A GAS–DYNAMIC CALCULATION OF TYPE II SHOCK PROPAGATION THROUGH THE CORONA

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ABSTRACT An approximate analytic theory of acoustic shock propagation in nonuniform media is used to determine the motion of a flare-generated shock wave in the corona. The shock is followed from the time it strikes the chromosphere-corona transition region (density interface) out to $5 R_{\odot}$ under the assumption that the corona in this region is approximately in hydrostatic equilibrium. In the actual corona, the situation would apply to the case of slow-mode shock propagation along a radially diverging magnetic field.

The strength of the shock incident on the transition region from below determines the ejection velocity of eruptive prominence material, as well as the initial velocity of the coronal shock. The calculation is applied to one well-documented case of a related flare spray, moving type IV isolated source, and type II burst. It is shown that a chromospheric shock of the appropriate strength to produce the observed prominence and type IV velocities strengthens as it moves out in the corona by an amount sufficient to account for the observed high velocity of the type II burst.

INTRODUCTION

It is now regarded as a well-established fact that type II solar radio bursts are the direct result of flare-generated shock waves traveling outward through the corona [*Wild*, 1969, 1970]. Whether such waves are primarily acoustic (slow mode) or magnetohydrodynamic (fast mode) in character, however, is still an open question. Many workers have relied on magnetic effects to explain certain features of the bursts, such as band-splitting [*Sturrock*, 1961], although nonmagnetic explanations are frequently available as well [*McLean*, 1967].

If the waves are acoustic in nature, then rather large shock Mach numbers are required to produce the observed burst velocities of the order of 1000 km/sec [*Weiss*, 1965]. Nevertheless, at least low in the corona the material motions following a flare, as evidenced by the underlying eruptive prominence (or so-called "flare spray"), appear to be largely channeled along magnetic lines of force. Observational evidence for magnetic fields influencing the motion of a type II source at higher levels has been presented by *Kai* [1969]. However, at least in some cases shock propagation must occur across field lines if one is to explain the Moreton-wave phenomenon as the intersection of a type II wave front with the chromosphere [*Wild*, 1969; *Uchida*, 1970].

Occasionally associated with a type II burst and prominence eruption, but occurring later in time and having a much longer duration, is a moving burst of type IV. *Smerd and Dulk* [1970] distinguish three subclasses of type IV: the isolated source, the expanding arch, and the advancing front. Here we are concerned only with the first of these, for which it appears that the source moves outward along predominantly open magnetic field lines [*Smerd and Dulk*, 1970; *Dulk and Altschuler*, 1971].

Type IV bursts of the "isolated source" variety are relatively infrequent; *Smerd and Dulk* [1970] have identified possibly eight examples observed with the 80-MHz radioheliograph during the period February

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1968 to April 1970. Particularly well observed was the event of March 21, 1970, reported by *Sheridan* [1970]: Following a flare of importance 2N near the east limb at $00^h\,23^m$, a large eruptive prominence was seen in $H\alpha$ to leave the flare location with a velocity of 260 km/sec. It reached a height of $0.3\,R_\odot$ above the photosphere before fading from view. Simultaneously, radiospectrograph records indicated a type II burst with a velocity of order 1500 km/sec [*Smerd and Dulk,* 1970]. Type II bursts are frequently observed to occur in pairs after large flares, presumably the result of multiple shocks being emitted by the flaring region. This was the case with the event being described. We refer here to the second, and stronger, burst.) A short time after the prominence faded, a type IV source appeared at $r = 1.9\,R_\odot$ and subsequently moved outward to a distance of more than $5\,R_\odot$ with a constant 290-km/sec velocity, whereupon it fragmented and slowly faded away. The source velocity was comparable to that measured for the prominence eruption; extrapolation backwards in time showed that both left the sun during the flash phase of the flare.

Sheridan [1970] suggested that the outward-moving prominence material excited the type IV source near the $r = 1.9\,R_\odot$ level. However, such an interpretation does not account for the nearly equal velocities of the source and the prominence ejecta. Alternatively, we may conjecture that the flare spray and the type IV source are simply different manifestations of the same material, the ejected "blob" becoming visible at 80 MHz only after the prominence fades in $H\alpha$ and rises significantly above the 80-MHz plasma level in the corona.

