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3. A Control-Theory Model for Human Decision-Making*

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The optimal-control model for pilot-vehicle systems has been extended to handle certain types of human decision tasks. The model for decision-making incorporates the observation noise, optimal estimation, and prediction concepts that form the basis of the model for control behavior. Experiments are described for the following task situations: **(1)** single decision tasks, **(2)** two decision tasks, and **(3)** simultaneous manual control and decision tasks. Using fixed values for model parameters, we can predict single-task and two-task decision performance scores to within an accuracy of 10 percent. The experiment on simultaneous control and decision indicates the presence of task interference in this situation, but the results are not adequate to allow a conclusive test of the predictive capability of the model.

INTRODUCTION

Considerable effort has been devoted to understanding how a pilot controls his aircraft, and reasonably accurate models for the pilot as a feedback controller have been developed. Continuous control, however, is but one of the functions required of the pilot; he must also make various decisions during the course of a flight. As flight-control systems become more sophisticated, monitoring and decision-making tasks will play an increasingly important role in the pilot's management of the aircraft.

This paper summarizes a theoretical and experimental study recently conducted for NASA-Ames Research Center, to develop a model for human decision-making. In order to provide a common model structure for both decision-making and continuous control, the model for decision-making is based on the existing optimal-control model for pilot/vehicle systems developed by Bolt Beranek and Newman Inc. The optimal-control model contains the concepts of observation noise, optimal prediction, and optimal estimation that can be applied to certain types of decision problems. In addition, the existing pilot/vehicle model is able to account for interference among tasks performed in parallel. The model developed in this study is intended to apply to situations in which the human bases his decision on his estimate of the state of a linear plant.

Considerations of space limit us to a presentation of only the highlights of this study. Additional details are given in reference **1.** The reader is directed to references **2** and **3** for a description of the pilot/vehicle model and to references **4** and **5** for the development and validation of the model for task interference.

DESCRIPTION OF THE DECISION TASK

The following three constraints were imposed on the selection of an experimental decision task: **(1)** the task should be compatible with the existing theoretical structure for optimal control and estimation, **(2)** the correctness or incorrectness of the subject's response should be unambiguous, and **(3)** the experimental task should bear some resemblance to a decision task encountered in flight situations. In addition, we desired to relate this work to a concurrent study of aircraft approach and landing conducted both at **NASA-**Ames Research Center (ref. **6)** and Bolt Beranek and Newman Inc. (ref. **7).** Accordingly, we designed and used the following decision task, which was intended as an idealization of the

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pilot's task of deciding whether or not he is within the "landing window."

The subject was presented with an oscilloscopic representation of a noisy glide-slope indicator along with two reference indicators showing the "target," or region of acceptable glide-slope error. The subject's task was to keep his response button depressed whenever he thought the true error was within the target area. In order to test the model for task interference, we provided two such decision tasks simultaneously in some of the experimental trials. In the two-task situation, two noisy indicators were presented on the same display and the subject manipulated two response buttons. The two "error" signals were linearly independent and were in no way affected by the subject's response. The display format for the two-task situation is shown in figure **1.**

The quantity displayed to the pilot was constructed as the summation of a "signal') plus a "noise" waveform. Thus,

$$y_d(t)=s(t)+n(t) \tag{1}$$

where $y_d(t)$ was displayed to the subject, $s(t)$ was a low-frequency random waveform that we defined as the "signal" (say, glide-slope error), and $n(t)$ was a random waveform of higher frequency that we defined as "instrument noise."

Both $s(t)$ and $n(t)$ were generated by simulated Gaussian white noise processes. Signal shaping was accomplished primarily by second-order Butterworth filters. The "bandwidth" of $s(t)$ was fixed at 0.5 rad/sec,* arid the input amplitude was adjusted so that $s(t)$ would be within the target area half the time during the course of an experimental trial. The bandwidth of $n(t)$ was sufficiently greater than that of $s(t)$ to enable the subject to distinguish between the noise and signal components of the displayed variable $y_d(t)$. Noise power and bandwidth were experimental variables. The white noise forcing functions driving the signal and noise filters were linearly uncorrelated.

***For semantic convenience, we refer to the critical frequency of the Butterworth filter as the "bandwidth" of the filter output.**

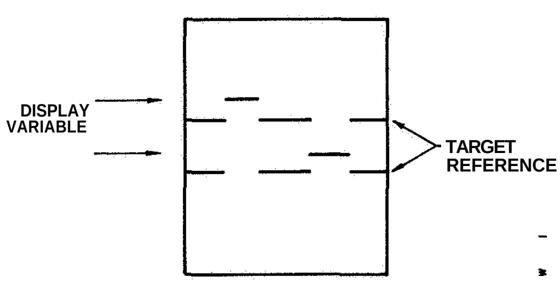

FIGURE 1.—Display format.

