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33. A Design Procedure and Handling Quality Criteria for Lateral-Directional Flight Control Systems*

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This paper describes a practical design procedure for aircraft augmentation systems based on quadratic optimal control technology and handling-quality-oriented cost functionals. The procedure is applied to the design of a lateral-directional control system for the F4C aircraft. The design criteria, design procedure, and final control system are validated with a program of formal pilot evaluation experiments.

INTRODUCTION

The work reported here is concerned with two problems in the area of optimal control and its application to the design of augmentation systems for fighter aircraft :

- Specification of performance criteria in terms of handling-quality requirements of the controlled vehicle
- Formulation (and solution) of the optimization problem such that practical control systems are obtained.

These problems have long frustrated efforts of flight control designers to exploit performance improvements and time savings offered by mathematical optimization. Even for simple performance criteria and system representations (notably quadratic cost functions and linear systema), optimization methods produce controllers too complex for flight control mechanization. When the varied requirements of handling qualities (MIL–F8785B) are imposed as criteria, the methods seem inapplicable altogether.

The objectives of this work were to alleviate these difficulties. Specifically, the objectives were: **(1)** to develop a practical controller design procedure based on quadratic optimal control technology and handling-quality-oriented cost functionals; **(2)** to apply the procedure to the design of a lateral-directional control system for the F4C aircraft; and **(3)** to validate the design criteria, design procedure, and final control system with a program of formal pilot evaluation experiments.

Briefly stated, the design procedure developed involves the computational solution of a reformulated optimal model-following control problem. The optimization problem enforces practicality by including constraints on the structure and scheduling requirements of the final controller, and it enforces handling quality requirements by minimizing quadratic functions of model-following errors. The models themselves are systems of differential equations which satisfy available handling-quality data.

More detailed descriptions of the models, the optimization problem and its computational solution, and their application to the F4 lateral directional axes are presented. Results of the pilot evaluation program which were used to validate the procedure are also presented here.

DESIGN CRITERIA

The first step of the design procedure is to specify handling-quality-oriented design criteria which are usable within the framework of quadratic optimal control technology. Such criteria are described in this section.

There are basically two approaches available for the specification of handling quality criteria.

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The first is empirical. Loosely speaking, it consists of measuring the suitability of many different kinds of aircraft configurations to the needs **of** the pilot and his mission. The configurations are parameterized and catalogued, and the most desirable parameter ranges identified. These desirable ranges define "ideal" configurations. Criteria are then expressed in terms of deviations of an actual airframe's parameters from the ideal. The second approach is analytical. It consists of treating the pilot not as the external judge and evaluator but rather as an integral part of the overall control loop. The loop is analyzed by representing the pilot in terms of mathematical models which are compatible with our descriptions of the vehicle. Optimality is then imposed with respect to some external performance criterion for the overall system, and the system is optimized, pilot model and all. The overall criterion thus provides implicit criteria for the controller/vehicle subsystem.

Needless to say, the second approach requires a very general pilot model which remains valid even under severe manipulations of the control loop. The judgment made here is that, at present levels of sophistication, pilot models fall short of this requirement. In consequence, we have used the first approach to the criteria question and have constructed "ideal" configurations, or handling-quality models, for the F4C lateral-directional axes. These were used as design criteria in an optimal-following control formulation. Our decision is an interim one, subject to change as the sophistication of pilot models grows.

Handling-Quality Models

Assuming that the dominant lateral-directional modes of response of our controlled vehicle configuration correspond to conventional aircraft dynamics, the configuration can be parameterized in the following way:

$$\frac{d}{dt}\begin{bmatrix} p_m \\ r_m \\ \beta_m \\ \phi_m \end{bmatrix} = \begin{bmatrix} L_p & L_r & L_\beta & 0 \\ N_p & N_r & N_\beta & 0 \\ Y_p & Y_r & Y_\beta & Y_\phi \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_m \\ r_m \\ \beta_m \\ \phi_m \end{bmatrix} + \begin{bmatrix} L_{\delta R} & L_{\delta AS} \\ N_{\delta R} & N_{\delta AS} \\ Y_{\delta R} & Y_{\delta AS} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_R \\ \delta_{AS} \end{bmatrix} \quad (1)$$

where p_m, r_m, β_m and ϕ_m are model roll rate, yaw rate, sideslip and bank angle, respectively, and where δ_R and δ_{AS} are rudder and lateral (coupled aileron and spoiler) commands.

The coefficients L_p, L_r, . . . , $L_{\delta AS}$, N_p, N_r, . . . , $N_{\delta AS}$, and Y_p, Y_r, . . . , $Y_{\delta AS}$ are the parameters of the configuration. Their values are such that the modes of response consist of two exponential modes (spiral and roll subsidence) and one oscillatory mode (dutch roll). Moreover, to correspond to the actual aircraft, certain of the coefficients have nearly constant known values:

$$Y_p \doteq 0, \quad Y_{\delta R} \doteq 0, \quad Y_{\delta AS} \doteq 0.$$

$$Y_r \doteq -1, \quad Y_\phi = g/U_0$$

This leaves 11 free coefficients which characterize the configuration.

Note that the coefficients of equation (1) are not given in conventional notation (ref. 2). Instead, they lump together both aerodynamic and inertia terms. This is consistent with our F4 data source (ref. 4).