Here we show, by means of a simple model, that a relationship exists between the velocities of the eruptive prominence and moving type IV isolated source (here assumed to be the same) and that of the type II burst. We postulate that these three phenomena are the natural consequence of a single flare-initiated shock traversing the solar atmosphere. In the model, the velocity of the flare spray is taken as a measure of the strength of the shock at the top of the chromosphere. With this information and knowledge of the coronal density structure, the shock strength and velocity may then be calculated as functions of height in the corona. As an example, we apply the calculation to the event observed by *Sheridan* [1970].

SHOCK-WAVE CALCULATIONS

The shock wave emitted by the flare during its flash phase is assumed to propagate vertically upward, or nearly so, through the chromosphere. Here we will be concerned with its motion only after the shock reaches the "top" of the chromosphere. By that time we assume it to be a slow mode shock traveling along the magnetic field.

The atmosphere ahead of the shock is characterized by large gradients of temperature and density. It is convenient to divide the calculation of the subsequent shock motion into two parts, corresponding roughly to regions with large or small temperature gradients.

Shock Interaction with the Transition Region

The shock immediately encounters the chromosphere-corona transition region, where the preshock atmospheric density drops abruptly by more than 2 orders of magnitude over a radial distance of less than a scale height. We idealize this region to be a sharp density interface (infinite temperature gradient), with equal pressures above and below.

The interaction of a slow-mode shock with a density interface is shown schematically by an *(x, t)* diagram in figure 1. Regions 0 and 3 represent the undisturbed chromosphere and corona, respectively, separated by the contact discontinuity CC'. When the chromospheric

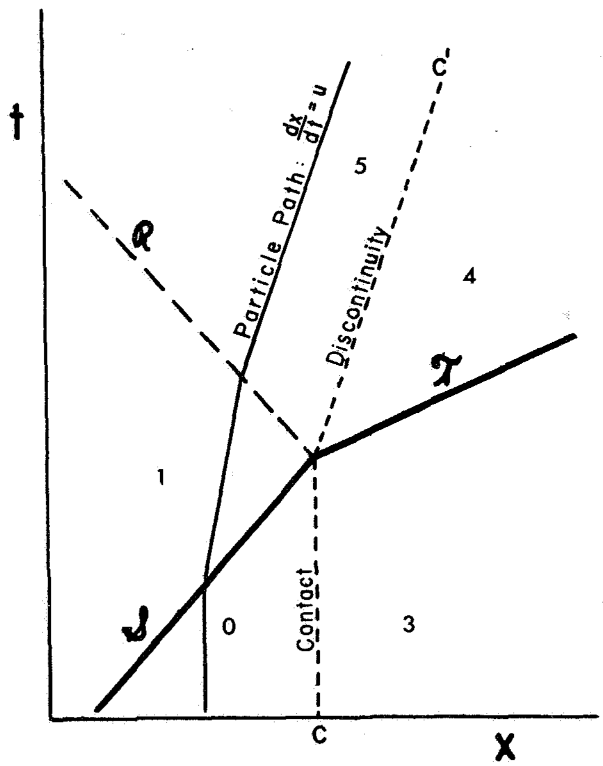

Figure 1. *A distance-time (x,t) diagram of a plane shock wave, $\mathcal{J}$, impinging normally upon a contact discontinuity CC'. The case shown here is that of the shock entering a region of lower density, for which (assuming equal specific heat ratios) the reflected wave, $\mathcal{R}$, is a rarefaction.*

shock $\mathscr{I}$ strikes the interface, it sets the latter in motion. At the same time a transmitted shock $\mathscr{T}$ and a reflected rarefaction wave $\mathscr{R}$ propagate away from the contact surface in the directions shown. Region 1 indicates the shocked chromospheric gas, region 4 the shocked coronal gas, and region 5 the shocked chromospheric gas, which has been somewhat recooled by the downward-propagating rarefaction wave.

In this picture the *Hα* prominence eruption is identified with chromospheric material immediately below the ascending interface (region 5); the velocity of the interface v_5 corresponds to that of the prominence.

This shock wave-density interface model of a flare spray is similar to that proposed by *Osterbrock* [1961] for spicules, except that a much stronger disturbance is involved here. There are six nontrivial, independent algebraic relations between the state variables in the five regions of the *(x, t)* diagram (the trivial ones being $p_0 = p_3$, $v_0 = v_3 = 0$, $p_4 = p_5$, and $v_4 = v_5$). Apart from differences in notation, they have been given by Osterbrock. Together with the known properties of the undisturbed chromosphere and corona, these relations completely determine the shock-interface interaction in terms of the strength of the incident shock. Solution by iteration is straightforward.

