N73-

Preceding page blank

32. Serial Segment Method for Measuring Remnant

R. E. MAGDALENO

Systems Technology, Inc.

For tracking tasks where a sum of sine waves forcing function is used it is often difficult and/or expensive to obtain the pilot's remnant in the vicinity of the sine waves. For the case where each sine wave has at least four times an integer number of cycle per run length, this paper illustrates the Serial Segments method for measuring remnant power spectral density in a frequency band centered on each sine wave. This method can be implemented on digital, hybrid, or analog Fourier coefficient analyzers, and is particularly advantageous on the latter since properties of Fourier coefficients are exploited to yield both a remnant measure and an improved estimate of the correlated component.

INTRODUCTION

There are many techniques available (see especially ref. 1) for measuring the pilot's describing function Y_p and remnant n_c in the compensatory task of figure 1. Generally the remnant is more predictable when referred to the input to the pilot, n_e (refs. 2 and 3). However, if a digital or hybrid computer is not available then remnant measurements with analog equipment (i.e., analog power spectral analyzers) may be expensive and/or lack the accuracy needed to separate signal from noise. For the case where the input is a sum of sine waves, reference 4 describes input-referenced-Fourier coefficient measurements as mechanized on "standard" analog computer equipment as well as on a describing function analyzer (DFA) where the latter uses synchronous motors to drive sine-cosine pots and multiplying pots. The inherent timing precision in the describing function analyzer approach is such that suitable processing of the Fourier coefficients (as they evolve in time) can yield an estimate of both the remnant and the describing function at the same frequency, thus enabling the remnant n_c to be reinterpreted as occurring at n_e by $\Phi_{nn_e} = \Phi_{nn_c}/|Y_p|^2$.

To illustrate the methods to be used, assume that we have a signal which is sine wave plus noise

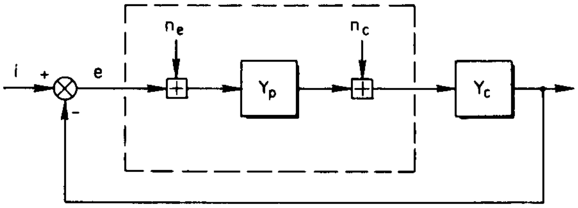

FIGURE 1.—Compensatory situation.

$$x(t) = A \sin(\omega_1 t + \varphi) + n(t) \tag{1}$$

where

A = peak amplitude
φ = phase (arbitrary)
ω_1 = frequency in rad/sec
$n(t)$ = noise component.

The relationship between variance and power spectral density (PSD) used in this paper is

$$\sigma_x^2 = \int_0^\infty \Phi_{xx}(f)\, df \tag{2}$$

where $\Phi_{xx}(f)$ = PSD of x; units of power/Hz. The PSD for $x(t)$ is

$$\Phi_{xx}(f) = \frac{A^2}{2}\delta(f - f_1) + \Phi_{nn}(f) \tag{3}$$

For a finite run length T_R the signal xt can be represented by a Fourier series form

PRECEDING PAGE BLANK NOT FILMED

$$x(t) = \bar{x} + \sum_{n=1}^{\infty} a_n \cos \omega_n t + \sum_{n=1}^{\infty} b_n \sin \omega_n t \qquad (4)$$

where the coefficients are evaluated from

$\bar{x}$ = mean value of x

$$a_n = \frac{2}{T_R}\int_0^{T_R} x(t) \cos \omega_n t \, dt$$

$$b_n = \frac{2}{T_R}\int_0^{T_R} x(t) \sin \omega_n t \, dt$$

$$\omega_n = n\omega_0$$

$$\omega_0 = \frac{2\pi}{T_R}$$

Where the fundamental frequency is one cycle per run length, the power in the frequency estimate at ω_n is

$$P_n = \frac{a_n{}^2 + b_n{}^2}{2}. \qquad (5)$$

For cases of interest in this paper the sine wave in equation (1) is assumed to be a harmonic of the fundamental frequency, $1/T_R$. When dealing with data taken over a finite run length T_R, a more useful definition of PSD is to assume that the power in the spike is distributed over the smallest resolvable bandwidth, $1/T_R$(Hz). This bandwidth follows from expanding $x(t)$ in Fourier coefficients which are orthogonal at that frequency spacing. Thus the PSD of $x(t)$ is

$$\Phi_{xx}(f) = \frac{A^2}{2} T_R + \Phi_{nn}(f); \frac{(\text{units of } x)^2}{\text{Hz}}. \qquad (6)$$

This is sketched in figure 2 in terms of each fundamental frequency band with the sine wave power centered on $f = f_1$ and assumed distributed over $1/T_R$ (Hz). The shaded area is the total power in the sine wave. The remnant sketched in indicates that it fluctuates about on either side of the sine wave portion as well as having a component centered there. The problem is to estimate both the sine wave component and a measure of the average remnant height.