A MODEL FOR THE DECISION SITUATION

A block diagram of this decision situation is shown in figure 2. The portion of the model relating to the human's response is denoted by the dotted line. This model is identical to the optimal-control model for continuous tracking except that optimal control activity is replaced by optimal decision behavior.

The equations of motion of the system (i.e., '(system dynamics") are assumed linear and are expressed in state-vector notation, with the state vector denoted as $\underline{x}(t)$. The quantities displayed to the subject are denoted by the vector $\underline{y}(t)$ which is generated by a linear operation on the state vector. The human's inherent limitations are represented by an equivalent perceptual time delay τ and an observation noise process $\underline{v}_y(t)$.

The observation noise process $\underline{v}_y(t)$ is intended to account for the various sources of human randomness (or "internal noise"). The vector $\underline{v}_y(t)$ contains white Gaussian noise terms which account for the noise processes associated with the perception of indicator displacement and indicator velocity. These noise processes are assumed to be linearly independent of each other and of input driving noises. Furthermore, we assume that the power density level of each noise term is proportional to the variance of the corresponding perceptual variable. The constant of proportionality is termed the "noise/signal ratio". This treatment of human randomness is parallel to our treatment of human controller remnant (refs. **8** through 10).

The human's estimation strategy is represented in the model by the operations of optimal prediction and optimal (Kalman) filtering, the joint output of which is $\hat{\underline{x}}(t)$, the best estimate of the state of the system. The optimal predictor and estimator may be used to predict the variances

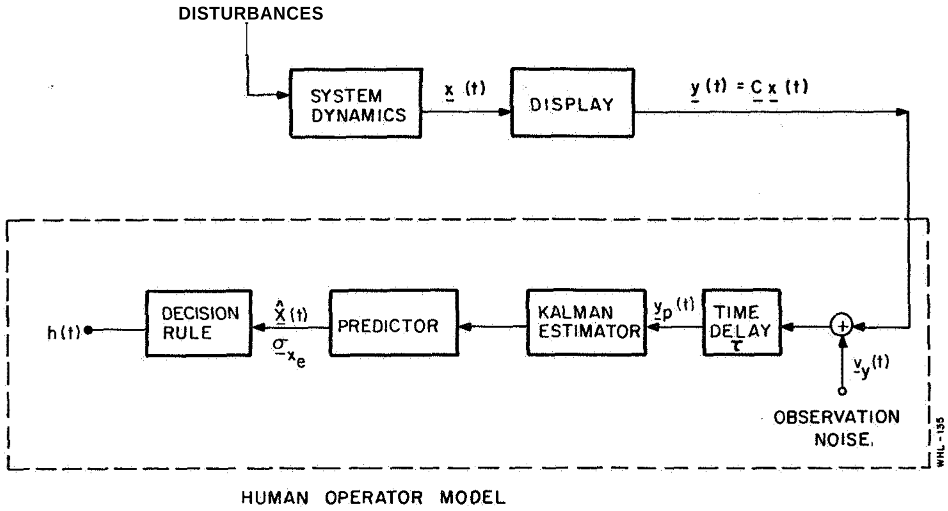

FIGURE 2.—Model for the decision situation.

(or rms levels) of the best estimate of the state vector and of the estimation error. These quantities are needed in order to predict the human's average decision performance.

Since $y_d(t)$ is the summation of linearly filtered white noise processes, this signal, by definition, is a sample function of a Gauss-Markov process (ref. **11).** This type of random process has the following properties which justify our assumptions of optimal estimation and prediction:

(1) The current "state" of the process contains all the useful information about the process. Thus, for most practical purposes, the entire past history of $\underline{y}(t)$ is "summarized" by the current value of the state vector, $\underline{x}(t)$.

(2) The best estimate of the state vector is given by a Kalman filter cascaded with an optimal predictor which operates on the noisy input variable $\underline{y}_p(t)$ (ref. **11).** This filter is linear and time-invariant, and the difference between the instantaneous value of the state vector and the best estimate of it is a time-stationary random process. (This difference, or "estimation error", has a variance which is denoted by $\underline{\sigma}_e^2$.) The estimate of the state vector, $\hat{\underline{x}}(t)$, is "best" in the sense that it is the minimum-variance as well as the maximum-likelihood estimate (ref. **12).**

(3) The pair $(\hat{\underline{x}}(t),\underline{\sigma}_e)$ constitutes a sufficient statistic to test hypotheses about $\underline{x}(t)$ based on the noisy data $\underline{y}_p(t)$. This is so because all the relevant information that can be extracted from $\underline{y}_p(t)$ is contained jointly in $\underline{x}(t)$ and $\underline{\sigma}_e$ (ref. **12).**

The model that we have described thus far is a model for the human's monitoring behavior. That is, this model will allow us to predict the way in which the human will process the noisy information that is available to him. In order to predict decision behavior, the model for monitoring must be coupled with a rule for generating the appropriate decision response.