Specification of Free Coefficient

The recently-updated Military Specification, MIGF-8785 (ref. 2) and the handling quality data which support it (ref. 3) provide data summaries for specifying ranges of desirable and undesirable values of the model coefficients. It is reasonable to assume that this summary is based upon enough careful analysis of the numerous sources of raw handling-quality data and enough Consideration for various expert opinions to have isolated the results of the most reliable experiments and studies pertinent to any particular specification. Accordingly, 11 handling-quality parameters were taken from references 2 and 3 which implicitly define the remaining coefficients of equation (1).* These are summarized in table 1, together with their approximate literal expressions as functions of the coefficients. The table also indicates the pertinent MIL

***The parameters of table 1 are the end results of several iterations through the available data, each iteration guided by the objective of uniquely specifying the free coefficients. It is quite possible to find additional conditions. However, these will either be redundant or they will overspecify the free coefficients.**

TABLE *1.—Parameters Which Specify Free Model Coefficients*

Handling-quality parameters	Parameter	Literal expression	Ref. 2, paragraph
Dutch roll frequency	$s_1=\omega_d^2$	$=N_\beta+N_r Y_\beta$	3.3.1.1
Dutch roll damping	$s_2=2\zeta_d\omega_d$	$=-\left[Y_\beta+N_r+\frac{L_\beta}{N_\beta}\left(N_p-\frac{g}{U_0}\right)\right]$	3.3.1.1
Roll time constant	$s_3=1/T_R$	$=-L_p+\frac{L_\beta}{N_\beta}\left(N_p-\frac{g}{U_0}\right)$	3.3.1.2
Spiral time constant	$s_4=1/T_s$	$=T_R\frac{g}{U_0}\left(-L_r+\frac{L_\beta}{N_\beta}N_r\right)$	3.3.1.3
Dihedral effect	$s_5=L_\beta$		ref. 3 data, pp. 215, 353
Zero location of p/δ_{AS}	$s_6=(\omega_\phi/\omega_d)^2$	$=1-\frac{L_\beta}{N_\beta}\frac{N_{\delta AS}}{L_{\delta AS}}$	3.3.2.2 and ref. 3 data
Transfer function	$s_7=2(\zeta_\phi\omega_\phi-\zeta_d\omega_d)$	$=\frac{L_\beta}{N_\beta}\left(N_p-\frac{g}{U_0}\right)+\frac{N_{\delta AS}}{L_{\delta AS}}L_r$	3.3.2.2 and ref. 3 data
Sideslip increment	$s_8=\Delta\beta_{\max}$	$L_{\delta AS}\delta_{AS_{\max}}\frac{T_R}{\omega_d^2}\left[2\frac{g}{U_0}\left(\frac{N_{\delta AS}}{L_{\delta AS}}L_r-N_r\right)+\frac{N_{\delta AS}}{L_{\delta AS}}L_p-N_p+\frac{g}{U_0}\right]$	3.3.2.4
Steady-state roll rate	$s_9=\left.\frac{p}{\delta_{AS}}\right\vert_{ss}$	$=L_{\delta AS}T_R s_6$	airframe characteristics
Steady-state sideslip	$s_{10}=\left.\frac{\beta}{\delta_R}\right\vert_{ss}$	$=N_{\delta R}/\omega_d^2$	airframe characteristics
Roll rate due to rudder	$s_{11}=\left.\frac{p}{\delta_R}\right\vert_{ss}$	$=T_R\left(L_{\delta R}-N_{\delta R}\frac{L_\beta}{N_\beta}\right)$	3.3.4.5

F8785 paragraphs or other data sources which define numerical values of each parameter.

Using the 11 parameters of table 1, four handling-quality models were developed for the F4C aircraft. The models are ordered monotonically in terms of predicted handling qualities "goodness"—ranging from poor, model 1, to excellent, model 4. This predicted ranking was later used in the validation experiments to verify the design criteria.

The process of developing models consists simply of choosing numerical values for the 11 parameters and solving the resulting nonlinear algebraic equations (table 1) for the model coefficients. Details of these steps can be found in reference 1. It suffices here to say that we chose parameter values for model 4 which achieve best pilot ratings and chose parameters for the remaining four models such that each successive model was about one-half to one rating unit worse than the previous one. The latter degradations were taken individually along each parameter axis (or, at most, along two axes simultaneously), so the total effective degradation cannot be predicted. It is reasonable, however, to expect that the total degradation will exhibit the intended monotonic ordering of models.

Transient responses of the resulting four models for both lateral and rudder commands are shown in figures 1 through 4.

THE OPTIMIZATION PROBLEM

The second step of the design procedure is to develop a mathematical model for the aircraft which includes all of the dynamic elements judged important for control (e.g., rigid body,

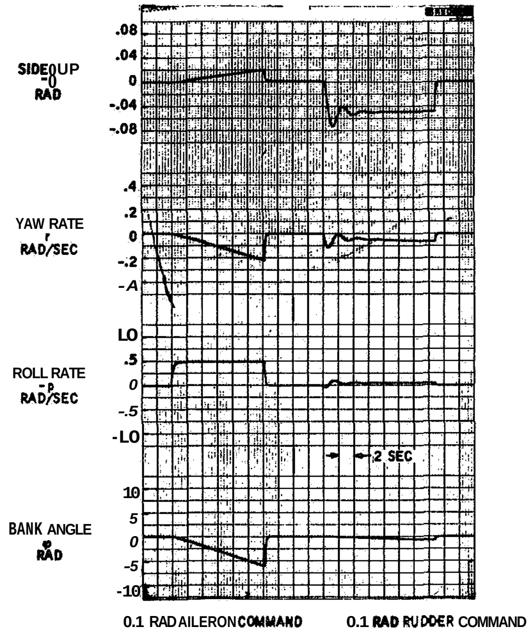

FIGURE 1.—step responses—model 1.

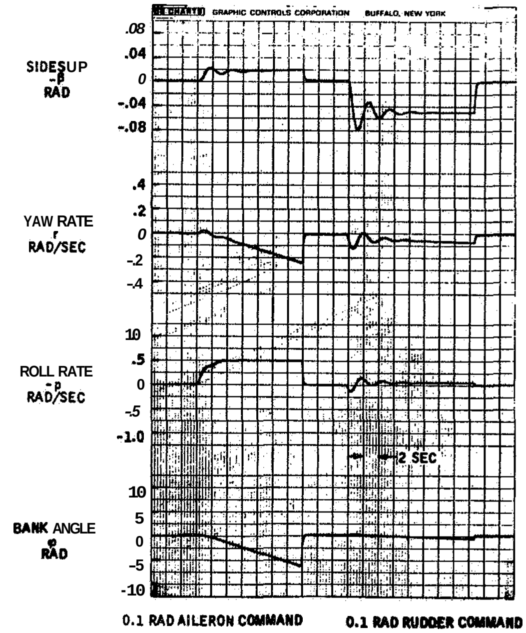

FIGURE 2.—Step responses—model 2.

actuators and servos, flexure modes, sensors, gusts). This vehicle model is then combined with the handling-quality model discussed above, and a controller is computed which satisfies a reformulated version of the usual quadratic optimal model-following control problem (see, for example, refs. 5, 6, and 7). The reformulated optimization problem and its computational solution are discussed in this section.

