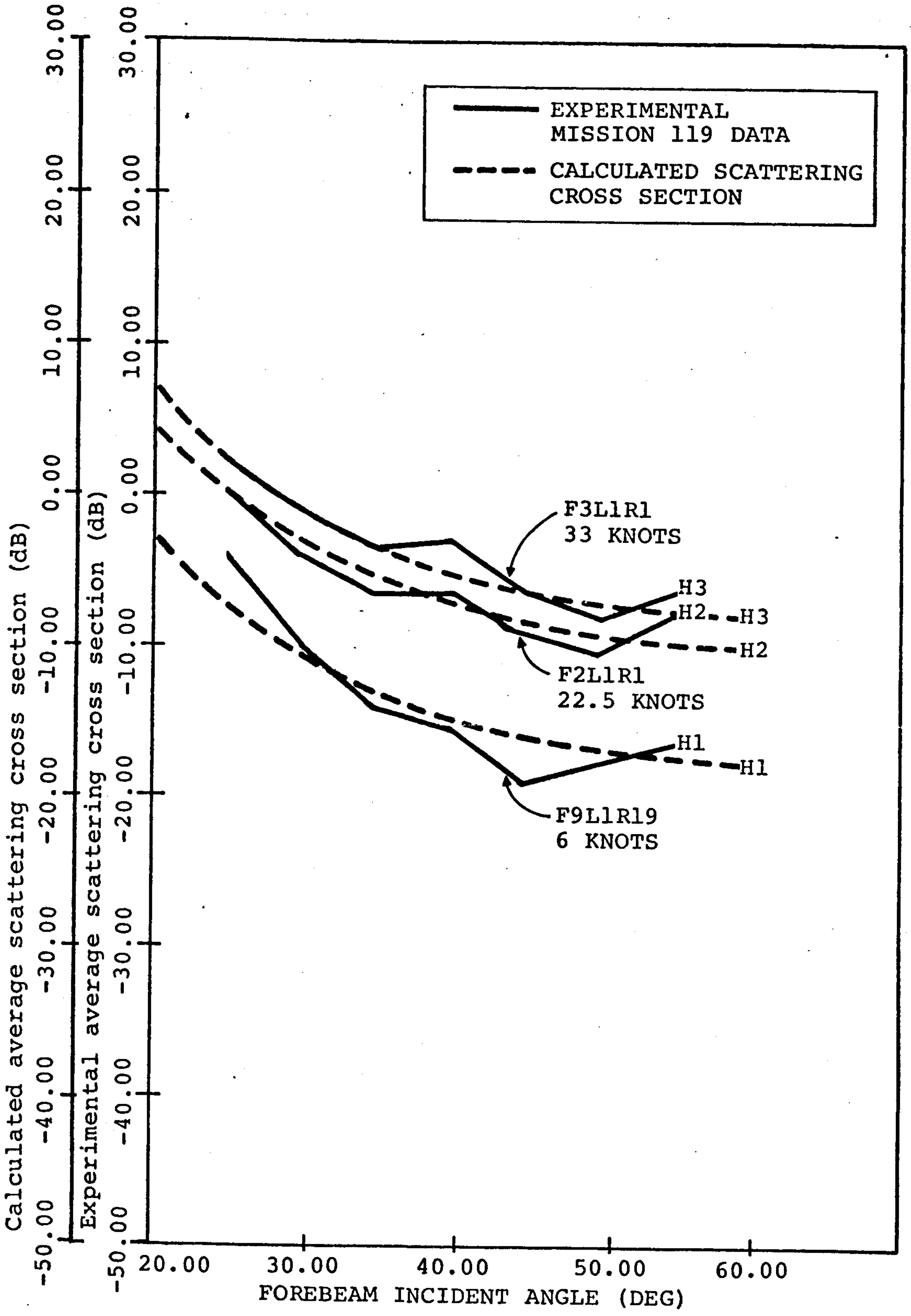


Figure 4. – Comparison of calculated and experimental scattering cross section for Mission 119.

