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A PARAMETRIC APPROACH TO IRREGULAR
FATIGUE PREDICTION

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SUMMARY

The method proposed consists of two parts: empirical determination of certain characteristics of a material by means of a relatively small number of well-defined standard tests, and arithmetical application of the results obtained to arbitrary loading histories. The following groups of parameters are thus taken into account: the variations of the mean stress, the interaction of these variations and the superposed oscillating stresses, the spectrum of the oscillating-stress amplitudes, and the sequence of the oscillating-stress amplitudes. It is pointed out that only experimental verification can throw sufficient light upon possibilities and limitations of this (or any other) prediction method.

FUNDAMENTALS OF PARAMETRIC APPROACH

The fundamental procedural scheme of the method evolved in this paper consists of the following phases:

- (1) Determination of a number of characteristics of the material by means of standard tests
- (2) Prediction of the fatigue life of the material on the basis of these characteristics and an analysis of the expected loading history.

The problem thus raised can, in principle, always be solved since for a specified accuracy there will always be a finite number of tests from which the necessary data for a satisfactory processing of the second phase can be drawn. There is, consequently, a problem of interpolation which can be solved with a finite number of base points. The question remains how to attain the goal economically. In view of the present state of development of servo-hydraulic testing equipment and digital computers, neither irregular stress sequences in the standard tests nor extensive algorithms in analysis and evaluation are prohibitive.

The progress of our knowledge of fatigue strength has in the last few decades become an alarming "parameter explosion." Thus, in order to avoid undue complications, no mention will be made of parameter groups connected with notch effect or environmental influences and nothing but the loading history will be considered.

Consequently, the following effects will be taken into account:

- (1) Mean stress effect (index M), influence of the chronological curve of the mean stress
- (2) Interaction effect (index J), influence of systematically occurring correlations between the effects of mean and oscillating stresses
- (3) Spectral effect (index P), effect of the statistical distribution of the oscillating-stress variations
- (4) Sequence effect (index Q), effect of the sequence in which the individual stress variations follow one another.

The parameters linked to these effects will be called in this paper "M.I.S.S. parameters" from their initial letters.

Owing to the inherent complexity of the problem, a certain number of simplifying assumptions must be made. In particular it is assumed that

- (1) Miner's rule is applicable with sufficient accuracy to sufficiently narrow sections of a stress spectrum
- (2) The influence of the mean-stress effect can be described with sufficient accuracy by the statistical amplitude distribution of the variations occurring in the mean stress
- (3) Every increment of a loading history produces an incremental interaction effect approximately proportional to its mean stress level and to its "linear-damage increment" (according to Miner's rule)
- (4) The effect of the sequence extends mainly to the "coarse sequence" (that is, to changes of the spectrum over longer periods of the loading history), whereas the "fine sequence" (that is, the individual sequence of the single-stress cycles) is of minor importance in practice, provided the process under consideration can be described with sufficient accuracy by stochastic characteristics
- (5) The effects of the M.I.S.S. parameters allow with sufficient accuracy a linear (in one exceptional case, a quadratic) interpolation when the logarithm of the "Miner sum"

$$\sum = \sum_i \frac{n_i}{N_i} \cdot \dots \quad (1)$$

(which according to Miner's rule should always be equal to 1) is used as the interpolation value. In equation (1), n_i is the number of cycles applied at stress level whereas N_i is the total number of cycles to cause failure at stress level.

It is not possible here to justify these assumptions. A justification of these assumptions is given in reference 1.

OUTLINE OF THE M.I.S.S. METHOD

The method outlined here starts from relation (1); it being clearly understood, however, that a sum not equal to 1 is permissible. In accordance with the assumptions made, the four M.I.S.S. parameters P_M , P_J , P_P , and P_Q are defined and

$$\sum = f(P_M, P_J, P_P, P_Q) \dots \quad (2)$$

is postulated. This postulation means that the spectrum, as in Miner's rule, is still the most important group of parameters, but not the only decisive one.

Thus, the first phase of the method results in the performance of an unequivocally defined series of standard tests with well-defined M.I.S.S. parameters and the determination of the resulting standard Miner sums \sum_S which are to be considered as characteristics of the material.

The second phase, that is, the application to an expected loading history with a given nominal fatigue life, is divided into the following partial phases:

- (1) Analysis of the loading history according to equation (1). The result is the Woehler-Miner sum \sum_W .
- (2) Analysis of the loading history according to well-defined formulae for determining their M.I.S.S. parameters.
- (3) Application of the M.I.S.S. parameters as a means of interpolation in the results field of the standard Miner sums. The result is the Effective Miner sum \sum_E .
- (4) Formation of the quotient \sum_E / \sum_W which indicates the chances of survival of the specimen. For $\sum_E / \sum_W > 1$, survival of the loading history is to be expected.