Reformulated Optimal Model-Following Control

Let the aircraft be represented at various points of the flight envelope and for various configurations and mass distributions by a collection of frozen-point linear plants:

$$\dot{x}_p(i) = F_p(i)x_p(i) + G_p(i)u_p + \Gamma_p(i)\eta_p \qquad i = 1, 2, \ldots, p \tag{2}$$

where $x_p(i)$ is the plant state vector, u_p is the vector of actuator or servo inputs, and η_p is white noise. The state x, and the system matrixes F_p, G_p, Γ_p, are indexed by the integer i, denoting a particular frozen-point plant. The index ranges from $i=1$ to $i=p$, meaning that p distinct plants will be handled simultaneously. Further, let

$$\dot{x}_m = F_m x_m + G_m u_m \tag{3}$$

represent the handling quality model [Equation (1)] with $x_m = (p_m, r_m, \beta_m, \phi_m)^T$ and $u_m = (\delta_R, \delta_{AS})^T$ and let

$$\dot{u}_m = F_u u_m + \Gamma_u \eta_u \tag{4}$$

represent a stochastic model for pilot commands; i.e., the commands δ_R and δ_{AS} are assumed to be sample functions of a linear system driven by white noise η_u, with magnitudes and spectral content determined by matrixes F_u and Γ_u.

We now look for a time invariant controller of the form

$$u_p(i) = K(i)\begin{bmatrix} x_p(i) \\ x_m \\ u_m \end{bmatrix}, \quad i = 1, 2, \ldots, p \tag{5}$$

such that the following composite performance index is minimized:

$$J = \sum_{i=1}^{p} \alpha_i J(i) \tag{6}$$

where

$$J(i) = E\{\|H_p(i)x_p(i) - H_m x_m\|^2_{Q(i)} + \|u_p(i)\|^2_{R(i)}\} \tag{7}$$

This cost functional is a generalization of the

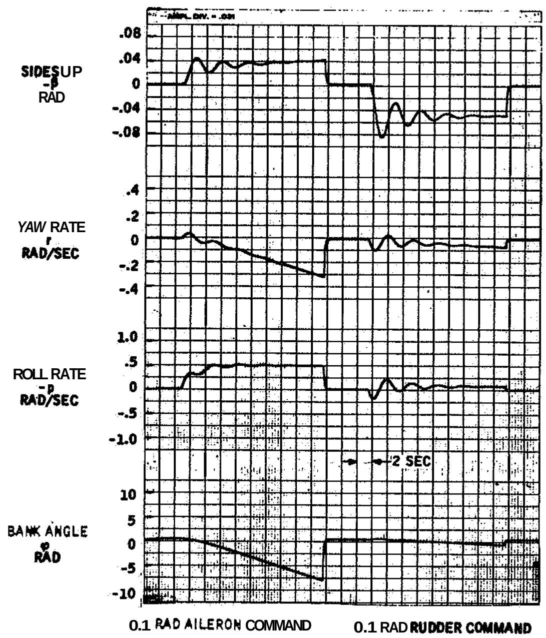

FIGURE 3.—Step responses—model 3.

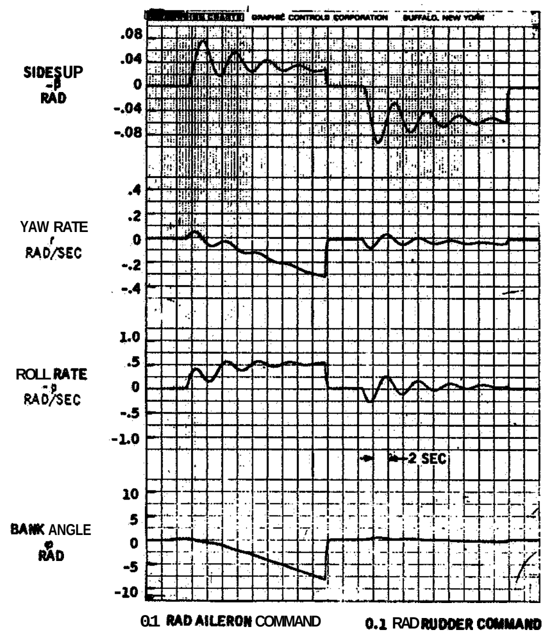

FIGURE 4.—Step responses—model 4.

usual quadratic index, representing weighted average performance over the flight envelope. The symbol $E\{-\}$ denotes mathematical expectation and $\|x\|_Q{}^2$ denotes the usual quadratic form x^TQx. The quantities $\alpha_iQ(i)$ and $\alpha_iR(i)$ are symmetric weighting matrixes, with $\alpha_iQ(i)\geq 0$ and $\alpha_iR(i)>0$ for all i. $H_p(i)$ and H_m are matrixes which select certain combinations of plant and model states whose differences are to be minimized.