The optimal decision element of our model is based on Bayesian decision theory (refs. **13** and **14).** In general, the human's decision strategy will depend on the probabilities of the various correct and incorrect decision situations that can occur and on the "utility" (or "cost") associated with each possible situation. Since space does not permit a generalized analysis, we shall consider only the specific decision situation explored in this study.

Our subjects were instructed to minimize the "decision error," which was defined **as** the fraction of time during an experimental trial during which the subject's response was incorrect. Two types of decision error were possible: the "false alarm" (indicating that the signal $s(t)$ was within the target area when, in fact, it was outside), and the "miss" (the reverse type of decision error). Equal weighting was given to the two types of decision error; that is, the total decision error score consisted of the sum of the two component error scores. For this situation the subject's decision rule was quite straightforward; namely, he was to respond "in" whenever his best estimate of $s(t)$ was within the target boundaries. (The optimal strategy is somewhat less trivial when the two types of decision error are not equally costly. See reference 1 for a more general analysis of the decision problem.)

PREDICTED DECISION PERFORMANCE

The model described above was used to predict the human's average decision error score for the experimental situations that were explored. The analysis procedure consisted of three steps: (1) problem specification, (2) computation of the variances of the best estimate ($\sigma_{\hat{s}}^2$) and the estimation error ($\sigma_{s_e}^2$) for the signal $s(t)$*, and (3) prediction of the decision error. The problem was specified in terms of the state vector $\underline{x}$, the display vector y, the time delay and noise/signal ratio which represented the human's limitations, and the performance requirements (i.e., minimization of decision error as defined above). The model for optimal estimation was implemented on a digital computer, and predictions were obtained for $\sigma_{\hat{s}}^2$ and $\sigma_{s_e}^2$.

The sum of the probabilities of the two types of decision error was used as a prediction of the decision error score. Thus,

$$\text{Predicted decision error} = P(H_1,h_0)+P(H_0,h_1) \quad (2)$$

where $P(H_1,h_0)$ is the joint probability of the subject deciding "in" and the signal being "out." Each of these probabilities was formulated as a joint gaussian distribution of the best estimate $\hat{s}(t)$ and the estimation error $s_e(t)$. Numerical techniques were used to compute average decision error scores, using numerical values for $\sigma_{\hat{s}}$ and σ_{s_e} yielded by the previous step in the analysis procedure. Details of the computational procedure are given in reference 1.

Two experimental variables were considered: the bandwidth of the simulated instrument noise, and the ratio σ_s^2/σ_n^2. The time-delay parameter of the model was fixed at 0.2 see—a value that is typical of the effective delays inferred from studies of manual control behavior. The noise/signal parameter of the pilot model served as a variable of the analysis procedure.

Predicted decision error is shown as a function of noise/signal ratio in figure 3. Curves are shown for each of the single-task conditions that were investigated experimentally. The filter bandwidth for the noise process $n(t)$ and the ratio of signal power to noise power is given for each of the conditions in the legend accompanying the figure. For the most part, the theoretical curves behave as one would expect. Predicted decision error increases as the simulated instrument noise power increases and as the human's internal noise level increases. Noise bandwidth, on the other hand, is shown to have little effect on decision performance.

Using the model for task interference that has been developed and validated for multivariable control situations (refs. 4 and 5), we can predict the increase in decision performance that will occur when a multiplicity of decision tasks are to be performed. Task interference is assumed to manifest itself as an increase in the human's internal noise/signal ratio according to the following relationship:

$$P_i^{(M)} = P_{o_i}/f_i \quad (3)$$

where $P_i^{(M)}$ is the noise/signal ratio associated with the i^{th} component task when a total of M tasks are performed, P_{o_i} is the ratio corresponding

