TRANSITION FROM A SUPERSONIC TO A SUBSONIC SOLAR WIND

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ABSTRACT The transition from a supersonic to a subsonic corona was investigated by increasing the density N_o at the base of the corona (initially $N_o = 9.3 \times 10^7/\text{cm}^3$) while keeping the temperature T_o there constant ($T_o = 2.1 \times 10^6$ °K). For the initial values of N_o and T_o, the solution of the inviscid solar wind equations is of the Parker type. As N_o is increased, Parker type supersonic solutions cease to exist for $N_o = N_o^p \sim 1.17 \times 10^8/\text{cm}^3$. No subsonic solutions exist, however, for $N_o < N_o^s$ where $N_o^s > 3 \times 10^8/\text{cm}^3$. For $N_o^p < N_o < N_o^s$ the solar wind equations permit supersonic solutions which are not of the Parker type, with the temperature varying as $(1/r)^{4/3}$ for large distances.

The transition region separating supersonic expansions of the solar wind [*Parker*, 1958] from subsonic expansions [*Chamberlain*, 1961] has been considered by *Parker* [1965]. Here we study this transition region for a given value of the temperature at the base of the corona. *Chamberlain's* notation [1961] will be used throughout this paper; that is, the dimensionless values of the temperature, square of the velocity, and the inverse of the radial distance will be defined as

$$\tau = T/T_1 \qquad \psi = mw^2/kT_1 \qquad \lambda = GM_\odot m/kT_1 r \quad (1)$$

where T_1 is a reference temperature taken equal to 2×10^6 °K, w the radial expansion velocity, m the average mass (for a hydrogen-helium mixture 10:1), $M_\odot$ the mass of the sun, and G and k are the gravitational and Boltzmann constants, respectively. The usual momentum and energy equation can then be written as

$$\frac{d\psi}{d\lambda} = \frac{1 - 2\tau/\lambda - d\tau/d\lambda}{0.5(1 - \tau/\psi)} \qquad (2a)$$

$$\frac{d\tau}{d\lambda} = \frac{\epsilon_\infty - \frac{1}{2}\psi + \lambda - \frac{5}{2}\tau}{0.5\, A\tau^{5/2}} \qquad (2b)$$

For A we take [*Chamberlain*, 1961]: $A = 5.8 \times 10^6/C$ where C is the mass flow given in terms of N, ψ, and λ by $C = N\psi^{1/2}\lambda^{-2}$ (N is the total particle density). In equation (2b) ϵ_∞ is the residual energy per particle at infinity. For $\epsilon_\infty \neq 0$ a well-known solution of equations (2a) and (2b) is the Parker supersonic solution. Then for small values of λ

$$\tau = D_o\lambda^{2/7}\left(1 + \frac{77}{9}\frac{\lambda^{2/7}}{AD_o^{5/2}} + \ldots\right) \qquad (3)$$

The asymptotic expansion for ψ is obtained from equation (2b). The parameter D_o in equation (3) is left undetermined; this allows equations (2) to be solved as follows. Given C and ϵ_∞, the integration of (2) and (3) is started from small values of λ

$$\left(\frac{77}{9}\frac{\lambda^{2/7}}{AD_o^{5/2}} \ll 1\right)$$

toward the sun; the condition that the numerator and denominator of equation (2a)

$$\phi_1 = 1 - 2\tau/\lambda - d\tau/d\lambda \qquad \phi_2 = 0.5(1 - \tau/\psi) \qquad (4)$$

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should vanish for the same value of $\lambda(\lambda_c)$ determines then D_O. This solution is supersonic for large distances and subsonic for $\lambda > \lambda_c$. Therefore, given the values of C and ϵ_∞ the integration of equations (2) determines N_O and T_O at the base of the corona; that is, the curves ϵ_∞ = constant and C = constant are known in the N_O, T_O plane [*Kopp*, 1968]. In the present paper, C and ϵ_∞ were adjusted to give the desired value of T_O *($T_O = 2.1\times10^6\,^\circ$K)* and increasing values of N_O *($N_O > 9.3\times10^7/\mathrm{cm}^3$)*. In figure 1, D_O is plotted versus

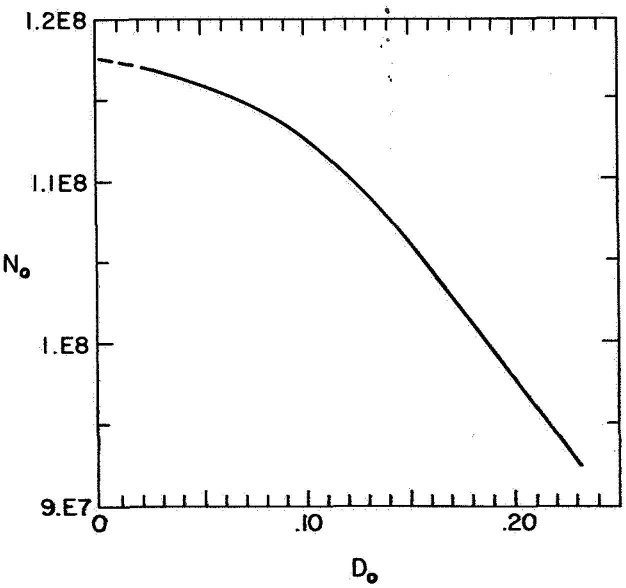

Figure 1. *Density at the base of the corona versus D_O. The dotted lines are interpolated values.*

