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Weighted Sum-Rate based Power Allocation for the Uplink of Nonregenerative Cooperative Multi-User MIMO Communication System

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Abstract: Amplify-and-forward (AF) is one of the most popular and simple approaches to transmit information over a cooperative multi-input multi-output (MIMO) relay channel. In this paper, we propose a novel power allocation method for the uplink of multi-user MIMO AF cooperative system, which is designed to optimise the weighted sum-rate of the cooperative system. This method provides similar sum-rate performance than algorithms that are designed to solely maximise the sum-rate of the system, and at the same time it increases the fairness of the rate distribution amongst the users. The performance of our method has been assessed against a sum-rate criterion based method, and results have shown clear improvement in terms of the fairness of the user rate distribution.

Keywords: Cooperative communication, amplify-and-forward, multi-input multi-output, multi-user, uplink

1. Introduction

Cooperative communication has recently attracted considerable research interests [1–6]. It uses one or several relays to improve the coverage and enhance the spectral efficiency of wireless communication. Relay node (RN) can be either used in a regenerative, i.e., decode and forward (DF), or in a non-regenerative way, i.e., amplify-and-forward (AF). In DF, the full decoding of the source message is first performed and is then followed by the forwarding of the whole message to the destination node (DN) via the RN. Whereas in AF, the RN amplifies and forwards the signal that is received from the source node (SN).

In a single-user multi-input multi-output (MIMO) cooperative scenario, the RN was first used as a simple equal gain amplifier, i.e., original AF scheme [6]. However, it has recently been shown in [7, 8] that it can also be utilised as a smart precoder by using a precoding matrix to fine-tune the power allocation over the relay channel, and thus, improve the spectral efficiency of the cooperative system. In the multi-user (MU) case, some methods have recently been proposed to efficiently design the precoding matrix at the RN but for the cases where users have a single antenna only [9, 10]. In this paper, we develop an efficient power allocation method for the uplink (UL) of MU MIMO nonregenerative cooperative system where all the nodes of the system have multiple antennas. Moreover, our method aims at maximising the weighted sum-rate (WSR) of the system instead of the sum-rate, as it is the case in the two previously cited works. In order to design our precoding matrix, we assume as in [7, 8] that the transmit signal covariance matrix and the RN to DN link channel state information (CSI) are known at the RN.

Our novel power allocation method is designed according to the UL cooperative MIMO system model, which is introduced in Section 2. In Section 3, we explain how to efficiently design the precoding matrix at the RN for maximising the sum-rate under total power constraint and indicate how the algorithm in [8] should be modified to perform the sum-rate maximisation. We then discuss the problem of maximising the WSR under total power constraint at the RN and design a constrained gradient search algorithm. In Section 4, we show the efficiency of our WSR based algorithm by comparing its performance against a sum-rate based method. The results show clear improvement in terms of the fairness of the user rate distribution while keeping similar sum-rate performance. Finally, conclusions are drawn in Section 5.

2. UL MU MIMO Cooperative System Model

We consider a cooperative UL MU MIMO communication system that is composed of $K+2$ nodes, where K SNs, which are equipped with n_k antennas, cooperates with a nonregenerative RN, which is equipped with q antennas, to transmit data to a DN, which is equipped with r antennas, as it is depicted in Fig. 1. The aggregate number of SN transmit antennas is defined as $n = \sum_{k=1}^{K} n_k$.

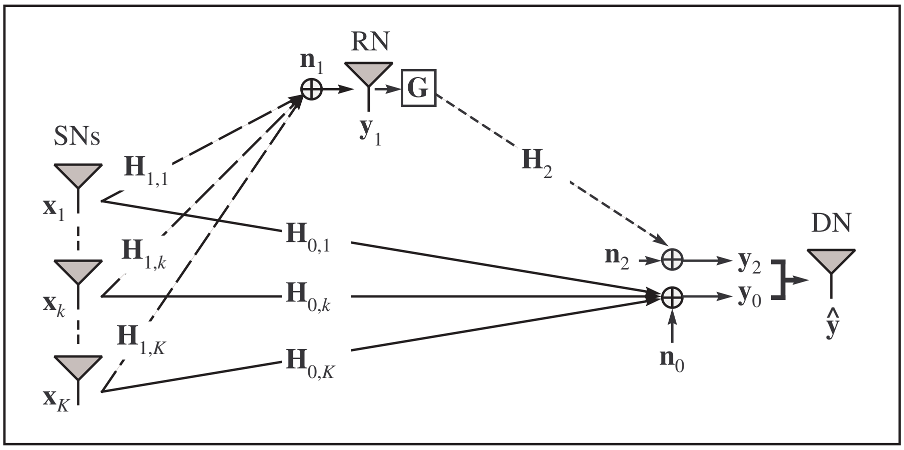

Figure 1: Nonregenerative cooperative UL MU MIMO communication system model.