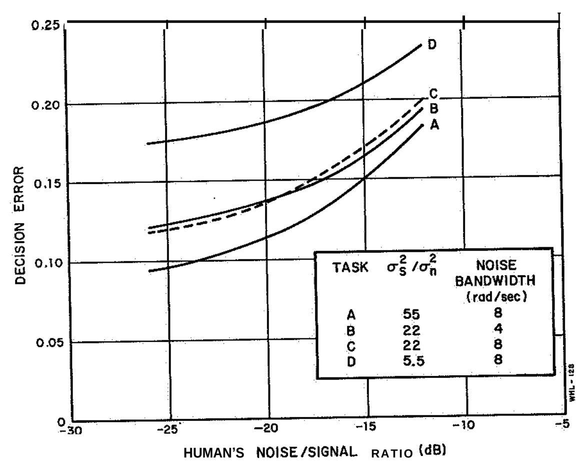

FIGURE 3.—Effect of noise/signal ratio on predicted decision error (signal bandwidth = 0.5 rad/sec.)

*The problem was formulated such that $s(t)$ was treated as one element of the state vector $\underline{x}(t)$.

to single-task performance of the i^{th} subtask, and f_i denotes the fraction of attention to that task in the multitask situation. In other words, we assume that the noise/signal ratio varies inversely with attention.

We further assume that the subject has a fixed amount of information-processing capability (or attention) that is allocated optimally among the various subtasks. The notion of fixed capacity is represented by the following mathematical constraint:

$$\sum_{i=1}^{M} f_i = \sum_{i=1}^{M} \frac{P_{o_i}}{P_i^{(M)}} = 1 \qquad (4)$$

The following steps are required to predict the human's decision performance in a multiple-task decision situation. First, we define a total performance measure that is to be minimized (say, a weighted sum of the decision errors associated with the component decision tasks). We then obtain theoretical curves relating decision error to the noise/signal parameter of the model. Finally, using a suitable iteration technique, we find the noise/signal ratios associated with each task which minimize the total performance measure, subject to the constraint of equation **(4).** We thus obtain predictions of total-task performance, performance on the component tasks, and the allocation **of** attention among the tasks as defined by equation **(3).** This analytical procedure may also be used to predict performance when decision and control tasks are performed concurrently, provided a measure of total-task performance can be defined.

THE EXPERIMENTAL PROGRAM

An experimental program was undertaken to test the validity of the model for decision-making presented above and to provide further tests of our model for task interference. The following task situations were explored: **(1)** single decision tasks, **(2)** multiple decision tasks, and **(3)** simultaneous manual control and decision-making. In this section of the paper we briefly describe these experiments and present the principal experimental results. Discussion of results appears in the subsequent section.

Single Decision Tasks

This experiment was conducted to determine the effects of changes in task parameters on decision error and on inferred noise/signal ratios. The four decision tasks identified in figure **3** were explored. Our primary objective was to determine the extent to which decision performance on all tasks could be accounted for with fixed values for time delay and noise/signal ratio.

Four undergraduate engineering students served **as** the subjects for this experiment. The subjects were provided with *six* training trials on each of the four decision tasks.* Following training, three "data" trials of 4 min duration each were conducted per task per subject. The order of presentation **of** tasks was counterbalanced among subjects.

The average decision error was taken as the primary performance measure for each of the four tasks. The standard deviation of the average score was estimated and was defined as

$$\text{Standard deviation} = \left[\frac{\sum_{i=1}^{N} (DE_i - \overline{DE})^2}{N(N-1)}\right]^{1/2} \qquad (5)$$

where DE_i is the average score of the i^{th} subject for a particular task, $\overline{DE}$ is the average score for all subjects on that task, and N is the number of subjects (in this case, four). **A** mean noise/signal ratio was inferred for each task by reference to the appropriate theoretical curve. Standard deviations were estimated for the noise/signal ratio as follows. Ratios were found which corresponded to the mean decision error plus (and minus) one standard deviation; the absolute value of the difference between these noise ratios, divided by two, was taken as the approximate standard deviation.

Figure 4 shows the effects of task parameters on predicted and measured average decision performance. Predictions were obtained with nominal values of **0.2** sec and **–20** dB assigned to the

* For **the most part, the subjects appeared** to **reach a stable level of performance** on **a given decision task after** *six* **training** runs. **Considerably more training was provided in the following two experiments to assure stable levels of performance** in **the two-task decision and simultaneous decision and tracking situations.**