As a numerical example, we assume a chromospheric temperature $T_0 = 7\times10^3$ °K, a coronal temperature $T_3 = 1.5\times10^6$ °K, and a 10:1 hydrogen-helium ratio (mean molecular weight $\mu = 0.609$). The specific heat ratio γ is taken to be 5/3 on both sides of the interface. Table 1 shows the results of a calculation in which the strength of the chromospheric shock was adjusted to obtain an interface velocity of about 290 km/sec corresponding to the eruptive prominence-moving type IV source velocity of the event of March 21, 1970 [*Sheridan*, 1970].

Shock Propagation in the Corona

As the transmitted coronal shock moves away from the sun it traverses a medium with continuously varying properties, which cause additional changes in its strength. To describe the motion of the shock in the corona we use the method of *Whitham* [1958], which was applied to the case of a spherical shock in a gravitational atmosphere by *Kopp* [1968]. For a corona in hydrostatic equilibrium this theory yields the following total differential equation for shock Mach number M:

$$\frac{1}{M}\frac{dM}{dr} = \eta\left\{\frac{1}{\xi+\zeta}\left[(\zeta^2 + \xi\zeta - 1)\frac{g(r)}{a^2} - \xi\zeta\frac{1}{A}\frac{dA}{dr}\right] - \xi\frac{1}{a}\frac{da}{dr}\right\} \tag{1}$$

In this equation, r denotes distance from the center of the sun, $g(r) = GM_{\odot}/r^2$ is the gravitational acceleration, $a = (\gamma RT/\mu)^{1/2}$ is the velocity of sound, and $(1/A)/(dA/dr)$ is the rate of geometric spreading of the wave. In addition,

$$\xi = \frac{2}{\gamma+1}\left(M - \frac{1}{M}\right) = \frac{\text{postshock gas velocity}}{\text{preshock sound velocity}}$$

$$\zeta = \left[\frac{(\gamma-1)M^2 + 2}{(\gamma+1)M^2}\,\phi\right]^{1/2} = \frac{\text{postshock sound velocity}}{\text{preshock sound velocity}}$$

Table 1. *Shock-density interface calculation*

Preshock	Chromosphere	$T_0 = 7\times10^3$ °K	
	Corona	$T_3 = 1.5\times10^6$ °K	
Flare spray	$v_5 = 288.1$ km/sec		
	$T_5 = 1.10\times10^5$ °K		
	$\rho_{spray}/\rho_{chromosphere} = 0.47$		(ρ = density)
Chromospheric shock Mach number		M = 14.26	(v = 179.8 km/sec)
Coronal shock Mach number		M = 2.48	(v = 457.8 km/sec)

and

$$\eta = \frac{\gamma + 1}{2} \left(\frac{2\zeta M^2}{\phi} + \frac{1 + M^2}{M} \right)^{-1}$$

where

$$\phi = \frac{2\gamma}{\gamma + 1} M^2 - \frac{\gamma - 1}{\gamma + 1} = \frac{\text{postshock pressure}}{\text{preshock pressure}}$$

Since $\xi > 0$ and $\zeta > 1$, we have that $\zeta^2 + \xi\zeta \ 1 > 0$, so that the first term in equation (1) corresponds to a strengthening of the shock as it moves into regions of lower density. Similarly, the second term yields a weakening of the wave due to geometric attenuation, and the third term indicates the effect of a coronal temperature gradient on the shock.

Equation (1) predicts changes in shock strength resulting only from nonuniform properties of the pre-shock gas. In addition, changes occur in response to disturbances overtaking the shock from the rear. These include (1) re-reflected waves generated by the changing shock strength itself, and (2) waves produced by variations in the motions of the shock "driver." The basis of *Whitham's* [1958] method is that the first of these disturbances can be neglected in an approximate description of the shock motion. For the coronal shock under consideration here, we suggest that the "driver" waves can be neglected as well, since the observed constancy of the type IV velocity implies a continuously driven type II shock.

Equation (1) can be numerically integrated for a corona of known temperature and for a shock of known geometrical spreading. Again, as an example, we adopt a constant coronal temperature of 1.5×10^6 °K ($a = 184.6$ km/sec). Using the transmitted shock strength, from the interface calculation, $M_0 = 2.48$, as an initial condition, and assuming a slow-mode shock channeled along a radially diverging magnetic field $[(1/A)(dA/dr) = 2/r]$, we obtain the radial dependence of shock speed shown in figure 2. Then the path of the shock in the (r, t) diagram can be found from

$$t(r) = a^{-1} \int_{R_\odot}^{r} \frac{dr}{M(r)} \qquad (2)$$

where $t = 0$ corresponds to the time when the shock leaves the transition region ($r = R_\odot$). The shock path for the above example is shown in figure 3.