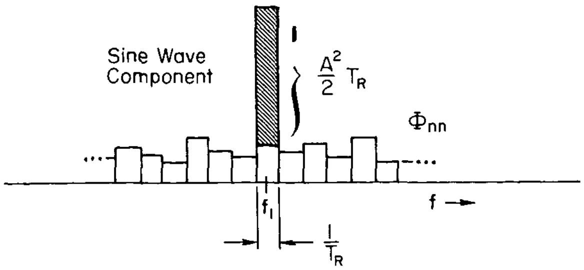

FIGURE 2.—Signal and noise power spectral densities.

METHOD

The DFA signal processing method can be modified so as to obtain remnant as well as describing function estimates for a 100 sec run length. The key point is to select forcing function frequencies that are orthogonal over 25 sec so that independent estimates of Fourier coefficients can be obtained at this run length as well as for 100 sec. Noting that each Fourier coefficient can be thought of as containing the power in a bandwidth of $(1/T_R)$ Hz where T_R is run length in seconds, then Fourier coefficients obtained over 25 sec have an effective bandwidth four times that of Fourier coefficients obtained over a 100 sec run length. Since the power correlated with the forcing function is common to coefficients obtained over 25 sec as well as those from 100 sec run lengths, then the difference in these powers is that due to remnant.

It will be convenient to deal with twice the power in a frequency band rather than the PSD,

$$2P_n = 2[\Phi_{xx}(f)]\, \Delta f \qquad (7)$$

where $\Delta f = 1/T_R$.
Inserting equations (5) and (6) into equation (7) yields (for a measurement frequency that falls on a sine wave)

$$a_n{}^2 + b_n{}^2 = A^2 + 2[\Phi_{nn}(f_n)]\left(\frac{1}{T_R}\right). \qquad (8)$$

Equation (8) relates the measured Fourier coefficients a_n, b_n to the sine wave and remnant descriptors A, Φ_{nn}. Two independent evaluations of equation (8) will allow a solution to be found.

Equation (8) evaluated over a 100 sec run length at a frequency corresponding to a forcing function frequency yields

$$M = A^2 + \frac{2\Phi_{nn}}{100} \qquad (9)$$

where $M = a_n{}^2 + b_n{}^2$.

Evaluating equation **(8)** over each **25** sec segment and then averaging the power in these four segments together yields

$$M_s = A^2 + \frac{2\Phi_{nn}}{25} \tag{10}$$

where

$$M_s = \frac{1}{4}\sum_{i=1}^{4}[a_{n_i}^2 + b_{n_i}^2]$$

a_{n_i} is the cosine component of the i^{th} segment
b_{n_i} is the sine component of the i^{th} segment

This method of calculating spectra from successive segments was suggested by Bartlett (ref. 5). The short segment coefficients are related to the full run length coefficients by

$$a_n = \frac{2}{100}\int_0^{100} x(t)\cos\omega t\,dt$$

$$= \frac{1}{4}\Bigg[\underbrace{\frac{2}{25}\int_0^{25} x(t)\cos\omega_n t\,dt}_{a_{n_1}} + \underbrace{\frac{2}{25}\int_{25}^{50}}_{a_{n_2}} + \underbrace{\frac{2}{25}\int_{50}^{75}}_{a_{n_3}} + \underbrace{\frac{2}{25}\int_{75}^{100}}_{a_{n_4}}\Bigg] \tag{11}$$

$$a_n = \frac{1}{4}\sum_{i=1}^{4} a_{n_i}$$

and similarly for b_n.

The right hand side of equation **(10)** shows that the average power in the four segments contains A^2 since the sine wave is identical in each of the four segments. The amount of measured remnant power is larger than in equation **(9)** since the effective bandwidth for the short run length segments is four times larger. However, the amount of remnant power in the 100 sec estimate is not exactly one-fourth of the remnant power in the averaged **25** sec segments. As shown in figure **2** the remnant component at $f = f_1$ (or any other component) may be higher or lower than the average height. Thus in equation (9) an additional factor ϵ should be added on to reflect the difference from the average remnant, Φ_{nn}.