For the simplicity of the introduction, we assume a half duplex relaying scenario with two equal duration phases as in [7, 8], where in the first phase each SN broadcasts the signal $\mathbf{x}_k$ to the DN and RN, and in the second phase, only the RN transmits to the DN. The signal $\mathbf{x}_k$ is received by the DN as $\mathbf{y}_0 = \sum_{k=1}^{K} \mathbf{H}_{0,k}\mathbf{x}_k + \mathbf{n}_0$ and by the RN as $\mathbf{y}_1 = \sum_{k=1}^{K} \mathbf{H}_{1,k}\mathbf{x}_k + \mathbf{n}_1$ at the end of the first phase, where $\mathbf{H}_{0,k} \in \mathbb{C}^{r\times n_k}$ as well as $\mathbf{H}_{1,k} \in \mathbb{C}^{q\times n_k}$ characterise the MIMO channel of each SN-DN and SN-RN links, respectively. During the second phase, the signal $\mathbf{y}_1$ is amplified by using the precoding matrix $\mathbf{G} \in \mathbb{C}^{q\times q}$, is then transmitted towards the DN and is received as $\mathbf{y}_2 = \mathbf{H}_2\mathbf{G}\mathbf{y}_1 + \mathbf{n}_2$ by the DN, where $\mathbf{H}_2 \in \mathbb{C}^{r\times q}$ characterises the MIMO channel of the RN-DN link. Moreover, each of the channel matrices $\mathbf{H}_{0,k}$, $\mathbf{H}_{1,k}$ $\mathbf{H}_2$ is a random matrix having i.i.d. complex Gaussian entries with zero-mean and unit variance. Furthermore, $\mathbf{n}_0 \in \mathbb{C}^{r\times 1}$, $\mathbf{n}_1 \in \mathbb{C}^{q\times 1}$ and $\mathbf{n}_2 \in \mathbb{C}^{r\times 1}$ are vectors of independent zero-mean complex

Gaussian noise entries with a variance of σ^2. The system model of UL MU MIMO cooperative communication that is introduced in Fig. 1 can be summarised as follows

$$\widehat{\mathbf{y}} = \begin{bmatrix} \mathbf{y}_0 \\ \mathbf{y}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{H}_0 \\ \mathbf{H}_2\mathbf{G}\mathbf{H}_1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{I}_r & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_2\mathbf{G} & \mathbf{I}_r \end{bmatrix} \begin{bmatrix} \mathbf{n}_0 \\ \mathbf{n}_1 \\ \mathbf{n}_2 \end{bmatrix}, \qquad (1)$$

where $\mathbf{x} = [\mathbf{x}_1^\dagger, \mathbf{x}_2^\dagger, \ldots, \mathbf{x}_K^\dagger]^\dagger \in \mathbb{C}^{n\times 1}$, $\mathbf{H}_1 = [\mathbf{H}_{1,1}, \mathbf{H}_{1,2}, \ldots, \mathbf{H}_{1,K}] \in \mathbb{C}^{q\times n}$, $\mathbf{H}_0 = [\mathbf{H}_{0,1}, \mathbf{H}_{0,2}, \ldots, \mathbf{H}_{0,K}] \in \mathbb{C}^{r\times n}$, $\mathbf{I}_r$ is a $r \times r$ identity matrix, and $(.)^\dagger$ denotes the conjugate transpose operator. The cooperative mutual information that is shared between each SN transmit signal $\mathbf{x}_k$ and the signal $\widehat{\mathbf{y}}$ can be expressed as [11]

$$I(\widehat{\mathbf{y}}; \mathbf{x}_k) = \frac{1}{2}\log_2 \left| \mathbf{I}_{2r} + \mathbf{H}_k \mathbf{R}_{\mathbf{x},k} \mathbf{H}_k^\dagger \mathbf{R}_{\mathbf{n},k}^{-1} \right| = \frac{1}{2}\log_2 \begin{vmatrix} \mathbf{A}_k & \mathbf{D}_k \\ \mathbf{C}_k & \mathbf{B}_k \end{vmatrix}, \qquad (2)$$