To achieve practicality, the form of the gain matrixes $[K(1), K(2), \ldots, K(i), \ldots, K(p)]$ cannot remain entirely arbitrary. Measurement realizability implies that certain elements of all gain matrixes must vanish, and elimination of gain scheduling requires that other elements of all matrixes be identical.* These conditions are imposed on the form of $K(i)$ at the outset, thus incorporating the controller simplification requirement directly into the optimization problem. In particular, let y denote a vector of readily observable signals, i.e.,

$$y=M(i)\begin{bmatrix}x_p(i)\\x_m\\u_m\end{bmatrix}, i=1, 2, \ldots, p \qquad (8)$$

where the matrixes $M(i)$ select sensed signals and account for such things as sensor position, orientation, and sensitivity as they vary with flight condition. Then the gains, $K(i)$, most satisfy

$$[K(1), \ldots, K(i), \ldots, K(p)]$$
$$=[KM(1), \ldots, KM(i), \ldots, KM(p)] \qquad (9)$$

for some constant matrix K.

Unfortunately, this new optimization problem does not yield a unique closed-form solution. Instead, several local minima are possible, each satisfying the necessary conditions of optimality. To solve for even one requires considerable computational effort. This is the price of practicality.

Computational Solutions

As is shown in reference **1,** necessary conditions of optimality for the gains, $K(i)$, can be expressed directly in terms of the performance index J, i.e.,

*** The latter constraint may well mean that adequate performance is impossible. While this is not the case in the F4 designs considered here, the possibility is certainly real and warrants research to remove the no-gain-schedule constraint.**

$$\frac{\partial}{\partial K}J[KM(1), KM(2), \ldots, KM(p)]=0 \quad \textbf{(10)}$$

or they can be obtained from the maximum principle. The latter approach leads to several iterative algorithms for the solution of our reformulated optimization problem. These include Axsater's algorithms (ref. 8), a modified gradient algorithm (ref. 9), and others (ref. 10). All require large computer budgets (for high-order system), and all suffer from initialization problems, i.e., choosing initial values for K. (Initial values determine which local minimum will finally be found.)

The design procedure described here uses a newly-developed algorithm which is based directly on equation (10). This algorithm exhibits comparable computational speeds but avoids the initialization problem. It proceeds by dividing the gain matrices, $K(i)$ (for the moment these are assumed to be arbitrary), into two components:

$$K(i)=(K^1+K^3)M(i)+\lambda K^2(i), \quad i=1, 2, \ldots, p \quad (11)$$

The first component, $(K^1+K^3)M(i)$, satisfies the constraint conditions [eq. (9)] and will be called "gains to be retained." The second component, $\lambda K^2(i)$, with scalar multiplier λ does not satisfy equation **(9)** and will be called "gains to be discarded." For generality, the first component is further subdivided into parts K^1 and K^3, where K^1 represents free gains to be optimized and K^3 is fixed.

In terms of these subdivisions, the necessary conditions (eq. (10)) become

$$\frac{\partial}{\partial K^1}J[(K^1+K^3)M(1)+\lambda K^2(1), \ldots, (K^1+K^3)M(p)+\lambda K^2(p)]=0$$

or more simply

$$\frac{\partial}{\partial K^1}J(K^1,\lambda)=0 \quad \textbf{(12)}$$

for fixed $K^2(i)$, $M(i)$, and K^3. This last equation implies that the optimal gains are functions of λ; i.e.,

$$K^1=K^1(\lambda)$$

where, according to the Implicit Function Theorem (ref. 11), $K^1(\cdot)$ is defined by the following differential equation:

$$\frac{dK^1(\lambda)}{d\lambda}=-\left[\frac{\partial^2 J(K^1,\lambda)}{\partial K^1\,\partial K^{1T}}\right]^{-1}\frac{\partial^2 J(K^1,\lambda)}{\partial K^1\,\partial\lambda}. \quad \text{(13)*}$$

From equation (11), however, it follows that the desired constrained gains correspond to $\lambda=0$. That is,

$$K(i)=[K^1(0)+K^3]M(i),\ i=1, 2, \ldots, p. \quad (14)$$

So it is possible to obtain solutions of equation (10) by solving equation (13), starting with any known terminal condition (satisfying eq. (12)) at $\lambda=1$, and integrating backwards toward $\lambda=0$. The choice of terminal condition and method of numerical integration is discussed below. The idea of parameterizing solutions of optimization problems and solving the resulting ordinary differential equations was developed by D. K. Scharmack (ref. 12).

Terminal conditions.—The most appealing terminal condition is the global optimum of the performance index, J. This corresponds to the gains $[K^*(1), K^*(2), \ldots, K^*(p)]$ which are obtained by optimizing the ith plant with respect to the ith criterion $J(i)$ without gain constraints, for all i. (These gains are readily computed by solving the standard quadratic optimization problem at each flight condition.)

The resulting K^1, K^2, K^3 values for $\lambda=1$ are:

$$K^3=\text{given} \qquad K^1 \text{ arbitrary} \quad (15)\dagger$$

$$K^2(i)=K^*(i)-(K^1+K^3)M(i), \quad i=1, 2, \ldots, p \quad \textbf{(16)}$$

Starting with these terminal conditions amounts to starting in the "deepest valley of J" and forcing K^2 to zero along the trajectory $[K^1(\lambda), \lambda K^2; 1\geq\lambda\geq 0]$. As long as the matrix of second partials in equation (13) remains nonsingular, the resulting solution, $K^1(0)$, is unique and represents a stable linear controller. Moreover we have the reassurance that it is a point "on the walls of the deepest valley." Together with the knowledge of $J(K^*)$ and $J[K^1(0)]$, this

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***Equations (12) and (13) make sense in vector-matrix notation only if the matrix K^1 is written out as a column vector. This is assumed.**

† Reference 1 suggests one possible way to choose K^1 such that $\|K^2(1) \ldots K^2(p)\|^2$ is minimized.

information could well suffice to terminate the search.