Equations **(9)** and **(10)** can now be solved for A^2 and Φ_{nn}. However, for generality we are interested in the case where there are Q short segments in the run length T_R. Thus the generalized versions **of** equations **(9)** and (10) are respectively

$$M = A^2 + \frac{2\Phi_{nn}}{T_R} + \epsilon \tag{12}$$

$$M_s = A^2 + \frac{2\Phi_{nn}}{T_R/Q} \tag{13}$$

where ϵ is the deviation of the remnant power from the average level. Since ϵ is a random variable and likely to either be positive **or** negative for any one run it is neglected in the solution of equations **(12)** and **(13)** for average values of A^2 and Φ_{nn}. Thus carrying ϵ through the solution so as to see its effects yields,

$$A^2 = \frac{QM - M_s}{Q-1} - \frac{Q\epsilon}{Q-1}$$

$$\Phi_{nn}(f_n) = (M_s - M)\frac{T_R}{2(Q-1)} + \frac{\epsilon}{Q-1}\frac{T_R}{2} \tag{14}$$

where

Q = number of short run lengths in the basic run length, T_R

$$M_s = \frac{1}{Q}\sum_{i=1}^{Q}(a_{n_i}^2 + b_{n_i}^2)$$

$$M = \left(\frac{1}{Q}\sum_{i=1}^{Q} a_{n_i}\right)^2 + \left(\frac{1}{Q}\sum_{i=1}^{Q} b_{n_i}\right)^2.$$

In addition the sine wave input frequency f_n must have an integer number of cycles in the short run length, T_R/Q. Note that the remnant estimate contains ϵ divided by $Q-1$, thus the remnant estimate is not sensitive to the scatter. The estimate for A^2 is contaminated by the ϵ term, yet A^2 is usually much larger than the remnant (or its scatter as given by ϵ) so that it is not sensitive either.

RESULTS AND INTERPRETATION

The serial segments method yields a remnant estimate (as well as describing function estimate) at each of the forcing function frequencies. This method of computing remnant was evaluated by simulating the calculations in equation (14), using BOMM (ref. *6*). Using actual human operator data we computed the Fourier coefficients