where

$$\mathbf{H}_k = \begin{bmatrix} \mathbf{H}_{0,k} \\ \mathbf{H}_2\mathbf{G}\mathbf{H}_{1,k} \end{bmatrix}, \mathbf{R}_{\mathbf{n},l,k} = \begin{bmatrix} \mathbf{R}_{\mathbf{n}_{0,k}} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_2\mathbf{G}\mathbf{R}_{\mathbf{n}_{1,k}}\mathbf{G}^\dagger\mathbf{H}_2^\dagger + \sigma^2\mathbf{I}_r \end{bmatrix},$$

the factor 1/2 accounts for the two-phase transmission, $\mathbf{A}_k, \mathbf{B}_k, \mathbf{C}_k, \mathbf{D}_k$ are matrices, $\mathbf{R}_{\mathbf{x},k} = \mathrm{E}\left\{\mathbf{x}_k\mathbf{x}_k^\dagger\right\}$ is the k-th transmit signal covariance matrix, $\mathrm{E}\{.\}$ stands for the expectation, $\mathbf{R}_{\mathbf{n}_{1,k}} = \sigma^2\mathbf{I}_q + \sum_{i=1,}^{k-1} \mathbf{H}_{1,i}\mathbf{R}_{\mathbf{x},i}\mathbf{H}_{1,i}^\dagger$ and $\mathbf{R}_{\mathbf{n}_{0,k}} = \sigma^2\mathbf{I}_r + \sum_{i=1,}^{k-1} \mathbf{H}_{0,i}\mathbf{R}_{\mathbf{x},i}\mathbf{H}_{0,i}^\dagger$ are noise plus residual interference covariance matrices when successive interference cancellation (SIC) is applied at the DN. In the UL, SIC can be applied to remove interference that is created by the i-th user towards the k-th user. Starting the SIC process from the K-th user in descending order, the interference from user i to k is then negligible for $i > k$. The direct and relay link mutual information, i.e., $I(\mathbf{y}_0; \mathbf{x}_k)$ and $I(\mathbf{y}_2; \mathbf{x}_k)$, can also be computed by using (2) for $\mathbf{H}_k = \mathbf{H}_{0,k}$, $\mathbf{R}_{\mathbf{n},k} = \mathbf{R}_{\mathbf{n}_{0,k}}$ and $\mathbf{H}_k = \mathbf{H}_2\mathbf{G}\mathbf{H}_{1,k}$, $\mathbf{R}_{\mathbf{n},k} = \mathbf{H}_2\mathbf{G}\mathbf{R}_{\mathbf{n}_{1,k}}\mathbf{G}^\dagger\mathbf{H}_2^\dagger + \sigma^2\mathbf{I}_r$, respectively, such that

$$\begin{aligned} I(\mathbf{y}_0; \mathbf{x}_k) &= \frac{1}{2}\log_2 |\mathbf{A}_k| = \frac{1}{2}\log_2 \left| \mathbf{I}_r + \mathbf{H}_{0,k}\mathbf{R}_{\mathbf{x},k}\mathbf{H}_{0,k}^\dagger \mathbf{R}_{\mathbf{n}_{0,k}}^{-1} \right| = \frac{1}{2}\log_2 \left| \mathbf{R}_{\mathbf{n}_{0,k+1}} \mathbf{R}_{\mathbf{n}_{0,k}}^{-1} \right| \\ I(\mathbf{y}_2; \mathbf{x}_k) &= \frac{1}{2}\log_2 |\mathbf{B}_k| = \frac{1}{2}\log_2 \left| \mathbf{I}_r + \mathbf{H}_2\mathbf{G}\mathbf{H}_{1,k}\mathbf{R}_{\mathbf{x},k}\mathbf{H}_{1,k}^\dagger\mathbf{G}^\dagger\mathbf{H}_2^\dagger (\mathbf{H}_2\mathbf{G}\mathbf{R}_{\mathbf{n}_{1,k}}\mathbf{G}^\dagger\mathbf{H}_2^\dagger + \sigma^2\mathbf{I}_r)^{-1} \right|. \end{aligned} \qquad (3)$$

Moreover, $I(\widehat{\mathbf{y}}; \mathbf{x}_k)$ in (2) can be simplified and re-expressed as $I(\widehat{\mathbf{y}}; \mathbf{x}_k) =$

$$I(\mathbf{y}_0; \mathbf{x}_k) + \frac{1}{2}\log_2 \left| \mathbf{I}_r + \mathbf{H}_2\mathbf{G}\mathbf{H}_{1,k}\mathbf{R}_{\mathbf{x},k}^{\frac{1}{2}} \widehat{\mathbf{A}}_k^{-1} \mathbf{R}_{\mathbf{x},k}^{\frac{\dagger}{2}} \mathbf{H}_{1,k}^\dagger \mathbf{G}^\dagger \mathbf{H}_2^\dagger (\mathbf{H}_2\mathbf{G}\mathbf{R}_{\mathbf{n}_{1,k}}\mathbf{G}^\dagger\mathbf{H}_2^\dagger + \sigma^2\mathbf{I}_r)^{-1} \right| \qquad (4)$$