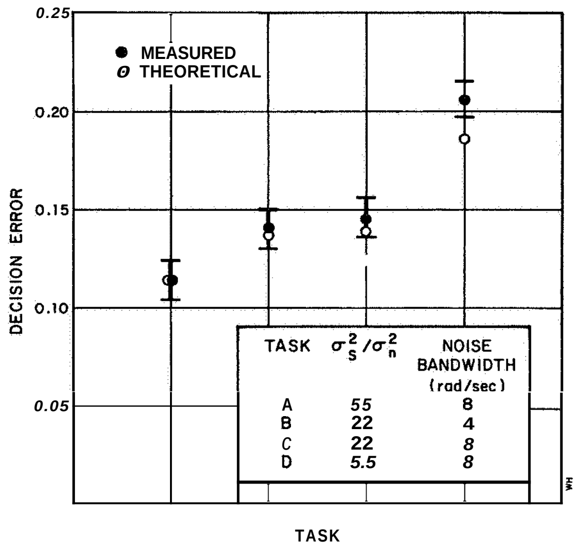

FIGURE 4.—Effect of task parameters on predicted and measured decision error (four subjects, three trials/subject).

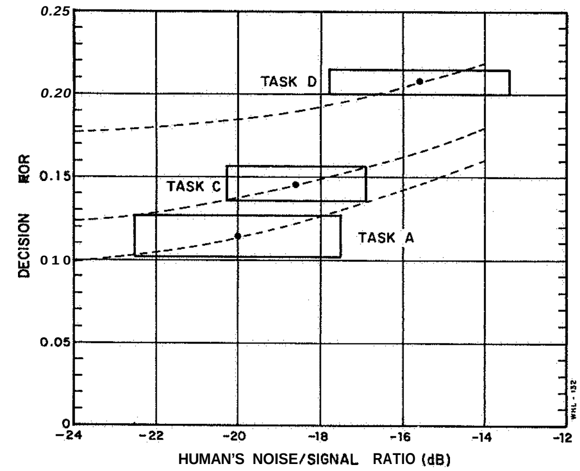

FIGURE 5.—Decision error scores and inferred noise/signal ratios for three tasks (four subjects, three trials/subject; dashed lines indicate theoretical relationships).

time delay and noise/signal ratio parameters of the model.

For the most part, predicted and measured scores were in very good agreement. For tasks *A*, *B*, and *C*, the measured decision error score varied by less than one standard deviation from the theoretical prediction. The decrease in "instrument noise" bandwidth from **8** to 4 rad/sec did not appreciably affect decision performance. (We had predicted that this would be the case if the human's noise/signal ratio were −20 dB.) The only notable discrepancy between theory and experiment occurred for the most difficult task (task *D*); in this case, the measured score was about **11** percent greater than the predicted decision error. **A** t-test performed on the subject means revealed that this difference, while small in absolute terms, was significant at the 0.05 criterion level.

Decision error versus inferred noise/signal ratio is shown graphically for tasks *A*, *C*, and D in figure 5. Rectangular boxes about each datum point indicate ± 1 standard deviation of both the error score and the noise/signal ratio. (The results of task *B* are not shown in this figure since they almost coincide with the results of task *C*.) This figure shows that the noise/signal ratio increases almost linearly with decision performance, ranging from −20.0 dB for task *A* to −15.6 dB for task *D*. Since the ratios inferred for tasks *A*, *B*, and *C* lie within one standard deviation of one another, we cannot ascribe any statistical significance to these differences. The noise/signal ratios associated with tasks *A* and D, however, differ by about two standard deviations; this difference is too large to dismiss simply as experimental variability. Factors which might account for the apparent variation of noise/signal ratio with task parameters are discussed later in this paper.

Multiple Decision Tasks

This experiment was performed to validate our model for task interference in a decision-making context. Decision error scores were obtained for tasks performed singly and two at a time, and the difference between the two-task and one-task scores was tested against the difference predicted by the model.

The subjects were provided with two decision tasks of the type identified as task *A* in figure **3.** The statistics of the left- and right-hand tasks were nominally identical, but the two display variables were linearly uncorrelated. When two tasks were performed concurrently, the subjects were instructed to minimize the sum of the decision errors associated with each component task. Each subject performed four data sessions consisting of the following three-task situations: (1) a single task for the left-hand, (2) a single task for the right-hand, and (3) left- and right-hand tasks together.

Average decision error score was 0.110 for the one-task situation and 0.130 for the two-task situation. The difference between these scores was found by a t-test to be significant at the 0.001 level.

If our model for interference is valid in this decision context, then the average increment in decision score should correspond to a doubling of the subject's noise/signal ratio. (That is, the subject devotes an average of half his attention to each task.) In order to obtain a theoretical prediction for the two-task decision error, we refer to the theoretical curve relating decision error to noise/signal ratio. From this curve we associate a noise/signal ratio of —20.8 dB with the average one-task score of 0.110. Taking this point as a reference, we derive the curve shown in figure **6** which relates the predicted increment in decision error to increments in noise/signal ratio (alternatively, to decrements in "attention").