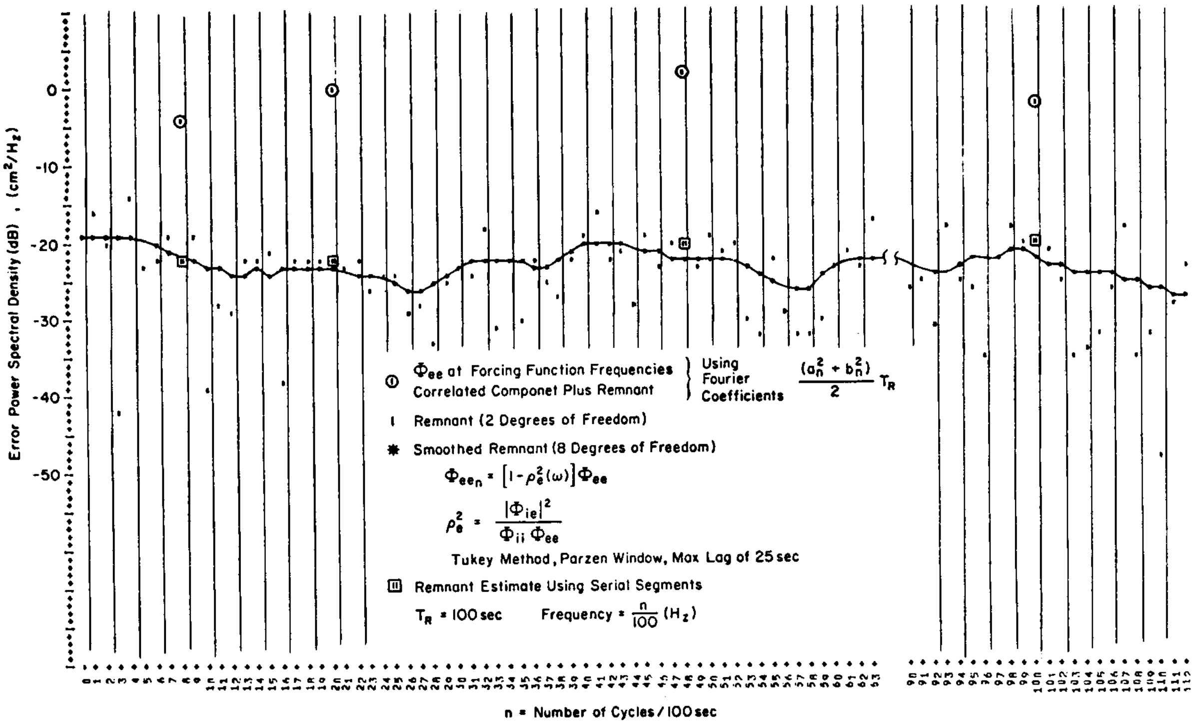

FIGURE 3.—Remnant computed from serial segments, $Y_c = 1/(s-1)$.

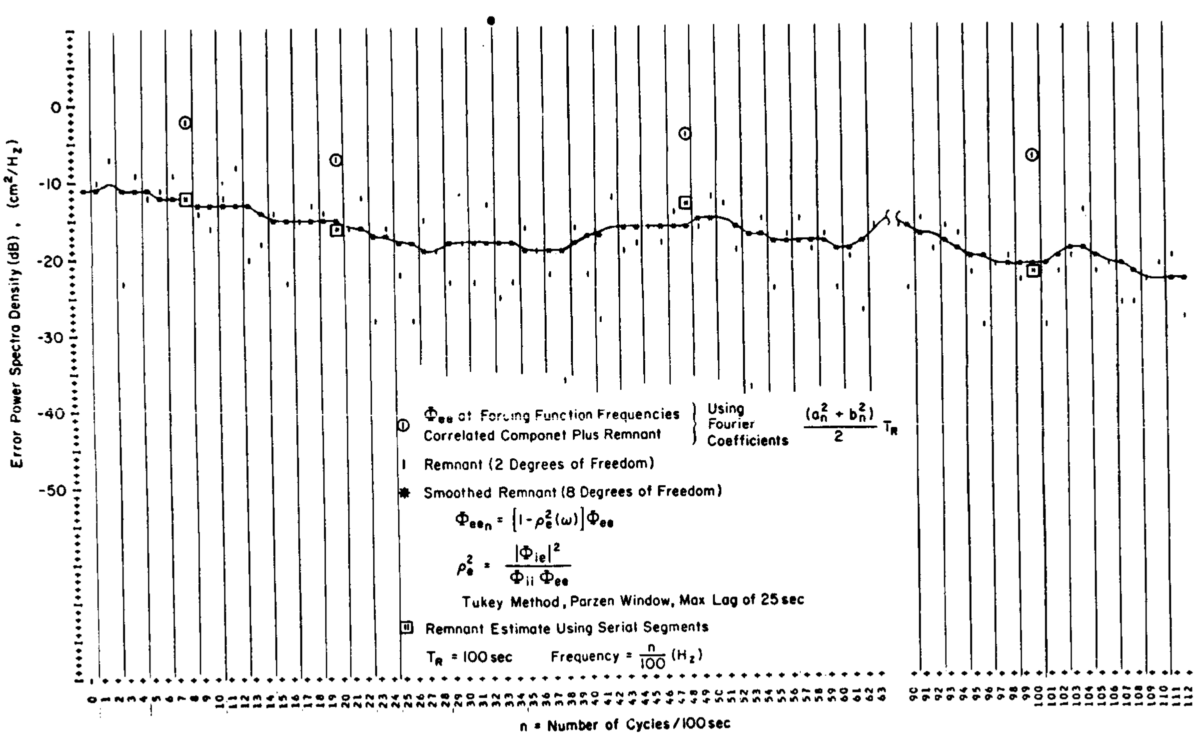

FIGURE 4.—Remnant computed from serial segments, $Y_c = 2/s(s-2)$.

of the error for $T_R = 100$ sec and for each **25** sec segment. These were processed according to equation **(14)** and compared with the remnant found by the Classical Tukey Method (ref. **7**). For the latter a Parzen data window was used with the number of lag values selected so that the effective bandwidth was close to that which results for the Serial Segments method when done four times per run length.