The most striking aspect of this calculation is the rapid strengthening of the shock below about $2 R_\odot$. Beyond $r = 2 R_\odot$ the shock speed varies by only about 20

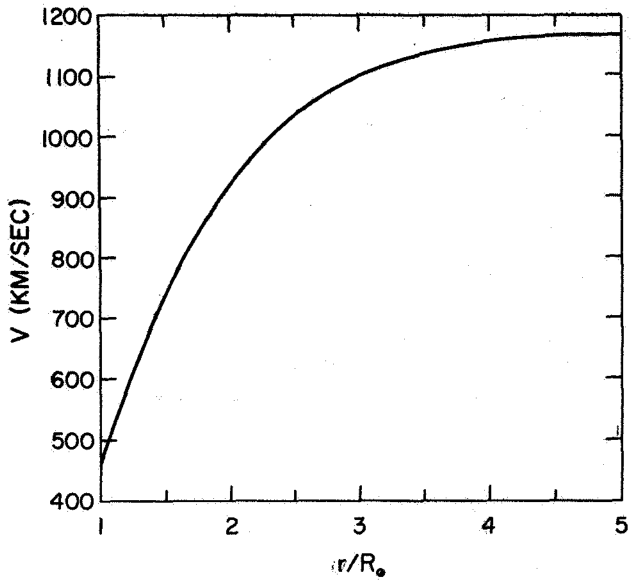

Figure 2. *Shock velocity versus height in a 1.5×10^6 °K corona, assuming a radially diverging geometry and a driven shock.*

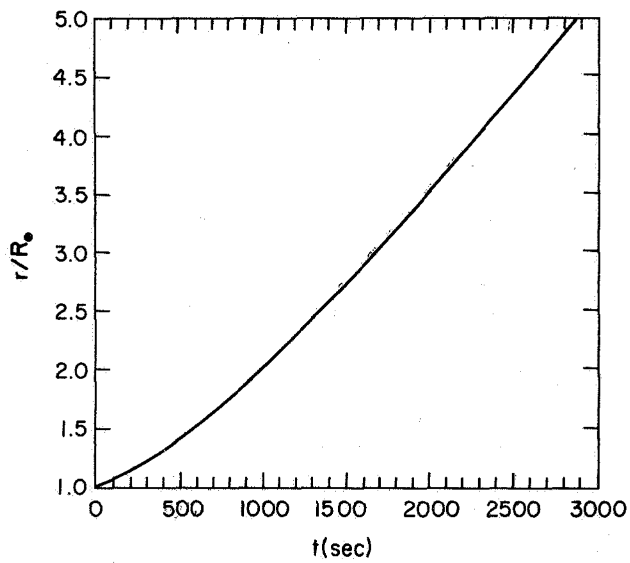

Figure 3. *Shock path r(t) corresponding to figure 2.*

percent of its maximum value of 1165 km/sec, which occurs near $4.8 R_\odot$. Evidently, the gravitational and area terms in equation (1) nearly balance each other over a rather broad region.

Considering the uncertainties inherent in deducing type II source velocities from dynamic spectra [*Zaitsev*, 1969; *Van Bueren and Kuperus*, 1970], the maximum shock speed obtained by this calculation is in reasonable agreement with the observed type II velocity for the event of March 21, 1970 [*Smerd and Dulk,* 1970; cf. previous section].

DISCUSSION

We have seen that the eruptive prominence, moving type IV isolated source, and type II burst associated with a particular flare may all be the result of a flare-initiated shock wave traveling outward along the magnetic field. When the shock strikes the transition region, the chromospheric material set in motion by the shock continues to rise into the corona and initially becomes visible as the ascending $H\alpha$ prominence, in the manner proposed by *Osterbrock* [1961]. Later, as the $H\alpha$ emission fades and the material passes the 80-MHz plasma level in the corona, it appears as a moving source of type IV emission.