by using [12], and where $\widehat{\mathbf{A}}_k = \mathbf{I}_{n_k} + \mathbf{R}_{\mathbf{x},k}^{\frac{\dagger}{2}} \mathbf{H}_{0,k}^\dagger \mathbf{R}_{\mathbf{n}_{0,k}}^{-1} \mathbf{H}_{0,k} \mathbf{R}_{\mathbf{x},k}^{\frac{1}{2}}$ is a positive definite matrix. Hence, $I(\widehat{\mathbf{y}}; \mathbf{x}_k) \le I(\mathbf{y}_0; \mathbf{x}_k) + I(\mathbf{y}_2; \mathbf{x}_k)$ according to (3) and (4). Moreover, it can easily be proved that $I(\widehat{\mathbf{y}}; \mathbf{x}_k) \ge \min\{I(\mathbf{y}_0; \mathbf{x}_k), I(\mathbf{y}_2; \mathbf{x}_k)\}$, Thus, $I(\widehat{\mathbf{y}}; \mathbf{x}_k)$ can be increased by maximising $I(\mathbf{y}_2; \mathbf{x}_k)$, or equivalently by optimising $\mathbf{G}$ at the RN, as it has been recently shown in [8] for the single user case. In the following, we consider that $\sigma = 1$ and $\mathbf{R}_{\mathbf{x},k} = (P_{1,k}/n_k)\mathbf{I}_{n_k}$ where $P_{1,k}$ is the total transmit power per SN.

3. WSR based Power Allocation at the RN

In the UL case, the relay link mutual information, which can be achieved by the weighted sum of the users, is given according to (3) as $\widehat{\Sigma}_{\mathbf{y}_2,\mathrm{UL}} =$

$$\sum_{k=1}^{K} w_k I(\mathbf{y}_2;\mathbf{x}_k) = \frac{1}{2}\left[\sum_{k=1}^{K}\Delta_k \log_2\left|\mathbf{I}_r + \mathbf{H}_2\mathbf{G}\mathbf{R}_{\mathbf{n}_{1,k+1}}\mathbf{G}^\dagger\mathbf{H}_2^\dagger\right| - w_1\log_2\left|\mathbf{I}_r + \mathbf{H}_2\mathbf{G}\mathbf{G}^\dagger\mathbf{H}_2^\dagger\right|\right], \tag{5}$$

where w_k is the k-th user weight, $\Delta_k = w_k - w_{k+1}$ and $w_{K+1} = 0$. The weights are used to introduce fairness in the user power distribution, they can be calculated according to system level criteria such as transmission buffer stabilisation [13] or link level criteria such as user channel quality. The decoding order of the users is fixed by the weights such that users K and 1 are first and last decoded, respectively. Moreover, the users are sorted according to their weights such that $w_1 \geq w_2 \geq \ldots \geq w_K \geq 0$, and consequently $\Delta_k \geq 0, \forall k \in [1, K]$ [13]. For the special case, where all weights are equal to one, the relay link mutual information that can be achieved by the sum of the users simplifies with (5) as

$$\Sigma_{\mathbf{y}_2,\mathrm{UL}} = \frac{1}{2}\log_2\left|\frac{\mathbf{I}_r + \mathbf{H}_2\mathbf{G}\mathbf{R}_{\mathbf{y}_{1,K}}\mathbf{G}^\dagger\mathbf{H}_2^\dagger}{\mathbf{I}_r + \mathbf{H}_2\mathbf{G}\mathbf{G}^\dagger\mathbf{H}_2^\dagger}\right|, \tag{6}$$

where $\mathbf{R}_{\mathbf{y}_{1,K}} = \mathbf{R}_{\mathbf{n}_{1,K+1}} = \mathrm{E}\left\{\mathbf{y}_1\mathbf{y}_1^\dagger\right\}$ is the relay received signal covariance matrix. The problem of maximising the sum mutual information, or sum-rate, under the constraint that the transmit power at the RN should not exceed P_2 is such that

$$\max_{\mathbf{G}} \Sigma_{\mathbf{y}_2,\mathrm{UL}} \text{ s.t. } \mathbf{G} \geq 0; \mathrm{tr}\left(\mathbf{G}\mathbf{R}_{\mathbf{y}_{1,K}}\mathbf{G}^\dagger\right) \leq P_2. \tag{7}$$