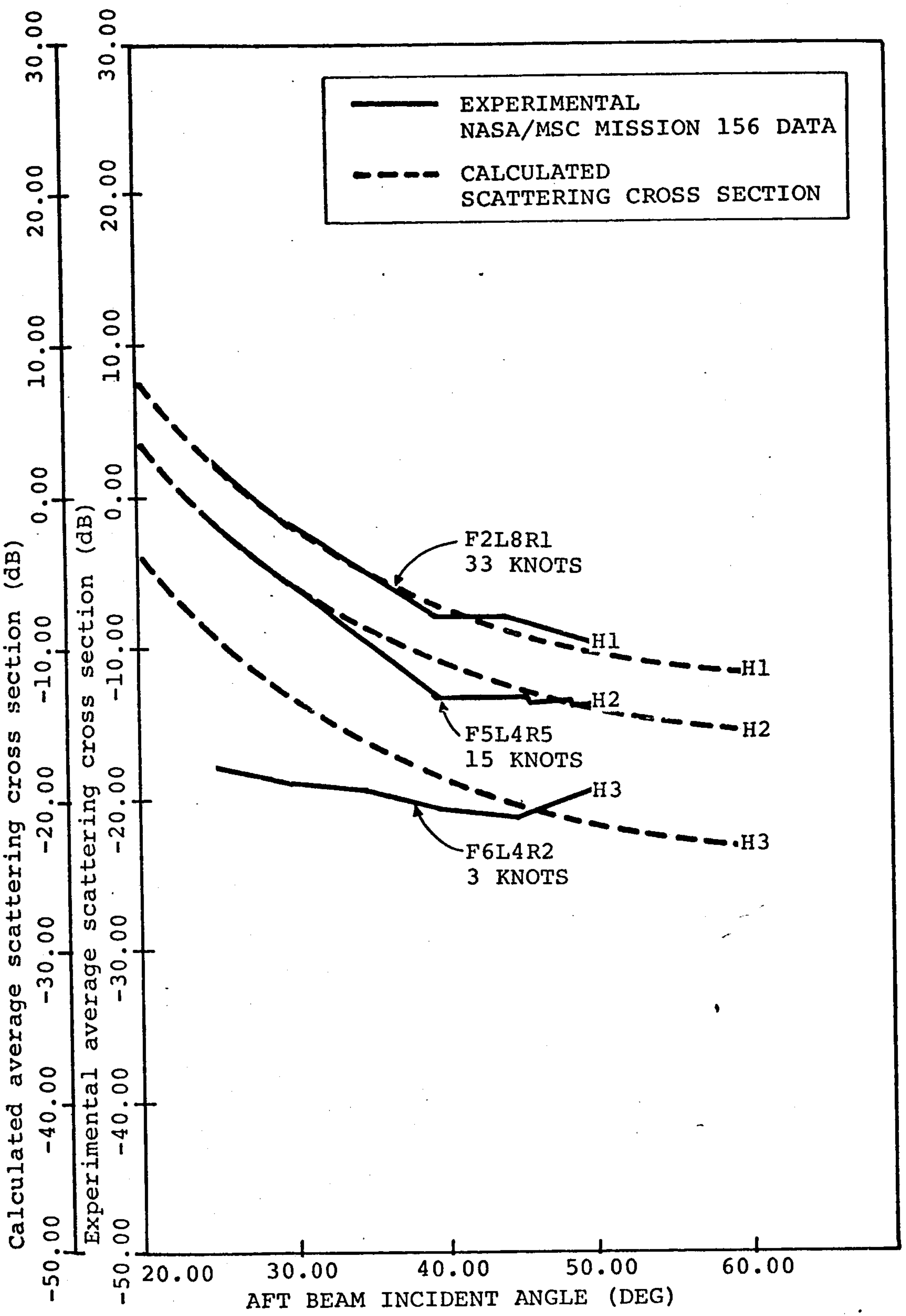


Figure 5. – Comparison of calculated and experimental Mission 156 data.