USE OF ALTIMETRY DATA IN A SAMPLING-FUNCTION APPROACH TO THE GEOID

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The planned operation of satellite-to-ocean altimeters will produce measurements that require mastery of particular data-analysis problems for the full utilization of the information in these measurements. Under the premises that the first altimeters will have an accuracy of ~ 1 m and that at this scale the ocean profile can be identified with an equipotential surface, the following problems are among those that must be examined:

1. Convenient mathematical representation of short-wavelength (eventually ~ 1°) features of the geoid or geopotential.

2. Utilization of detailed data from only part of the globe (i.e., the oceans).

3. Application of appropriate formalism to relate the sea-level equipotential below the atmospheric mass to the external potential above the atmosphere.

4. Mathematical applicability of an adopted geopotential representation on the surface of the physical geoid.

These topics are not independent, of course.

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The concept of using a sampling-function representation of the geoid and geopotential emerged from efforts to prepare for some of these problems, and the evolution of this concept can be followed in other papers (Lundquist and Giacaglia, 1969; 1971a, b; Giacaglia and Lundquist, 1971). The objective here is rather to review the current status of the sampling-function representation as a partial answer to the analysis problems posed by altimetry data.

With respect to the first problem – a convenient representation of short-wavelength features – the coefficients in an expansion in sampling functions are essentially tabular values of the geoid radius or potential at a grid of sampling points on a sphere or similar reference surface. The grid can be scaled as finely as desired. The sampling-function representation through some degree is equivalent to a spherical-harmonic expansion through the same degree, and the transformation from sampling functions to spherical harmonics and its inverse are expressed in analytical form (Lundquist and Giacaglia, 1971b). Therefore, no need arises to invert large matrices numerically, and this aspect of the altimetry problem is resolved.

In an oversimplified scenario for the treatment of altimeter data, each altitude measurement from a determinable position in orbit implies a geocentric radius to the ocean surface. All these measurements of radii in the neighborhood of a sampling point can be accumulated and averaged appropriately to give the radius at the point. This radius value is immediately the coefficient of the corresponding sampling function in the geoid representation. If the equivalent spherical-harmonic expansion is desired, this is obtainable by applying the analytically defined transformation.

Some recent progress toward implementing these calculations has been the preparation at the University of Texas of computer algorithms to evaluate the necessary analytical formulas for fairly high degree. Even though simpler than some other approaches, the calculations involved are extensive, owing to the great detail of the desired representation. In the interest of computer efficiency, the formulation of the analytical expressions and the computer algorithms have progressed through several steps of refinement.

Degree 36 has been selected for exploratory investigations, although a still higher degree might be more illuminating. In this case, features with wavelengths as short as 5° can be represented. For an expansion through degree 36, there are $(36 + 1)^2 = 1369$ terms in either a sampling-function or a spherical-harmonic expansion. The transformation matrices relating the equivalent forms have nearly two million elements.

As a trial application using the sampling points for degree 36, geocentric radii were calculated to an equipotential surface derived by use of the Smithsonian harmonic coefficients presented at the 1971 IUGG meeting (Gaposchkin, Kozai, Veis, and Weiffenbach, 1971). This calculation at the University of Texas followed the procedure discussed by Lundquist and Giacaglia (1971a). Also, geocentric radii were calculated (Girnius, 1971) for 45 sampling points in the North American Datum, by use of the Army Map Service 1967 Map of Geoid Contours in North America from Astrogeodetic Deflections (Fischer, 1966). Figure 1 shows the 45 sampling points. The geoid heights were transformed to geocentric radii in 1969 Smithsonian Standard Earth (II) coordinates by using the Lambeck (1971) parameters, assuming the Smithsonian and North American Datum axes are parallel.

The radius values from the astrogeodetic geoid could contain somewhat shorter wavelength information than the values from the Standard Earth. To generate a sampling-function representation corresponding to the astrogeodetic geoid in North America, it is only necessary to replace the Smithsonian values with those from the geoid map for the sampling points in North America. This has been done. If one wants the equivalent spherical-harmonic representation, the analytically defined linear transformation can be applied.