The formulations of the relay link mutual information in the single user case in [8] and of the sum mutual information, or sum-rate, in the UL MU case in (6) are the same, except that the term $\mathbf{R}_{\mathbf{y}_{1,K}}$ is replaced by $\mathbf{R}_{\mathbf{y}_1} = \mathbf{I}_q + \mathbf{H}_1\mathbf{R}_{\mathbf{x}}\mathbf{H}_1^\dagger$ in the single user case. Therefore, the $\mathbf{G}$ matrix that optimise $I(\mathbf{y}_2;\mathbf{x})$ in the single user case [8] has the same structure in the UL MU case, and is given by

$$\mathbf{G} = \mathbf{V}_2\widetilde{\mathbf{G}}\mathbf{U}_1^\dagger, \tag{8}$$

when assuming that the transmit signal covariance matrix $\mathbf{R}_{\mathbf{y}_1}$ and the RN-DN link CSI $\mathbf{H}_2$ are known at the RN [7, 8]. In equation (8), $\mathbf{V}_2$ is a column matrix that contains the q right-singular vectors of $\mathbf{H}_2$, $\widetilde{\mathbf{G}}$ is a $q \times q$ diagonal matrix with diagonal elements $\sqrt{p_i}$ and $\mathbf{U}_1$ are the eigenvectors of $\mathbf{R}_{\mathbf{y}_1}$. Here, we can use the same $\mathbf{G}$ matrix structure but where $\mathbf{U}_1$ are the eigenvectors of $\mathbf{R}_{\mathbf{y}_{1,K}}$ and $\mathbf{R}_{\mathbf{y}_{1,K}} = \mathbf{U}_1\boldsymbol{\Lambda}\mathbf{U}_1^\dagger$ with $\boldsymbol{\Lambda}$ is a $q \times q$ diagonal matrix that contains the eigenvalues λ_i of $\mathbf{R}_{\mathbf{y}_{1,K}}$, which are sorted in descending order [7]. Then, the problem in (7) simplifies as

$$\max_{\mathbf{p}} \frac{1}{2}\sum_{i=1}^{q}\log_2(1 + p_i\omega_i\lambda_i) - \log_2(1 + p_i\omega_i) \text{ s.t. } \mathbf{p} \geq 0; \sum_{i=1}^{q} p_i\lambda_i \leq P_2 \tag{9}$$

by inserting (8) in (7), and where $\mathbf{p} = \{p_1, p_2, \ldots, p_q\}$ and ω_i are the q eigenvalues of $\mathbf{H}_2^\dagger\mathbf{H}_2$, which are sorted in descending order as in [8]. The problem in (9) is concave and it can be directly solved by employing the low-complexity water-filling type of algorithm in [8].

In general case, the problem of maximising the WSR under the constraint that the transmit power at the RN must not exceed P_2 can be defined as follows

$$\max_{\mathbf{G}} \widehat{\Sigma}_{\mathbf{y}_2,\mathrm{UL}} \text{ s.t. } \mathbf{G} \geq 0; \mathrm{tr}\left(\mathbf{G}\mathbf{R}_{\mathbf{y}_{1,K}}\mathbf{G}^\dagger\right) \leq P_2, \tag{10}$$

The $\mathbf{G}$ matrix structure in (8), which conveniently diagonalises the matrices in (6) and allows to simplify (7) into (9), can not be utilised in this case since each $\mathbf{R}_{\mathbf{n}_{1,k+1}}$ term in (5) for $k \in [1, K-1]$ are most likely to not have eigenvectors that are proportional to $\mathbf{U}_1$, since $\mathbf{R}_{\mathbf{n}_{1,k+1}} = \mathbf{R}_{\mathbf{n}_{1,k}} + \mathbf{H}_{1,k}\mathbf{R}_{\mathbf{x},k}\mathbf{H}_{1,k}^\dagger$ and all the $\mathbf{H}_{1,k}$ matrices are randomly generated. Therefore, we do not assume any particular structure for $\mathbf{G}$ and we design a constrained gradient search algorithm to solve the problem in (10), where the gradient of $\widehat{\Sigma}_{\mathbf{y}_2,\mathrm{UL}}$ is given by