The increments in decision error and inferred noise/signal ratio that we obtained experimentally are shown in figure **6** for comparison with the theoretical curve. The range of decision error and noise/signal ratio corresponding to ± 1 estimated standard deviation are also indicated. The increase of **0.020** in decision errors score that we measured corresponds to an increment of 3.3 dB in the inferred noise/signal ratio. This increase is within one standard deviation of the 3 dB increment predicted by our model for task interference. Similarly, the assumption of a **3** dB increment in noise/signal ratio leads to a predicted increase in error score of 0.018. **A** t-test of the average two-task, one-task difference scores shows that this prediction is not significantly different from the measured increase of 0.020. On the basis of this very good agreement between theory and experiment, we conclude tentatively that our model for task interference is applicable to the type of decision task explored in this study.

Simultaneous Control and Decision-Making

The third and final experiment was conducted to determine the extent to which the model would

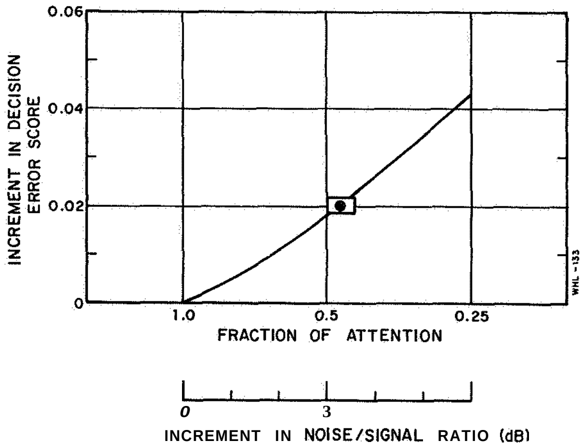

FIGURE 6.—Effect of attention on decision error score, task A (four subjects, three trials/subject).

account for interference between a decision task and a continuous control task performed concurrently. The decision task employed in this experiment was of type ***A*** as described above; the tracking task was a conventional K/s (i.e., velocity control) compensatory tracking task of the type used in previous studies (refs. *5* and 8 through 10). The display format shown in figure 1 was used, with the decision variable displayed on the left and the tracking error displayed on the right. The experimental procedure was similar to that employed in the previous experiment.

The subjects' instructions were to minimize decision error (DE) when performing the decision task alone, to minimize mean-squared tracking error (σ_e^2) when performing the tracking task alone, and to minimize a weighted sum of decision and tracking errors when performing the two tasks concurrently. The total performance measure for the two-task situation was $J = \sigma_e^2 + 3 \cdot \mathrm{DE}$. This combination of decision and tracking errors was selected on the basis of pre-experimental analysis in an attempt to have decision and tracking performance scores contribute roughly equally to the total score.

Each subject yielded higher (worse) performance scores in the two-task situation than in the one-task situation. This was true not only for the total performance measure but also for the component scores as well. The *magnitude* of these differences, however, varied widely from subject-to-subject. The fractional increase in total score ranged from about 5 percent for one subject to about 45 percent for another subject. For reasons which are detailed in reference 1, the data from these two subjects were considered unreliable and were not subjected to further analysis. The remaining two subjects exhibited an increase of about 15 percent in total score; these results were used to test the model for interference.

The procedure for testing the model for interference was similar to that used in the preceding experiment. Theoretical curves of decision error versus noise/signal ratio and mean-squared tracking error versus noise/signal ratio were used to determine the noise/signal ratios that corresponded to single-task performance. Additional parameters of the model for manual control (time-delay, motor noise ratio, and lag time constant) were chosen partly on the basis of previous experience and partly to provide a good match to the mean-squared error-rate and control scores. Using the rules set forth in equations (3) and (4), we then obtained theoretical curves for total- and component-task performance as a function of attention.

Figure 7 shows the predicted two-task performance scores for combined decision and tracking for two subjects. Total performance, tracking error, and weighted decision error scores are shown as a function of the fraction of attention paid to the tracking task. We see from this figure that optimal performance (i.e., minimum total score) for each subject corresponds to a nearly equal division of attention to the decision and tracking tasks. We note, however, that total score is relatively insensitive to attention in the vicinity of this theoretical optimum.

Superimposed on the theoretical curves in figure 7 are the theoretical two-task performance scores which correspond to no interference and to full interference. Also shown are the two-task scores that were obtained experimentally. The "no interference" theoretical scores are simply the scores obtained in the one-task experiments. (The no-interference total score is the weighted sum of the one-task tracking and decision error scores.) The "full interference" scores are the ones predicted by our model for task interference. These scores are obtained from the theoretical curves of figure 7 for a 50 percent allocation of attention to the tracking task, (which is the allocation of attention that yields the lowest predicted total score).