Because a very similar operation is envisioned when satellite-to-ocean altitudes are available, a study of the properties of this modified geoid representation should indicate the utility of this method. Such a study is in progress. Partial answers to both problems 1 and 2 are expected as a result of the trial application to the North American geoid, since this test involves features of both problems.

For problem 1, a crucial aspect is the ability of the sampling-function representation to reproduce short-wavelength features in North America. For problem 2, the crucial question is whether extraneous short-wave detail is introduced with significant amplitude for the geoid outside North America. The desired result should be a geoid in North America resembling the astrogeodetic contours in its 5° and longer wavelength features, with the properties of the satellite-determined field elsewhere. Also, the corresponding geopotential should have essentially the Smithsonian coefficients for the lower degree and order spherical harmonics. An iterative scheme may be necessary to achieve these properties.

The discussion and procedures above have been based on the implicit assumption that the geopotential derived from satellite observations is also applicable at the surface of the earth. While this is an acceptable simplification for exploratory studies, it certainly must be reconsidered for accurate treatment of actual altitude measurements. Problems 3 and 4 recognize the need to proceed with caution.

The mass of the atmosphere is given by Verniani (1966) as 8.594×10^{-7} of the mass of the earth. Clearly this mass contributes differently to the gravitational field at satellite altitudes than it does at sea level. The first step to accommodate this situation would seem to be a decomposition of the external potential into a major portion due to the mass of the solid earth and oceans and a minor portion due to the mass of the atmosphere.

The leading term in the usual spherical-harmonic expansion is proportional to total mass, so that its coefficient can be decomposed into two fractions – respectively, 0.999, 999, 140, 6 and 0.000, 000, 859, 4 of the total. Such an adjustment was made by Veis (1967) in a determination of the equatorial radius and gravity of the earth. This effect was noted also by Rapp (1970) in a discussion of methods for the computation of geoid undulations from potential coefficients and by others in other contexts (Ecker, 1968; Ecker and Mittermayer, 1969).

Since the atmosphere is constrained to a nearly ellipsoidal lower boundary by the shape of the solid earth and oceans, its mass must make a contribution to the total J_2 of the earth. A first crude estimate of the size of this contribution is obtained by considering the total mass of the atmosphere concentrated in a uniform ellipsoidal shell with the same semimajor axes as the earth. This crude estimate gives J_2 (atmosphere) = 0.002×10^{-6} as compared with the Kozai value $J_2 = (1082.637 \pm 0.001) \times 10^{-6}$ for the total earth system. Thus, the contribution of the solid earth and of the oceans would be J_2 (solid earth and oceans) = 1082.635×10^{-6}. This very small change would not seem to be important until geoid accuracies in the centimeters are obtained.

On the other hand, Kozai reports an annual variation of amplitude $\delta J_2 = 0.0013 \times 10^{-6}$, presumably due to mass displacements somewhere in the earth system (Kozai, 1970). A more accurate calculation of the atmospheric contribution to J_2 would be instructive, to improve the crude estimate above. Kelly (1971) has assembled the atmospheric models and formulas for such a calculation.

In principle, there is a further complication associated with the atmosphere — namely, the gravitational field at sea level due to the nearly elliptical atmospheric shell above. This contribution should be added back into the potential after the external atmospheric contribution has been subtracted from satellite information to isolate the field due to the solid earth and oceans. However, this internal field of the atmosphere is probably even less important than correction of the J_2 value.

The fourth problem, the mathematical applicability of an adopted geopotential representation at sea level, is a perplexing one in potential theory (see, for example, Hotine, 1969; Madden, 1971). It has been argued that the convergence uncertainties expected with a spherical-harmonic expansion could be largely alleviated by the use of ellipsoidal harmonics (see, for example, Madden, 1968; Walter, 1971), presumably because the ellipsoidal functions can better conform to the shape of the earth.