$$\begin{aligned}\frac{\partial \widehat{\Sigma}_{\mathbf{y}_2,\mathrm{UL}}}{\partial \mathbf{G}} = \frac{1}{\ln(2)} \Bigg[& \sum_{k=1}^{K} \Delta_k \mathbf{H}_2^\dagger \left(\mathbf{I}_r + \mathbf{H}_2\mathbf{G}\mathbf{R}_{\mathbf{n}_{1,k+1}}\mathbf{G}^\dagger\mathbf{H}_2^\dagger\right)^{-1} \mathbf{H}_2\mathbf{G}\mathbf{R}_{\mathbf{n}_{1,k+1}} \\ & - w_1 \mathbf{H}_2^\dagger \left(\mathbf{I}_r + \mathbf{H}_2\mathbf{G}\mathbf{G}^\dagger\mathbf{H}_2^\dagger\right)^{-1} \mathbf{H}_2\mathbf{G}\Bigg],\end{aligned} \tag{11}$$

since $\partial \ln\left|\mathbf{I} + \mathbf{X}\mathbf{Y}\mathbf{X}^\dagger\right| / \partial \mathbf{Y} = 2\left(\mathbf{I} + \mathbf{X}\mathbf{Y}\mathbf{X}^\dagger\right)^{-1}\mathbf{X}\mathbf{Y}$ if $\mathbf{Y}$ is an Hermitian matrix.

Algorithm 1 : UL-MU-WSR

Input: ϵ, $\mathbf{R}_{\mathbf{n}_{1,k+1}}$ and $w_k \forall k \in [1, K]$,
Set $m = 1$ and $\mathbf{G} = \mathbf{G}_{\text{init}}$;
Compute $f = \widehat{\Sigma}_{\mathbf{y}_2,\mathrm{UL}}(\mathbf{G})$ in (5);
repeat
  Evaluate $\delta\mathbf{G} = (\partial \widehat{\Sigma}_{\mathbf{y}_2,\mathrm{UL}} / \partial \mathbf{G})\mathbf{R}_{\mathbf{y}_{1,K}}^{-1}$ by using (11),
  Set $\delta\mathbf{G} = \sqrt{P_2 / (\delta\mathbf{G}\mathbf{R}_{\mathbf{y}_{1,K}}\delta\mathbf{G}^\dagger)}\delta\mathbf{G}$;
  Set $\widehat{\mathbf{G}} = \mathbf{G} + t^{-1}\delta\mathbf{G}$;
  Set $\widehat{\mathbf{G}} = \sqrt{P_2 / (\widehat{\mathbf{G}}\mathbf{R}_{\mathbf{y}_{1,K}}\widehat{\mathbf{G}}^\dagger)}\widehat{\mathbf{G}}$;
  Compute $\widehat{f} = \widehat{\Sigma}_{\mathbf{y}_2,\mathrm{UL}}(\mathbf{G} = \widehat{\mathbf{G}})$
  Set $a = \widehat{f} - f$;
  Set $m = m + 1$;
  if $(a < \epsilon)$ **then**
    Set $t = t + 1$;
  else
    Set $\mathbf{G} = \widehat{\mathbf{G}}$ and $f = \widehat{f}$;
  end if
until $(|a| < \epsilon$ or $m > 1/\epsilon)$
Output: $\mathbf{G}$.

Our novel AF power allocation based on WSR criterion is implemented via a gradient search algorithm, which often exhibits approximately linear convergence [14], i.e., the variable a converges to zero at least as fast as a geometric series. The overall complexity of the algorithm is fairly-low, however, in our case extra computation is needed to ensure that the searched $\mathbf{G}$ matrices are always within the search space, which slightly increases the complexity. Furthermore, the trade-off between accuracy and complexity can be fine-tuned by appropriately modifying the value of ϵ.

4. Results

Our novel AF power allocation method, which is summarised in **Algorithm 1** and is refereed as UL-MU-WSR, is here compared against the sum-rate criterion based method, which is described in (7) and (9) and has been implemented by employing the algorithm in [8] but with $\mathbf{R}_{\mathbf{y}_{1,K}}$ instead of $\mathbf{R}_{\mathbf{y}_1}$ as an input of the algorithm. We refer this algorithm as UL-MU-SR.

In our simulations, we denote SNR_0 as the SNR of the SN-DN link, SNR_1 as the SNR of the SN-RN link and SNR_2 as the SNR of the RN-DN link. Note that we define SNR_1 as $\mathrm{SNR}_1 = \log_{10}(P_{1,k})$, where all the $P_{1,k}$ are equal to each other, and $\mathrm{SNR}_2 = \log_{10}(P_2)$. A single-tap independent and identically distributed (i.i.d) Rayleigh fading channel is assumed between the various links, SN-DN, SN-RN, and RN-DN. We considered 5×10^3 realisations of each channel for our simulations. Note that the parameter ϵ, which is used to fine-tune the accuracy in **Algorithm 1**, has been set to $\epsilon = 10^{-5}$. Finally, we utilise the output of the UL-MU-SR algorithm to set $\mathbf{G}_{\text{init}}$ in **Algorithm 1**.