The major predictions of the shock-interface calculation are that:

1. A strong slow-mode shock is required in the upper chromosphere to produce a flare spray of the observed velocity. Such a shock could result either from intense heating in the flaring region or from rapid growth of a weak flare-initiated shock as it runs outward through chromospheric layers of successively lower density.
2. The kinetic temperature of the rising prominence ($\sim 10^5$ °K) greatly exceeds that of the chromosphere, whereas the density is somewhat less. This may account for the eventual disappearance of the spray in $H\alpha$ as a new statistical equilibrium becomes established at the higher temperature.
3. The transmitted shock is of intermediate strength. In particular, its velocity is too low to match that of a type II burst.

The flare-excited shock continues into the corona, where the outward-decreasing density leads to its restrengthening and acceleration to typical type II velocities. The observed constancy of the type IV source velocity out to distances of several solar radii lends credence to the postulate that the type II shock is steadily driven from below.

The derived shock speeds depend, of course, on the assumed values of the coronal temperature and the geometrical spreading factor. For example, a higher coronal temperature would decrease the initial rate of strengthening of the shock through the "scale-height" term in equation (1). Moreover, the shock Mach number would then reach its maximum value closer to the sun. In spite of the increased speed of sound in the corona, the result would be a lower shock velocity.

It would be extremely interesting to follow the motion of such a shock to large distances from the sun ($r > 5 R_\odot$), as it is possible that type II shocks may be detected as collisionless interplanetary shock waves at 1 AU. Such a calculation is tractable at the present time, although it requires explicit consideration of the expansion state of the preshock atmosphere.

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REFERENCES

Dulk, G. A.; and Altschuler, M.: A Moving Type IV Radio Burst and Its Relation to the Coronal Magnetic Field. Submitted to *Solar Phys.*, 1971.

Kai, K.: Radio Evidence of Directive Shock-Wave Propagation in the Solar Corona. *Solar Phys.*, Vol. 10, 1969, p. 460.

Kopp, R. A.: The Dynamical Structure of a Shock-Heated Corona. Ph.D. Thesis, Harvard University, 1968.

McLean, D. J.: Band Splitting in Type II Solar Radio Bursts. *Proc. Astron. Soc. Australia,* Vol. 1, 1967, p. 47.

Osterbrock, D. E.: The Heating of the Solar Chromosphere, Plages, and Corona by Magnetohydrodynamic Waves. *Astrophys. J.,* Vol. 134, 1961, p. 347.

Sheridan, K. V.: 80 MHz Observations of the Movement of a Type IV Source Out to 6 $R_\odot$ from the Centre of the Sun. *Proc. Astron. Soc. Austr.,* Vol. 1, 1970. p. 376.

Smerd, S. F.; and Dulk, G. A.: 80-MHz Radioheliograph Evidence on Moving Type IV Bursts and Coronal Magnetic Fields. IAU Symposium No. 43 (Solar Magnetic Fields, Paris, Aug. 31-Sept. 4, 1970).

Sturrock, P. A.: Spectral Characteristics of Type II Solar Radio Bursts. *Nature,* Vol. 192, 1961, p. 58.

Uchida, Y.: Diagnosis of Coronal Magnetic Structure by Flare-Associated Hydromagnetic Disturbances. *Publ. Astron. Soc. Jap.,* Vol. 22, 1970. p. 341.

Van Bueren, H. G.; and Kuperus, M.: On the Determination of the Velocity of the Exciters of Type II Solar Radiobursts. *Solar Phys.,* Vol. 14, 1970, p. 208.

Weiss, A. A.: The Nature and Velocity of the Sources of Type II Solar Radio Bursts. *Aust. J. Phys.,* Vol. 18, 1965, p. 167.

Whitham, G. B.: On the Propagation of Shock Waves Through Regions of Non-Uniform Area or Flow. *J. Fluid Mech.*, Vol. 4, 1958, p. 337.

Wild, J. P.: Radio Evidence of Instabilities and Shock Waves in the Solar Corona, in *Plasma Instabilities in Astrophysics,* edited by D. G. Wentzel and D. A. Tidman, Gordon and Breach, New York, 1969, p. 119.

Wild, J. P.: Some Investigations of the Solar Corona: The First Two Years of Observation with the Culgoora Radioheliograph. *Proc. Astron. Soc. Austr.*, Vol. 1, 1970, p. 365.

Zaitsev, V. V.: Parameters for Shock Waves Generating Type II Solar Radio Bursts and for Coronal Magnetic Fields. *Soviet Astron.-AJ,* Vol. 12, 1969, p. 610.