NASA TECH BRIEF
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Improved Procedures for Mass Matrix-Reductions in Eigenvalue Solutions

Determination of natural frequencies and mode shapes for structures with moderately large numbers of degrees of freedom (say, more than 200) requires solution of an eigenvalue problem for which mass matrix-reduction schemes are often employed to condense the order of the problem so that it is consistent with limitations of computer storage or available time. Prevalent mass matrix-condensation procedures can be considered in the context of the well-known Rayleigh-Ritz technique; that is, in order to solve a system of equations of high order, the order of the system is reduced by assuming that the solution can be expressed in terms of a much smaller subset of preselected displacement functions. These displacement functions are customarily derived to incorporate unit displacements or unit external loads at selected "indicator" degrees of freedom. Straightforward mathematical procedures are then applied to generate a reduced mass matrix with order equal to the number of displacement functions. Then, the order of the problem solution can be condensed to the order of the reduced mass matrix. In general, accuracy of the solution will depend to some extent upon validity of the selected functions.

Analytical models of three structures (antenna pedestal and reflectors) were used to test mass matrix-reduction schemes using the Rayleigh-Ritz procedure. These models were of sufficiently small orders (165 to 175 degrees of freedom) so that it was feasible to solve the eigenvalue problems without reduction for comparison with solutions obtained with reductions for various indicator modes. The accuracy of the four lowest mode shapes and frequencies was tracked through successive mass matrix-reductions with diminishing numbers of indicator degrees of freedom. These were chosen as carefully as would be expected in normal practice. Results for all three structures were approximately equivalent in accuracy and were consistently disappointing. The errors in natural frequency were higher than expected, and it was also found that retaining as many as 30 to 40 displacement functions (20% to 25% of the original orders) in the condensed problem did not always guarantee recovery of the second (from the lowest) mode shape.

Subsequently, two new procedures were developed and tested in an attempt to improve accuracy and regain validity for the previous results. The first procedure was to augment an original set of displacement functions generated by indicators with six new displacement functions. These were developed by determining the six independent rigid body displacements of the structure for unit motions (3 translations and 3 rotations) at the foundation. Loading applied at each degree of freedom was equal to the product of the mass and rigid motion at the location, and the six sets of displacements for these loads were included as additional displacement functions. Accuracy of the solutions with indicator displacements was found to be greatly improved.

The second procedure provided dramatic improvements for a given, and possibly poorly chosen, set of displacement functions. The procedure involves the use of loads, computed as the product of the full mass matrix and mode shapes found in the initial solution, to generate displacement functions that are the basis for a new solution; implementation is by the same

(continued overleaf)

procedure used to generate the rigid body loading displacements described previously. The remainder of the computation steps to complete the eigenvalue problem solution are identical to the initial steps. The additional computation time required, although possibly insignificant in comparison with the solution time required without condensation, approaches the initial computation time. Some of the additional computation time could be saved by making the number of mode shapes retained for the second cycle equal to only about two or three times the number of accurate mode shapes desired. Furthermore, although the results clearly show a great improvement is achieved with only this second pass, additional improvements in accuracy can be obtained by continuing through several additional cycles. Indications are that with repeated cycling the results tend to converge toward the exact results for the number of mode shapes retained.

Note:

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