Numerical Integration.—The full gamut of integration techniques is available for this problem. In reference 12, a predictor-corrector scheme was found to be particularly useful. It consists of the following equations:

- Adams-Moulton Predictor

$$K^p = K^1(\lambda_k) + \frac{\Delta\lambda}{24}\left[55\frac{dK^1}{d\lambda}(\lambda_k) - 59\frac{dK^1}{d\lambda}(\lambda_{k-1}) + 37\frac{dK^1}{d\lambda}(\lambda_{k-2}) - 9\frac{dK^1}{d\lambda}(\lambda_{k-3})\right] \quad (17)$$

- Newton-Raphson Corrector

$$K^c = K^p - \left[\frac{\partial^2 J(K^p,\lambda_{k+1})}{\partial K^1\, \partial K^{1T}}\right]^{-1} \frac{\partial J(K^p,\lambda_{k+1})}{\partial K^1} \quad (18)$$

The corrector is cycled repeatedly, each time replacing the old value K^p with the new value K^c, until a convergence criterion of the form $|\partial J/\partial K^1| < \epsilon$ is satisfied. The final value of K^c becomes $K^1(\lambda_{k+1})$, where

$$\lambda_{k+1} = \lambda_k + \Delta\lambda,\ \Delta\lambda < 0$$

$$k = 0,\ 1, 2,\ \ldots\ ,\frac{1}{|\Delta\lambda|}$$

The integration is initiated at $\lambda_0 = 1$, with $K^1(1)$ given by equation (15).

The utility of these equations depends, of course, on the computer time and memory required to evaluate first- and second-partial derivatives. Equations for this purpose are developed in reference 1. It turns out that all necessary derivatives for a single predictor or corrector step can be obtained by solving $(\ell+3)$ n-dimensional Liapunov-type equations where ℓ is the number of gain components in K^1. These equations can be solved very quickly even for large systems (ref. 13). So it becomes possible, with the algorithm discussed above, to solve some fairly complex control problems. This is shown in the next section with the F4C design applications.

THE F4C TEST CASE

The design procedure discussed in the last two sections was used to design several control systems for the lateral-directional axes of the F4C aircraft. While space does not permit a detailed account of these designs, some of their basic features are discussed briefly below. Greater detail can be found in reference 1.

Mathematical Model

A 20th order dynamic model was used to represent the optimization problem. This model included 14 states for thg plant proper (the aircraft), plus three states for the handling quality model (eq. (1) with the spiral mode extracted), and one state each for the rudder and lateral command models and for a lateral wind gust model. The aircraft model itself included four rigid body states, one actuator state and two servo states in each of the control channels, two states for the lateral accelerometer, and two states for an approximated first asymmetric flexure mode. While flexure is not a primary control consideration on the F4, the first mode was included in order to constrain any tendency of model-following controllers to excite and/or destabilize flexure degrees of freedom.

A total of 11 flight conditions was considered in the design process, providing broad coverage of the flight envelope. Not all conditions were included in the computational procedure, however. Instead, the algorithm was run for a few selected flight conditions treated simultaneously, and the resulting controllers were checked at the remaining conditions.

Sensed signals available for feedback included body axis roll rate, yaw rate, and bank angle, all with unity sensor dynamics, and lateral acceleration near the pilot station, passed through second-order instrument dynamics. Of course, the handling-quality model states and command states were also available for control.

Summary of Designs

Using the problem representation above, the computational algorithm was first used to carry out several single-flight condition designs $(p = 1)$. These served to debug the algorithm, to explore sensor complements, and to identify critical flight conditions. Typical run times for these single-flight-condition cases were 20 min on an H–1800 computer. (On present machines this means about **2** min.)

Two designs were then carried out for five flight conditions treated simultaneously ($p=5$). The first of these proved disappointing in that too little dutch roll damping was achieved. This was later traced to an improper choice of command magnitudes in the optimization formulization. The second design, however, proved successful and was "flown" in the validation program. Run times for these multi-flight-condition cases came close to **160** min on the **H–1800,** or about **16** min on current machines.

THE VALIDATION PROGRAM

The design criteria and the controllers developed as discussed earlier were validated with a formal program of piloted experiments. These were conducted on a fixed-base simulator at Honeywell Inc., Minneapolis, Minnesota, and later completely replicated on a limited moving-base simulator at the Air Force Flight Dynamics Laboratory. Our rationale for the validations, the experimental design, procedures, and equipment setup are discussed in this section. Results of the experiments are summarized later.

The objectives of the experimental program were threefold: **(1)** to validate handling-quality design criteria used in the design phase of the program; **(2)** to evaluate the resulting practical lateral-directional control system using pilot performance measurement and rating scale values; and **(3)** to cross-validate pilot opinion measures with performance measures. A basic premise underlying the program is that neither rating data nor performance data alone can provide adequate evaluations of the "quality" of a controller/vehicle configuration. Simply stated, pilot ratings tell us how a pilot feels about flying a system, while performance data tell us how well he can satisfy mission requirements with the system. Both pieces of information are important for evaluation. It is reasonable to expect, of course, that the two measures are interrelated. It was the intent of the third objective to explore this relationship and to add to its data base. Accordingly, the validation program was designed to collect both kinds of data in a form which can be analyzed and interpreted.

The Experimental Design

A $2\times5\times3$ mixed-design analysis of variance (ref. **14)** was used to combine factorially five systems and two flight conditions. Three pilot subjects performed under each of the resulting **10** experimental conditions, giving a total of **30** experimental cells.

In the case of performance data, **10** trials were flown in each cell, where a trial was defined as a single flight profile three minutes long. The order of presentation of conditions was randomized to minimize order effects across subjects. In the case of pilot rating data, only two trials were flown in each cell.

Independent variables.—The following fixed variables were of interest: **(1)** system type, and **(2)** flight condition.

Five different systems were evaluated, each consisting of an F4C aircraft with a particular lateral-directional controller. The controller for systems **1, 2, 3,** and 4 was a quadratic-optimal, model-following controller (no practicality constraints) designed to follow handling-quality models **1, 2, 3,** and **4,** respectively. As developed in Design Criteria, these models are predicted to exhibit an ascending order of handling-quality "goodness." It was anticipated that systems **1, 2, 3,** and **4** would exhibit a consonant ranking, and, in fact, if they did, the design criteria would be validated (the first validation objective).