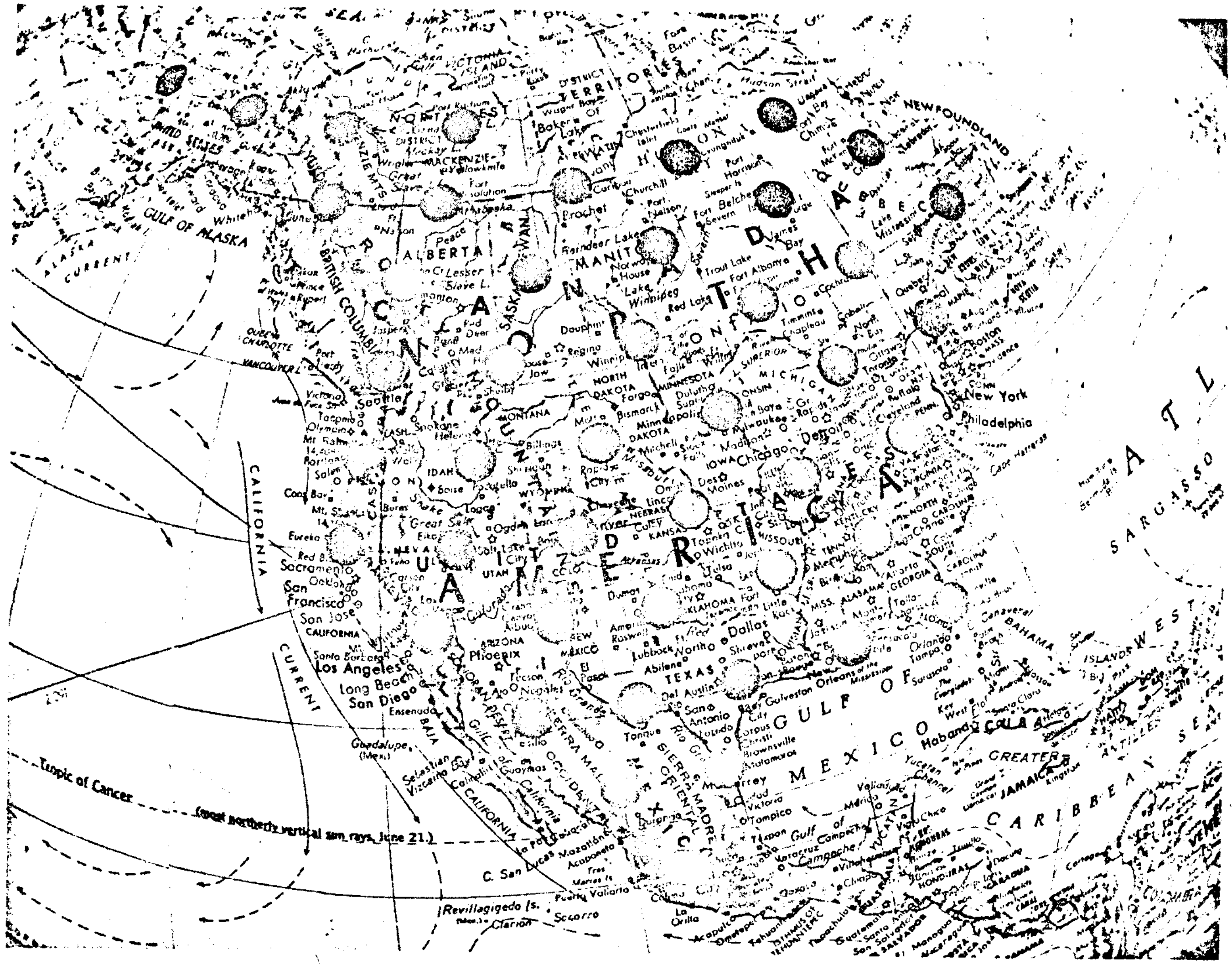

Figure 1. The 45 sampling points in North America used in a trial application of sampling-point representation of the geoid.

The sampling functions can be defined on an ellipse about as easily as on a reference sphere, and if the elliptical formulation is used, it would seem that they should accrue the same benefits as ellipsoidal harmonics. Still further, the sampling functions can also be defined on a surface conforming still more closely to the geoid. It is an open question whether this would still further alleviate the convergence uncertainty.

In summary, although many questions remain to be answered, a sampling-function representation of the geoid still promises to be a useful tool in utilizing satellite-to-ocean altitudes.

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