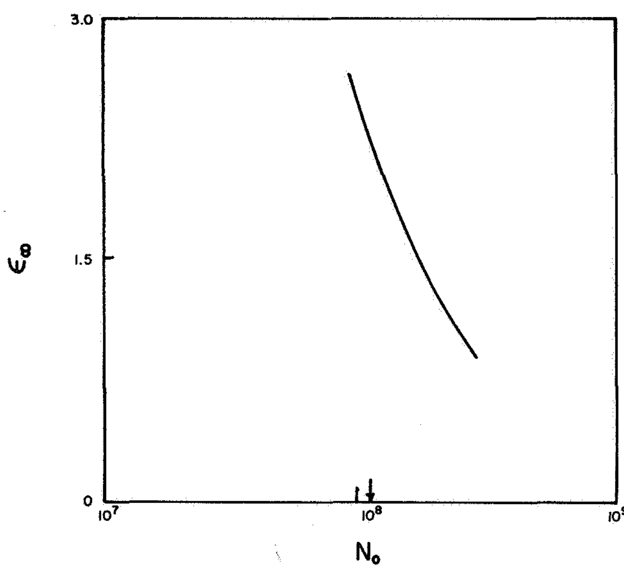

Figure 2. *Residual energy at infinity versus N_O for $T_O = 2.1\times10^6\,^\circ K$.*

N_O; the dashed curve represents interpolated values. In figure 2, ϵ_∞ is plotted versus N_O. Thus, from figures 1 and 2 it is clear that D_O vanishes for $N_O = N_O^B = 1.17\times10^8/\mathrm{cm}^3$, that is, for a finite value of ϵ_∞. It is easily seen that the vanishing of D_O implies that the conductive flux at infinity is zero. Therefore, supersonic solutions of the Parker type cease to exist for $N_O > N_O^B$. However, since for $N_O = N_O^B$ the total energy flux at infinity, ϵ_∞, is not zero, it is to be expected from physical grounds that as N_O is increased the flow will remain supersonic. This was confirmed by numerical calculations. No subsonic solutions were found for $N_O < 10^9/\mathrm{cm}^3$.

The above considerations led to the search for a supersonic solution of equations (2) with an asymptotic expansion different than that given by equation (3). It was indeed found that the full equations (2) also admit a solution with the following asymptotic expansions

$$\tau = D_O\lambda^{4/3}[1 + D_1\lambda + D_2\lambda^{4/3} + D_3\lambda^{5/3} + D_4\lambda^2 + D_5\lambda^{7/3} + \ldots] \tag{5a}$$

$$\psi = \psi_\infty + 2\lambda + C_O\lambda^{4/3}[1 + C_1\lambda + C_2\lambda^{4/3} + C_3\lambda^{5/3} + C_4\lambda^2 + C_5\lambda^{7/3} + \ldots] \tag{5b}$$

with

$$\left.\begin{aligned}
&\epsilon_\infty = \frac{1}{2}\,\psi_\infty\,;\ C_O = -5D_O\,;\ C_1 = D_1 = -\frac{2}{3\psi_\infty}\,;\\
&C_2 = D_2 = \frac{5}{3}\frac{D_O}{\psi_\infty}\,;\ C_3 = D_3 = 0\\
&C_4 = D_4 = \frac{8}{9{\psi_\infty}^2}\,;\\
&D_5 = -\frac{2D_O}{63}\left[\frac{175}{{\psi_\infty}^2} + 22\,A{D_O}^{3/2}\right];\\
&C_5 = \frac{4A}{15}\,{D_O}^{5/2} + D_5
\end{aligned}\right\} \tag{6}$$

As in the Parker solutions the parameter D_O in equation (5) is left undetermined, and equations (2) can again be solved by integrating from small values of λ toward the sun. An iteration procedure was used to determine D_O so that $\phi_1(\lambda) = \phi_2(\lambda) = 0$ (cf. eq. (4) for the same value of $\lambda = \lambda_c$). This determines the critical point. The iteration procedure is based on the fact that if ϕ_1 vanishes first ($\phi_1(\lambda_c) = 0$ and $\phi_2(\lambda) \neq 0$ for $\lambda < \lambda_c$), then D_O is too