The controller of system **5** was a practical model-following controller which was designed to follow handling-quality model 4 but involved only practical feedback signals and fixed gains. We did not predict a performance or rating measurement for this system. However, since the four optimal systems were selected along a continuum of "goodness," the data of this experiment would locate the practical system along the same continuum and thus serve to evaluate the practical controller (the second validation objective).

For the second independent variable, two flight conditions—low-speed (Mach 0.5; **328** kt) and high-speed (Mach **1.2; 787** kt)—were selected **as** representative points of the speed range of the F4 vehicle. The three subjects were treated as random variables.

Dependent variables.—For each performance data trial, the following variables were recorded on magnetic tape, at the rate of three sample points per second: (1) altitude error, (2) velocity error, (3) lateral error, **(4)** bank angle, (5) sideslip, **(6)** pilot aileron stick command, and (7) pilot rudder command. Several summary scores were also computed on-line and displayed to the experimenter at the end of each trial. For pilot rating trials, the Cooper Harper scale (ref. 15) and the Global scale (ref. 16) were administered after each trial. Voice recordings of pilot commands were collected for all rating and performance trials.

Both performance data and rating data were formally analyzed to evaluate the independent variables. Conclusions reached and numbers obtained were compared to cross-validate the two types of data (the third validationobjective).

The Task

The task consisted of a series of flight profiles of the type illustrated in figure 5. Each profile was composed of twelve 15-sec intervals during which constant bank angle turns or wings-level flight were commanded. Bank angles were randomly selected from the values ± 15, ± 12.5, **f** 10, ± 7.5, ± 5, and 0, with approximatelyhalf of the angles taking the zero value. All profiles were initiated at 2000 ft mean sea level and required a 2000-fpm climb to approximately 8000 ft.

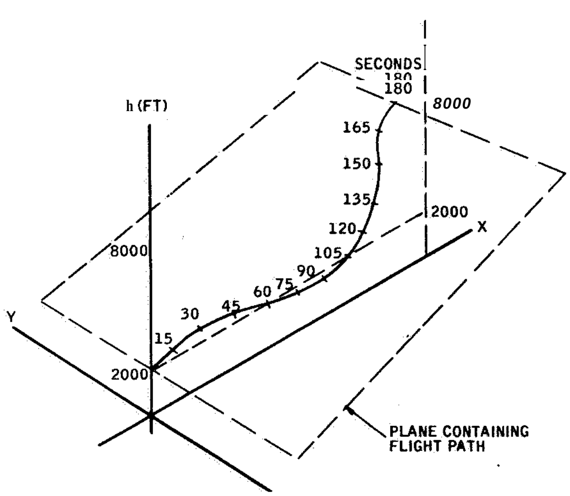


FIGURE 5.—Typical flight profile.

The primary display of the flight path course to the pilot was the flight director. The glide slope portion of the display (pitch command bar) was used to display deviations from the commanded 2000-fpm climb, while the localizer portion of the display (lateral command bar) was used to display deviations from the commanded course centerline. The perceptual motor task for the pilot was simply to detect any deviations from pitch and lateral command bar center and to make appropriate corrections to keep the needles centered.

Subjects

A total of nine pilots were involved in various phases of the validation program. At Minneapolis, two (former) Navy and one (former) Air Force pilot participated. The Air Force subject repeated the experiment at WPAFB along with two current Air Force test pilots. In addition, a second group of four Air Force pilots participated in the Rating Data Phase at WPAFB. The qualifications and experience of the subjects are documented in reference 1.

Simulation Equipment

The experiments were conducted on a fixed-base simulator in Minneapolis and replicated on the limited-moving-base simulator at the Air Force Flight Dynamics Laboratory. In Minneapolis the simulation was mechanized on a **40K** memory, SDS Sigma–5 digital computer with associated A/D and D/A links, and a PACE 231–R–5 analog computer. The simulation was basically all-digital, with analog portions used only to drive instruments of the cockpit mockup. The AFFDL simulation was hybrid, with navigation and control equations mechanized on an EAI 8400 digital computer with associated A/D, D/A links, and aircraft dynamics and cockpit motion mechanized on four PACE 231–R analog computers.

Both simulations used nonlinear 6-degree-of-freedom equations of motion and had comparable complements of cockpit controls and displays. Controls included stick, rudder pedals, and throttles, and flight instruments included airspeed/Mach meter, altimeter, angle-of-attack meter, flight director, horizontal situation indicator, and a sideslip **or** lateral acceleration indicator.

At each facility, the experiment was divided into two complete phases plus a familiarization phase. Phase I consisted of measuring pilot performance under the restrictions of the $2\times5\times3$ experimental design, and Phase II consisted of pilot rating measurements under the same experimental design. Phase II followed the completion of phase **I.**

RESULTS OF THE VALIDATION PROGRAM

Parametric statistical analyses were performed on the pilot rating scores and on several summary error scores computed from time histories **of** each performance data trial. Results of these analyses lead to the following general findings:

(1) Systems **1** through 4 exhibit the predicted monotonic ordering of "goodness" both when evaluated on the basis of pilot opinion data and when evaluated on the basis of pilot performance data. This verifies that our original design criteria—the handling quality models—do in fact differentiate on a scale of handling-quality goodness, and further, it verifies that the F4 can be made to exhibit the properties of the models, at least with optimal control.

(2) The ranking of system 5 shows an ordering of 5 better than 4 for rating data and 4 better than 5 for performance data. However, neither of these differences is statistically significant. This says that a practical controller designed with the procedure dc ,eloped here is sufficient to achieve the required level of model-following.

(3) The effect of flight condition is not significant on all dependent measures analyzed save one—lateral stick activity. This means that the model-following controllers achieved a high degree of performance invariance for the two flight conditions investigated.

Reference **1** contains numerous histograms, graphs and analysis of variance tables to substantiate these findings. Here we will present results for two of the primary dependent measures—RMS lateral error from course centerline and Cooper-Harper ratings, both obtained from the Wright-Patterson AFB (WPAFB) simulation.

