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Marshall Space Flight Center



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A New Algorithm for Finding Survival Coefficients Employed in Reliability Equations

The problem:

Product reliabilities are predicted from past failure rates and a reasonable estimate of future failure rates. Even with this information, useful predictions are difficult to make. Nevertheless, many industries require such predictions. For instance, electronic firms may wish to predict the reliability of a developmental memory or storage product as a function of time. This information may then be used to establish a product price that considers requirements for a field engineering force.

The solution:

A new algorithm is used to calculate the probability that a product will function correctly. The algorithm sums the probabilities of each survival pattern and the number of permutations for that pattern, over all the possible ways in which the product can survive.

How it's done:

In stores and memories, error-correcting encoding is employed to improve the reliability and reduce the service costs. Whenever a single error correcting code is employed, an error in a single-bit position is corrected and the store "survives." Whenever multiple failures occur and adversely affect only single bits, there is a calculable probability that they will occur in different words, and hence the store has a probability of surviving even with multiple failures. Specifically, the store reliability (if other failure modes are ignored) at any specified time is:

$$R_{\text{store}} = \sum_{e=0}^{\infty} P_e H_e$$

where P_e is the probability that there are exactly e bit errors in store and the H_e 's are the survival coefficients. The problem of calculating any specific H_e is cast into classical form as follows: Take an urn containing ℓ labeled balls each of n different colors (ℓn balls). (In a store ℓ

would be the number of words each of length n bits.) Pick e balls. Determine the probability that no two (or more) of them with the same label have different colors.

The algorithm, for e balls chosen, consists of four steps, (a) generating a generic survival pattern, (b) calculating the number of permutations which are still survival patterns, (c) calculating the probability of the generic pattern and multiplying it by the number of permutations, and (d) summing these products over all generic survival patterns.

The rules that are used to generate the set of generic survival patterns for a given n , ℓ , and e are as follows:

- (a) Let the number of picked balls of color $i \leq n$ and label $j \leq \ell$ be indicated by S_{ij} thus:

$$(S_{11}, S_{12}, S_{13}, \dots, S_{1\ell}) (S_{21}, S_{22}, \dots, S_{2\ell}) \dots (S_{n1}, S_{n2}, \dots, S_{n\ell})$$

$$\text{Then define } R_i = \sum_{k=1}^{\ell} S_{ik}$$

Define A_m to be equal to the number of R_i equal to m .

Define B_{im} to be equal to the number of S_{ik} equal to m for $m \geq 0$

$$\text{Define } G_j = \sum_{i=1}^j \sum_{k=1}^{\ell} B_{ik}$$

then the rules are:

- $R_1 \geq R_2 \geq \dots \geq R_n$
- $S_{ik} = 0$ for $k \leq G_{i-1}$
- $S_{ik} \geq S_{i(k+1)}$ for $k > G_{i-1}$

(continued overleaf)

(b) The number of permutations of each generic survival pattern is

$$\frac{n!}{\sum_{m=0}^c A_m! \prod_{m=1}^e B_{1m}! \prod_{m=1}^e B_{2m}! \dots \prod_{m=1}^e B_{nm}! (\ell - G_n)!} \times \frac{\ell!}{\prod_{m=1}^e B_{1m}! \prod_{m=1}^e B_{2m}! \dots \prod_{m=1}^e B_{nm}! (\ell - G_n)!}$$

(c) The probability of each pattern occurring is

$$\frac{e!}{(n\ell)^e \prod_{i=1}^n \prod_{k=1}^{\ell} (S_{ik})!}$$

(d) The probability of survival is found by summing the products of: the number of permutations [from (b)] and the probability of the pattern [from (c)] over all generic survival patterns.

Note:

Requests for further information may be directed to:
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Patent status:

Title to this invention has been waived under the provisions of the National Aeronautics and Space Act [42 U.S.C. 2457 (f)], to the IBM Corp., Huntsville, Alabama 35805.

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