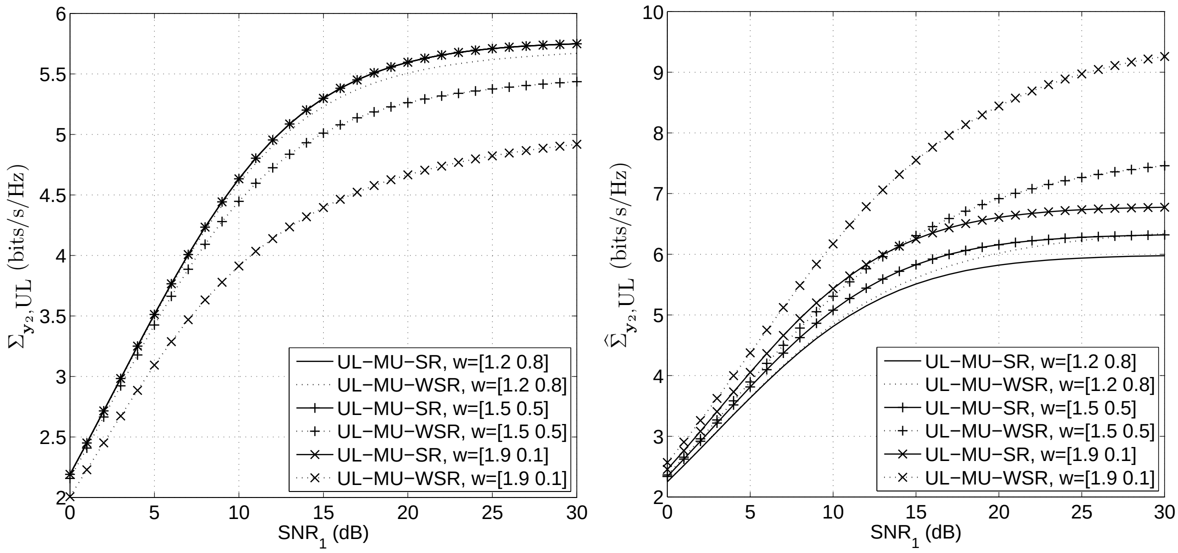

Figure 2: Relay sum-rate (left) and weighted sum-rate (right) performance of the UL-MU-SR and UL-MU-WSR algorithms in function of SNR_1 (dB) for SNR_0=0 dB, SNR_2=10 dB, $K = 2, n_k = 2, q = 4$ and $r = 4$.

On the left and right hand-side of Fig. 2, we display the relay link sum-rate and WSR performances, i.e., $\Sigma_{\mathbf{y}_2,\mathrm{UL}}$ and $\widehat{\Sigma}_{\mathbf{y}_2,\mathrm{UL}}$, respectively, of the UL-MU-SR and UL-MU-WSR algorithms. We consider a two-user UL MU MIMO scenario where $n_k = 2$, $q = 4$, $r = 4$ and we assume different weight values, i.e., $w_1 = 1.2, 1.5, 1.9$ and $w_2 = 0.8, 0.5, 0.1$. Notice that the UL-MU-SR algorithm aims at finding a $\mathbf{G}$ that maximises $\Sigma_{\mathbf{y}_2,\mathrm{UL}}$ whereas the UL-MU-WSR algorithm aims at finding a $\mathbf{G}$ matrix that maximises $\widehat{\Sigma}_{\mathbf{y}_2,\mathrm{UL}}$. Therefore, we expect our UL-MU-WSR method to outperform the UL-MU-SR in terms of WSR but at the expense of lower sum-rate performances. The results in Fig. 2 confirm our expectations and clearly indicate that our UL-MU-WSR method can be utilised to improve the WSR performance, and consequently increase the fairness of the system, but at the expense of a reduced sum-rate. Moreover, the WSR performance increases as w_1 increases.