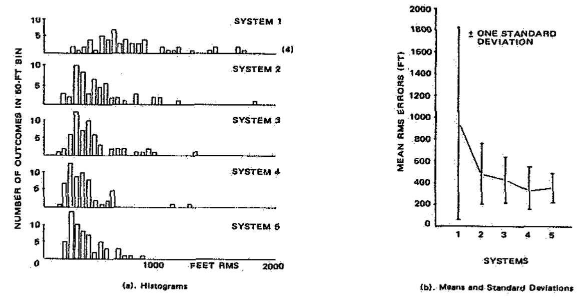
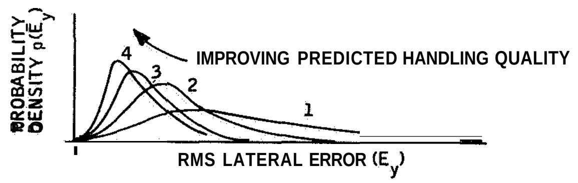


FIGURE 6.—Rms lateral errors at **WPAFB.**

Lateral Error at WPAFB ($\bar{E}_y$)

Figure 6(a) shows some histogram plots of the **300** lateral rms error scores obtained at WPAFB. These plots give the number of experimental outcomes which fell into 50-ft "bins," for bin locations between zero and 2000 ft.* The plots thus constitute crude experimental probability density functions of the random variable $\bar{E}_y$ at WPAFB. The samples are grouped into five plots according to the system with which they are obtained.

The plots of figure 6(a) show qualitatively how the probability density function of $\bar{E}_y$ changes as a function of the independent systems variable, i.e., the main effect of systems. We see first of all that all of the five distributions seem to have the same characteristic shape, something approaching chi-square if we indulge our imagination a little. The average value and spread of the distributions decrease steadily from the predicted worst system **(1)** to the predicted best system (4), and little difference exists between systems 4 and 5. Qualitatively, then, the main effect of systems looks something like this



Mean values and standard deviations of the lateral error histograms are presented in Figure

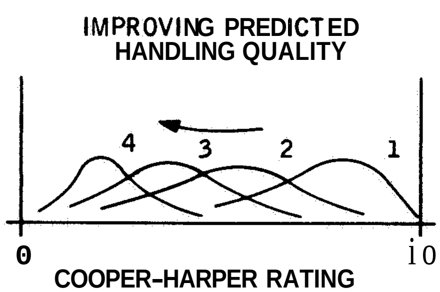
* **As** indicated on the plots by a parenthetical number at the right extreme of the abscissa, four trials fell outside of the 2000-ft range. These are 2068, 2998, 4065, and 5632 ft in magnitude.

6(b). These statistics are again broken down to show the main effects of systems. A monotonic ordering is apparent for systems **1** through **4,** with a slight deterioration for system 5. Note that this is precisely the predicted ordering. The range of error from system **1** to system **4** is approximately 589 ft rms. Likewise, there appears to be a consistent decrease in variability from **1** to **5.** System **1** differs most noticeably from the others both in terms of mean error value and standard deviations.

The statistical significance of these apparent differences was assessed with an analysis of variance (ref. **14)** of the lateral error data and with individual comparisons of systems performed via Scheffe's test (ref. **17).** The analysis of variance showed a highly significant main effect of systems ($F=113$), with subjects and systems-by-flight condition interactions also significant. All other effects were not significant beyond the **0.05** level. Individual comparisons showed that all differences of system means were significant except for adjacent pairs. In particular, the difference between systems **4** and **5** was not statistically significant.

Cooper-Harper Data at WPAFB

A similar analysis procedure was carried out for the Cooper-Harper rating data. This begins with figure 7 which shows histograms and mean values and standard deviations for this dependent measure. The histograms again show a qualitative main effect of systems which this time appears to match the sketch below:



The mean values show a monotonic ordering of systems, with system **1** being the least acceptable and system 5 the most acceptable The average Cooper-Harper value for system **1** is 7.8 with a decreasing trend across systems to an average of 2.7 for system 5. Note that system *5* obtained a better average rating than system **4.** This is contrary to the performance data which showed that system 5 errors were greater than system **4** and contrary to our original predictions. However, the difference between systems **4** and 5 turns out to be statistically insignificant. For all systems, the standard deviations appear comparable.

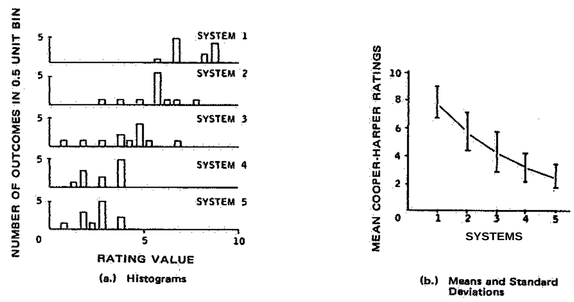


FIGURE 7.—Cooper-Harper ratings at WPAFB (3-subject group).

The analysis of variance for the Cooper-Harper data showed a significant systems effect ($F=23$) and also significant flight condition-by-subjects, systems-by-subject, and systems-by-subject-by-flight condition effects. Individual comparisons showed all differences of system means to be significant, except of course, the **4** versus **5** comparison.

Other Dependent Measures

Several other dependent measures were also analyzed. These include altitude errors, velocity errors, global rating scores, and secondary measures such as rms lateral commands and rms rudder commands. All measures were separately analyzed for the Minneapolis and WPAFB versions of the experiment. The results show pretty much the same thing. Systems **1** through **4** are rank ordered as predicted with system *5* a bit better than system **4** for rating scores and a bit poorer for performance scores. In all cases, the latter difference was not statistically significant.