Both subjects achieved two-task total scores that fell between the theoretical scores associated with no interference and with full interference. Thus, the mutual interference between the tracking and decision tasks was not as severe as that predicted by the model.

Since we have reliable results from only two subjects, we cannot claim with a high degree of assurance that our model for interference either does or does not apply to the combined decision and control situation. There is little question that interference does occur: all four subjects yielded higher total and component scores in the two-task situation. The *degree* of interference remains in question. Accordingly, we must conclude at this stage that the model which we have proposed for combined decision-making and control shows promise, and that a conclusive set of experiments remains to be conducted.

DISCUSSION OF RESULTS

Experimental results agreed very closely with predicted performance scores in situations involving decision-making only. Using fixed values for human time delay and noise/signal ratio, we were able to predict both one-task and two-task decision error scores to within an accuracy of about 10 percent. Agreement was less good for the simultaneous decision and control situation, with prediction errors on the order of 15 percent.

Although the differences between theory and experiment are relatively small, they cannot be attributed entirely to "experimental variability." We consider briefly certain methodological problems associated with the decision task that may account for some of these differences, and we suggest refinements which might improve the predictive accuracy of the model.

Methodological Considerations

Perhaps the most serious drawback of the decision task which we explored—at least with respect to testing our model—was the relative insensitivity of decision error to the human's noise/signal ratio. This insensitivity may be largely responsible for the wide range of noise/signal

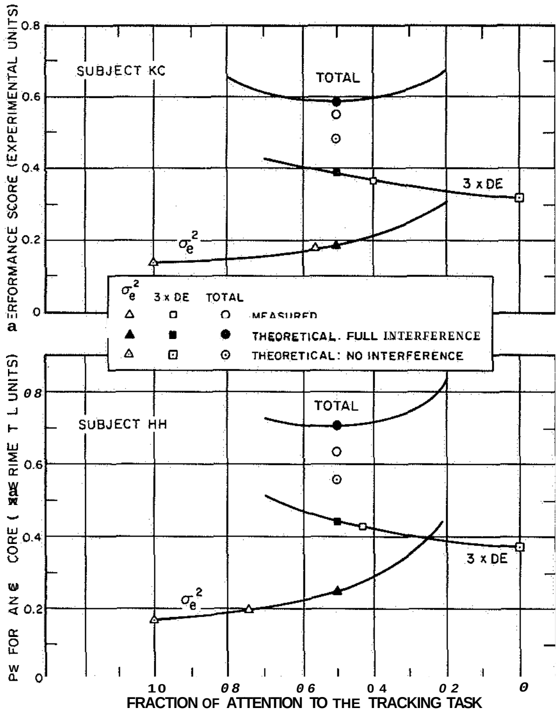

FIGURE 7.—Effect of division of attention on decision, tracking, and total performance scores.

ratios needed to match all single-task decision scores perfectly (fig. 5).

We have found in previous studies of manual control that, subjects will operate at an unusually low noise/signal level if, by so doing, they can substantially reduce their mean-squared tracking error scores (ref. 5). By the same token, we would expect subjects to operate at higher than usual levels of noise/signal ratio if task performance is particularly insensitive to noise/signal. From figure 3 we observe that a doubling of the noise/signal ratio from **−20** dB to **−17** dB theoretically produces only about a **5** percent increase in the decision error for task D. The same increase in noise/signal ratio accounts for a **17** percent increase in score for decision task **A** and a **44** percent increase in mean-squared error for the tracking task considered in this study. Thus, it is entirely possible that the subjects were insufficiently motivated to maintain a **−20** dB noise/signal ratio when performing task D.

We suspect that the inability to obtain reliable, conclusive data in the experiment on simultaneous decision and control was also due, in part, to a relative insensitivity of performance score to noise/signal ratio. Note that we can predict the total-task performance score to within **15** percent with either the full-interference or no-interference concept incorporated into the model. In order to obtain a more conclusive set of results, a decision task should be explored which is more sensitive to pilot parameters.

The subjects may have encountered difficulty in learning the strategies appropriate to the various decision tasks because of inadequate knowledge of results during the training period. The only knowledge of performance given to the subject was the decision error score that was given him at the end of each trial. Thus, if a subject were to try various estimation strategies during the course of a single trial, he would not know which strategy was best. Various methods of presenting relatively instantaneous knowledge of performance were considered, but these ideas were rejected because of the high probability that the subject would learn to respond to the performance indicator and not to the signal on the primary display. To some extent, then, the relatively large noise/signal ratio inferred for decision task ***D*** may reflect an inappropriate estimation strategy on the part of the subject.