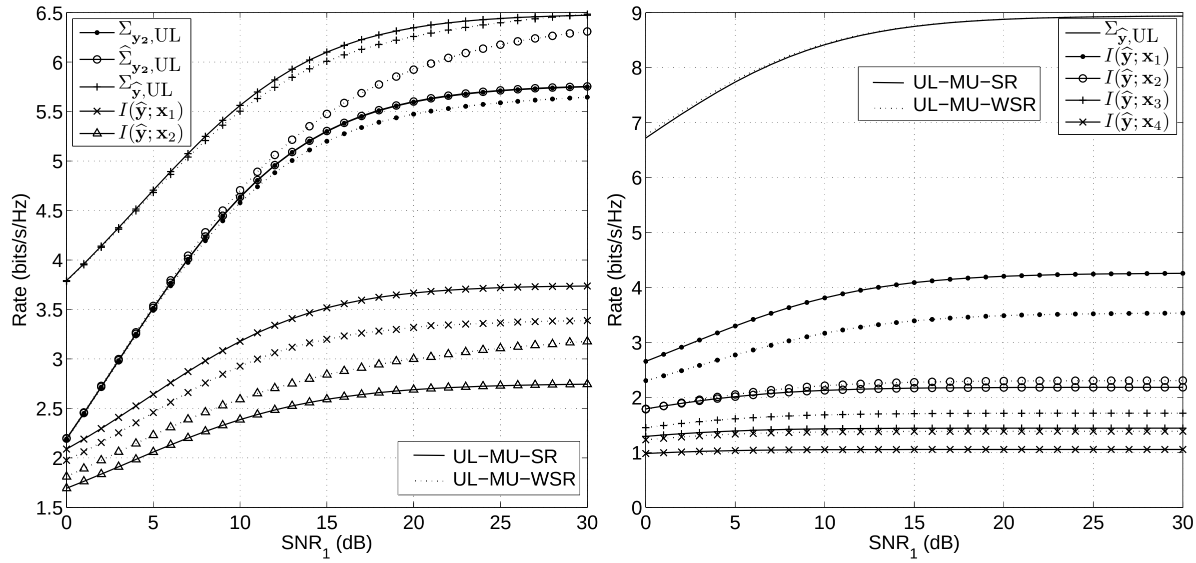

Figure 3: Performance comparison of UL-MU-SR and UL-MU-WSR algorithms in function of SNR_1 (dB) for SNR_0=0 dB, SNR_2=10 dB, $K = 2, n_k = 2, q = r = 4$ (left) and $K = 4, n_k = 4, q = r = 4$ (right).

On the left-hand side of Fig. 3, we compare the relay sum-rate, i.e., $\Sigma_{\mathbf{y}_2,\mathrm{UL}}$, relay WSR, i.e., $\widehat{\Sigma}_{\mathbf{y}_2,\mathrm{UL}}$, cooperative sum-rate, i.e., $\Sigma_{\widehat{\mathbf{y}},\mathrm{UL}} = \sum_{k=1}^{K} I(\widehat{\mathbf{y}};\mathbf{x}_k)$, and cooperative user rate performances of the UL-MU-SR and UL-MU-WSR algorithms against SNR_1 (dB) and for SNR_0=0 dB, $\mathrm{SNR}_2 = 10$ dB, $K = 2, n_k = 2, p = 4$ and $q = 4$. Here, the weights have been computed according to the channel norm of each user, and the user K is the one with the best channel norm. Moreover, the ordering of the users has then been made according to their respective weights. Firstly, as in Fig. 2, the results show that the UL-MU-WSR algorithm outperforms the UL-MU-SR method in terms of WSR performance. Secondly, the results show that both algorithms provide similar cooperative sum-rate performance, i.e., the performance of the UL-MU-WSR algorithm are slightly below the one of the UL-MU-SR method for medium range of SNR_1 (dB). However, the UL-MU-WSR algorithm provides a fairer cooperative user rate distribution, since the rates of each user gets closer to each other. On the right-hand side of Fig. 3, we perform the same comparison but with an increased number of users and antennas, i.e., for $K = 4, n_k = 4, p = 4$ and $q = 4$. The results indicate that both UL-MU-SR and UL-MU-WSR provide similar cooperative sum-rate performance but with a different cooperative user rate distribution. Part of the rate of the strongest users is shared amongst the other users when using the UL-MU-WSR method, which increases the fairness of the rate distribution.

5. Conclusions

In this paper, we have introduced a novel power allocation method for nonregenerative cooperative UL MU MIMO communication, which is designed to maximise the weighted sum-rate of the cooperative system. We have shown how to efficiently design the precoding matrix at the RN for maximising the sum-rate under total power constraint and indicate how the algorithm in [8] should be modified to to perform the sum-rate maximisation. We then discuss the problem of maximising the WSR under total power constraint at the RN and design a constrained gradient search algorithm. The performance of our WSR based method has been assessed against a sum-rate based method and the results have shown that our novel power allocation method can provide similar cooperative sum-rate performances than the sum-rate based method, but with an increase fairness in terms of the user rate distribution. Future work could be carried out by considering the joint power allocation at SNs and RN under WSR to further increases the fairness of the user rate distribution.

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