Tables **2** and **3** present the significance levels for the main effects and interactions and the individual comparisons, respectively, for all dependent measures. Entries in these tables are

TABLE 2.—*Summary of Levels of Significance for Main Effects and Interactions of the Dependent Variable Analysis of Variance*

Data source	(A) Systems	(B) Flight condition	(C) Subjects	AXB	AXC	BXC	AXBXC
Lateral error, WPAFB	$p<0.01$	.....	$p<0.05$	$p<0.05$	.....	.....	.....
Lateral error, Minneapolis	$p<0.05$	.....	$p<0.05$	$p<0.05$	$p<0.01$	.....	.....
Altitude error, WPAFB	$p<0.05$	.....	$p<0.01$	$p<0.05$	$p<0.05$	.....	...
Altitude error, Minneapolis	$p<0.05$	.....	.....	.....	.....	.....	.....
Velocity error, WPAFB	$p<0.05$	.....	$p<0.05$	.....	.....	$p<0.01$	.....
Velocity error, Minneapolis	.....	.....	$p<0.05$	.....	.....	$p<0.01$	.....
Cooper-Harper, WPAFB	$p<0.01$	.....	.....	.....	$p<0.01$	$p<0.01$	$p<0.01$
Cooper-Harper, Minneapolis	$p<0.01$	.....	.....	.....	.....	.....	.....
Cooper-Harper, 4-subject group, WPAFB	$p<0.01$	.....	$p<0.01$	.....	$p<0.01$	.....	.....
Global, handling qualities, WPAFB	$p<0.01$	.....	.....	.....	.....	$p<0.05$	$p<0.01$
Global, handling qualities, Minneapolis	$p<0.01$	.....	$p<0.01$	.....	.....	.....	.....
Global, demands on pilot, WPAFB	$p<0.01$	.....	.....	.....	.....	$p<0.05$	$p<0.01$
Global, demands on pilot, Minneapolis	$p<0.01$	.....	$p<0.01$	.....	.....	.....	.....
Global, handling qualities, 4-subject group, WPAFB	$p<0.01$	.....	$p<0.01$	.....	.....	.....	.....
Global demands on pilot, 4-subject group, WPAFB	$p<0.01$	.....	$p<0.01$	.....	.....	.....	.....
Lateral commands, WPAFB	$p<0.01$	$p<0.05$	$p<0.01$	$p<0.05$	$p<0.05$	.....	.....
Rudder commands, WPAFB	$p<0.05$	.....	$p<0.01$	.....	$p<0.05$	.....	.....

probability values indicating significant effects of the independent variables. In those cases where no entries are made there are no significant differences.

In terms of results, the two simulations—fixed base at Minneapolis and moving base at WPAFB—showed only small differences (fig. 8). Moreover, the two types of data—performance and rating—turned out to be complementary not contradictory. The rank orderings of systems are the same whether based on pilot opinion or pilot performance. This suggests a strong relationship between the two types of measures.

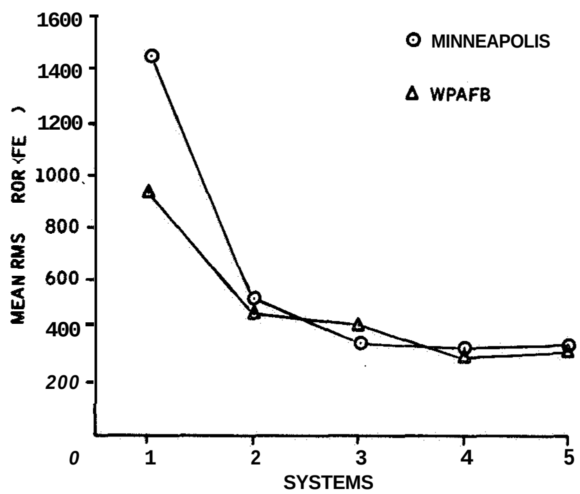


FIGURE 8.—Mean lateral error for both simulations as a function of systems.

CONCLUSION

The validation program has demonstrated that the design procedure developed here represents a powerful approach to the design of augmentation systems. The program has shown that handling-quality models for the controller/vehicle combination are a meaningful way to express design criteria, and that the **F4C** aircraft can be made to follow such models with a practical control system designed by the procedure. This is true at least to the resolution of pilot opinion and performance data gathered in the validation experiments.

TABLE 3.—*Summary of Levels of Significance for Individual Orthogonal Comparisons of All Pairs of Systems*

Data source	1 vs 2	1 vs 3	1 vs 4	1 vs 5	2 vs 3	2 vs 4	2 vs 5	3 vs 4	3 vs 5	4 vs 5
Lateral error, WPAFB	<0.01	<0.01	<0.01	<0.01	......	<0.01	<0.01	<0.05	......	...
Lateral error, Minneapolis	<0.01	<0.01	<0.01	<0.01	......	......	......	......	......	...
Altitude error, WPAFB	<0.01	<0.01	<0.01	<0.01	......	......	......	......	......	...
Altitude error, Minneapolis	<0.05	<0.01	<0.01	<0.01	......	......	......	......	......	...
Velocity error, WPAFB	<0.01	......	<0.01	......	......	......	......	<0.05	......	...
Velocity error, Minneapolis		......	......	......	......	......	......	......	......	...
Cooper-Harper, WPAFB	<0.05	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	......	<0.05	...
Cooper-Harper, Minneapolis	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	......	......	...
Cooper-Harper, 4-subject group, WPAFB	<0.01	<0.01	<0.01	<0.01	......	<0.05	......	......	......	...
Global, handling qualities, WPAFB	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	...
Global, handling qualities, Minneapolis	<0.01	<0.01	<0.01	<0.01	<0.05	<0.01	<0.01	......	<0.05	...
Global, demands on pilot, WPAFB	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	...
Global, demands on pilot, Minneapolis	<0.05	<0.01	<0.01	<0.01	<0.05	<0.05	<0.01	......	......	...
Global, handling qualities, 4-subject group, WPAFB	<0.01	<0.01	<0.01	<0.01		<0.05	<0.05			
Global, demands on pilot, 4-subject group, WPAFB	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	......	......	...
Lateral commands, WPAFB	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	<0.05	......	...
Rudder commands, WPAFB	<0.05	<0.05	<0.01	<0.05	......	......	......	......	......	...

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