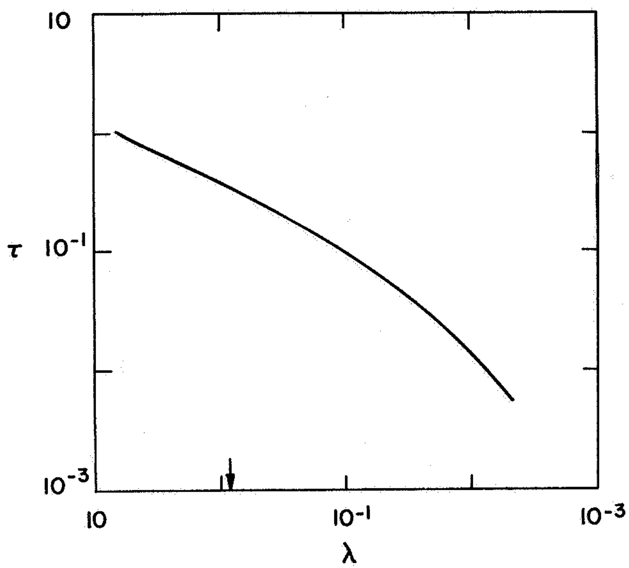

Figure 3. *τ versus λ for $C = 5.47\times10^4, \epsilon_\infty = 0.887$.*

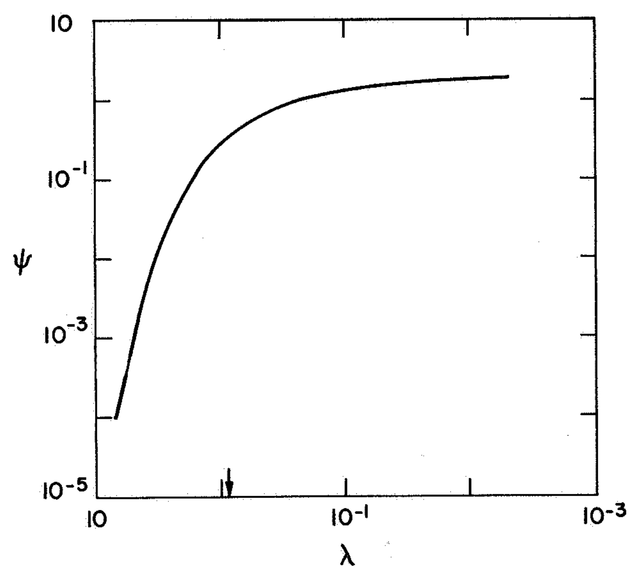

Figure 4. *ψ versus λ for $C = 5.47\times10^4, \epsilon_\infty = 0.887$.*

large and it should be decreased; if on the other hand ϕ_2 vanishes first, then D_O is too small and it should be increased. This method allows the critical point λ_c to be known as accurately as desired. Once λ_c is determined a Taylor expansion of τ and ψ in the neighborhood of λ_c can be used to integrate across the critical point. In figures 3 and 4, τ and ψ are plotted versus λ for $C = 5.47\times10^4$, $\epsilon_\infty = 0.887$. With the help of the asymptotic expansions (5) the integrations were started from $\lambda = 4.7\times10^{-3}$ toward the sun. The arrow indicates the position of the sonic point. For the above values of C and ϵ_∞ the values of the density and temperature at the base of the corona are $T_O = 2.1\times10^{6\circ}$K and $N_O = 2.88\times10^8/\text{cm}^3$, respectively. Therefore, to summarize, if the temperature at the base of the corona is assumed constant and equal to $2.1\times10^{6\circ}$K, the solution of the solar wind equations is of the Parker type for values of $N_O < 1.17\times10^8/\text{cm}^3$. For values of N_O such that $1.17\times10^8/\text{cm}^3 < N < N_O^S$ ($N_O^S > 3\times10^8/\text{cm}^3$) the flow continues to be supersonic but differs from the Parker solution. The asymptotic expansions of the velocity and temperature are now given by equations (5). For $N > N_O^S$ the flow becomes subsonic. The value of N_O^S was not determined with great accuracy; the numerical calculations suggest that $N_O^S \sim 2\times10^9/\text{cm}^3$. Since the integrations are time consuming no supersonic solutions were evaluated for $N_O > 3\times10^8/\text{cm}^3$.

As the density at the base of the corona is increased from an initial value of $N_O = 9.3\times10^7/\text{cm}^3$ and the temperature there is held constant ($T_O = 2.1\times10^{6\circ}$K) there is not a direct transition from a Parker type supersonic flow to a subsonic flow. As N_O is increased the conductive flux at infinity ϵ_∞^C decreases sharply and vanishes for a *finite* value of the total energy per particle at infinity ϵ_∞. This is not surprising, since as N_O increases the mass flow C also increases, and since T_O has been kept constant ϵ_∞ must decrease. The sharp decrease in ϵ_∞^C arises as more and more of the conductive flux at infinity is converted into kinetic energy of the particles. The Parker type supersonic solutions cease to exist when ϵ_∞^C vanishes. The total energy flux at infinity remaining, however, different from zero, it is to be expected from physical grounds that if N_O increases still further there will not be an immediate transition to a subsonic flow but rather a transition to a different type of supersonic flow (cf. eqs. (5) and (6)). The expansion becomes adiabatic [*Chamberlain,* 1961] and the heat conduction flux is much smaller than the internal energy flux [*Hundhausen,* 1971]. Any further increase in N_O must now be entirely accounted for by a decrease in the kinetic energy flux at infinity. The velocity reaches a maximum and decreases towards a finite value as $r \to \infty$. Finally, for values of N_O large enough, ϵ_∞ vanishes and the flow becomes subsonic.

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