Example calculations are shown in figures **3** and **4,** where the error power spectral density estimates are plotted versus frequency in units of cycles per 100 sec run length. The Fourier components at forcing function frequencies are circled. The pertinent computations using the Serial Segments method are enclosed with a box. Generally these are in good agreement with the remnant power spectral density computed via the Tukey Method, which is included in figures **3** and **4** to give an indication of remnant both at and between the forcing function sine waves. For the Tukey Method (ref. **7**), the maximum number of lags was selected to mimic the 8 degrees of freedom for the Serial Segments (the scatter for 8 degrees of freedom is such that 60 percent of the values should fall within a **4** dB range approximately centered on the true remnant PSD). Note that both the Tukey and Serial Segments methods give smoothed estimates of the remnant, and they are in good agreement with each other. The small differences are partly due to the scatter term ϵ in equation **(14)**.

The Serial Segments method was mechanized using the **DFA** by sampling the Fourier coefficient integrators at the **25, 50,** 75, and **100** sec times and computing the serial segment values per equation **(11).** Typical error spectra results for a pilot (taken from ref. 8) are shown in figure **5.** The measured remnant values are generally well above the worst case system noise (measured with an analog pilot). Even at the highest frequency, the measured remnant is above the actual value of the system noise (about **3** dB lower than the worst case system noise). The normalized injected remnant is consistent with the reference **2** and **3** findings.

Thus the Serial Segments method for measuring remnant is shown to yield excellent remnant estimates over a large dynamic range.

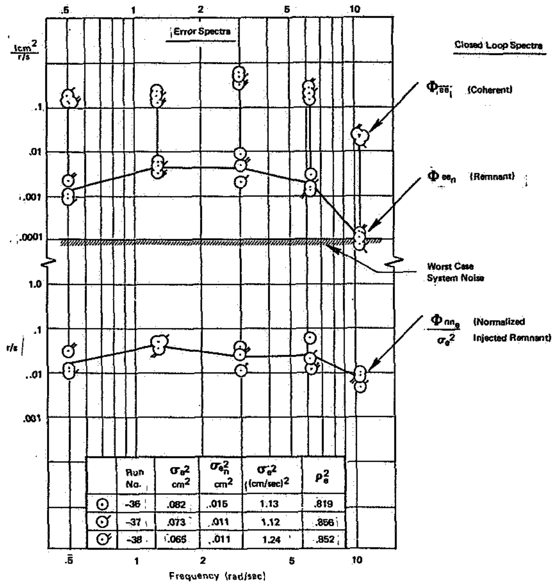

FIGURE 5.—Typical serial segments remnant data.

REFERENCES

1. SHIRLEY, RICHARD S.: A Comparison of Techniques for Measuring Human Operator Frequency Response. Proc. of the Sixth Annual Conference on Manual Control, Apr. 7–9, 1970, p. 803.
2. LEVISON, W. H.; BARON, S.; AND KLEINMAN, D. L.: A Model for Human Controller Remnant. IEEE Trans. on Man-Machine Systems, vol. MMS–IO, no. **4**, pt. I, Dec. 1969, p. 101.
3. JEX, H. R.; AND MAGDALENO, R. E.: Corroborative Data on Normalization of Human Operator Remnant. IEEE Trans. on Man-Machine Systems, vol. MMS–10, no. **4**, pt. I, Dec. 1969, **p.** 137.
4. ALLEN, R. WADE; AND JEX, HENRY, R.: A Simple Fourier Analysis Technique for Measuring the Dynamic Response of Manual Control Systems. Proc. of the Sixth Annual Conference on Manual Control, **Apr.** 7–9, 1970, p. 785.
5. BARTLETT, M. S.: Smoothing Periodograms from Time-series with Continuous Spectra. Nature, vol. 161, 1948, p. ***686.***
6. BULLARD, E. C.; ET AL.: A User's Guide to BOMM, **A** System of Programs for the Analysis of Time Series. Published by the Institute **of** Geophysics and Planetary Physics, Univ. of Calif. (San Diego), Jan. 1966.
7. BLACKMAN, R. **B.;** AND TUKEY, **J.** W.: The Measurement **of** Power Spectra. Dover Publications, Inc., 1958.
8. JEX, HENRY **R.;** ALLEN, R. WADE; AND MAGDALENO, RAYMOND E.: Display Format Effects **on** Precision Tracking Performance, Describing Functions, and Remnant. TR–191–1, Systems Technology, **Inc.,** Mar. 1971.