Refinements to the Model

The first problem discussed above suggests one obvious refinement to the model; namely, that the sensitivity of performance to noise/signal ratio be taken into account. The rules for selecting observation noise levels might be modified to show the noise/signal ratio as an explicit function of this sensitivity. There is enough experimental evidence to indicate that this is a reasonable idea, although further study would be needed in order to determine with any degree of precision what this function should be. A model refinement of this sort would improve the predictive accuracy of models for manual control as well as for decision-making.

Another possible model refinement is to consider the power density level of the observation noise as a time-varying quantity which scales with the instantaneous magnitude of the observed signal. Such a treatment would be consistent with our assumption that human randomness stems from underlying multiplicative noise sources (ref. **1).**

The treatment of observation noise as a time-stationary process is a mathematical convenience that has apparently worked very well for modelling manual control behavior, but one which might introduce non-negligible modelling error in the decision situation. Note that the relation between the magnitude of the estimation error and the instantaneous value of the "signal" $s(t)$ determines the effect that a given amount of estimation error has on decision performance. For example, if the signal is two target widths beyond the target boundary, the subject will make the correct decision even if the error in his estimate of the signal position is relatively large. On the other hand, relatively small estimation errors may cause an incorrect decision if the signal is very close to one of the target boundaries. **A** more accurate modelling procedure would take account of the relation between the instantaneous signal value and the variance of the accompanying estimation error.

REFERENCES

1. LEVISON, W. H.; AND TANNER, R. B.: A Control-Theory Model for Human Decision-Making. Rept. No. **2119,** Bolt Beranek and Newman Inc., Apr. **1971.**
2. KLEINMAN, D. L.; BARON, S.; AND LEVISON, W. H.: An Optimal Control Model of Human Response—Parts I and 11. Automatica, vol. **6,** pp. **357–369,** Pergamon Press, May **1970.**
3. KLEINMAN, D. L., AND BARON, S.: Manned-Vehicle Systems Analysis by Means of Modern Control Theory. Rept. No. **1967,** Bolt Beranek and Newman Inc., June **1970.**
4. LEVISON, W. H.: A Model for Task Interference. Proceedings of the Sixth Annual Conference **on** Manual Control, Air Force Institute of Technology, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Apr. **1970.**
5. LEVISON, **W. H.;** ELKIND, **J.** I.; AND WARD **J.** L.: Studies of Multivariable Manual Control Systems: **A** Model for Task Interference. Rept. No. **1892,** Bolt Beranek and Newman Inc., Dec. **1969.**
6. PALMER, **E.;** AND WEMPE, T.: Pilot Performance with a Simulated ILS-Independent Pictorial Landing Display. Proceedings of the Seventh Annual NASA-University Conference **on** Manual Control, University of Southern California (Los Angeles, Calif.), June **1971.**
7. KLEINMAN, D. L.; AND BARON, S.: Analytic Evaluation of Display Requirements **for** Approach and Landing. Rept. No. **2075,** Bolt Beranek and Newman Inc., Mar. **1971.**
8. LEVISON, **W.** H.; KLEINMAN, D. L.; BARON, S.: A Model for Human Controller Remnant. Rept. No. **1731,** Bolt Beranek and Newman, Inc., Oct. **1968.**
9. LEVISON, W. H.; ET AL.: A Model for Human Controller Remnant. IEEE, Trans. **on** Man-Machine Systems, vol. MMS–10, **no. 4,** Dec. **1969.**
10. LEVISON, **W.** H.: The Effects of Display Gain and Signal Bandwidth **on** Human Controller Remnant. AMRLTR–70–93, Wright-Patterson Air Force Base *(to* be published).
11. KALMAN, R. E.: A New Approach to Linear Filtering and Prediction Problems. Trans. ASME, vol. 82D, p. **35, 1960.**
12. BRYSON, A. **E.,** JR.; AND Ho, Y C.: Applied Optimal Control. Blaisdell Pub. Co. (Waltham, Mass.), **1969.**
13. EDWARDS, W.; Lindman, H.; AND PHILLIPS, L. D.: Emerging Technologies for Making Decisions. Published in New Directions in Psychology II, Holt, Rinehart and Winston **1965.**
14. GREEN, D. M.; AND SWETS, J. **A.:** Signal Detection Theory and Psychophysics. John Wiley and Sons, Inc., **1966.**