The first and fourth phases are carried out according to the known algorithms of Miner's rule. The interpolation in the third phase must mainly be carried out linearly for $\log \sum$. Figure 1 shows the interpolation for P_M and P_J , with P_P and P_Q held constant. The heights of the column at its four edges represent the results of four standard tests. The result of the interpolation is the height (heavy dotted line) for P_M and P_J . Only the standard tests and the stress-history analysis need to be considered in more detail.

MATHEMATICAL TREATMENT OF PARAMETER GROUPS

It would be impossible to give a complete review of the mathematical deductions applied to obtain appropriate equations for the calculation of the M.I.S.S. parameters. It must be referred, therefore, to more detailed works on the subject. (See refs. 1 and 2.) The following remarks are made in order to give a general idea of the logical structure of the method.

All the equations used in this connection are based solely on the simplifying assumptions made. For instance, the second assumption means only that the mean stress effect for given values of the other parameters is determined by the "spectrum of the mean stress" so that Miner's rule may be considered as applicable to the mean stress. Thus, an extremely simple definition of the mean-stress parameter is obtained; that is,

$$P_M = \sum \frac{m_\sigma}{N_\sigma \sum_W} \dots \quad (3)$$

where m_σ is the number of mean-stress variations of the value σ , and N_σ is the cycle number pertaining to σ according to the σ -N curve. The division by the Woehler-Miner sum \sum_W is used for normalizing purposes.

The spectrum of a loading history is represented in principle by a parabolic approximation so that theoretically, three parameters are needed for the mean value, the average slope, and the curvature of the parabola. The first of these parameters, however, is insignificant since in the chosen representation, the mean value of the parabola is given. It is a particular feature of the method that the ordinal numbers i of certain well-defined stresses σ_i are used as values of the independent variable of the approximation so that the respective numbers n_i are expressed by

$$\frac{n_i}{N_i \sum_W} = c_0 + c_1 \cdot i + c_2 \cdot i^2 \dots \quad (4)$$

where c_0 , c_1 , and c_2 are the spectral parameters P_P .

In a more or less analogous way, the parameters for interaction and sequence, P_J and P_Q , are deduced from the assumptions made. Both are connected with variations of the spectral parameter c_1 ; thus, predominance of high- or low-stress amplitudes under particular conditions is indicated. Although P_J establishes a correlation with the mean stress σ_m (taking into account such phenomena as the increased dynamic forces due to increased payload of a vehicle), P_Q accounts for variations undergone in

the course of the loading history as a whole (as encountered owing to more frequent overload when an aircraft is transformed from passenger to cargo transport); thus, it serves as a safeguard against unpleasant surprises, because many materials have a shorter fatigue life when first subjected to low- and then to high-stress amplitudes.

The influence of parameter variations upon loading histories is illustrated in figures 2, 3, and 4. In figure 2, the oscillating stresses are not present for negative mean stresses because of the choice of constants in the equations. In figure 3 the stress amplitudes are higher in section II than section I; thus, a higher value of c_1 is indicated. In figure 4, the variation of c_2 results in predominance either of extreme (high and low) or of medium amplitudes. It will be observed that in figure 3, section II shows higher stress amplitudes than section I; thus, a higher value of c_1 is indicated. Variation of c_2 results in predominance either of extreme (high and low) or of medium amplitudes (fig. 4).

STANDARD TESTS AND PRACTICAL APPLICATION

Since linear interpolation of $\log \sum$ has been accepted for all parameters (excluding c_1), variations through all possible combinations other than the meaningless ones ($P_M = 0_i$; $P_J + 0$) would give 36 base points which can be found from 36 tests (plus repetitions). The cost is considerable but is certainly justified for important materials.

Experience will show whether all 36 standard tests are really required. The number depends on how far the multidimensional functions of figure 1 can be approximated by plane surfaces. If, for example, it should be found that, apart from P_P , the function surface can be considered as sufficiently plane, then only the inclinations of this plane would have to be known; that is, for every additional parameter only one single base point would have to be determined and nine tests would suffice. Probably the truth lies between these two extremes.

An actual "cooking recipe" for the details of the method is found in reference 1. There the individual steps are not only commented on but are also compiled in tabular form so that the economic establishment of suitable computer programs is possible.

POSSIBILITIES AND LIMITATIONS

By the method described, it is possible to determine the probable fatigue life of a given material under a given loading history without having to carry out a large number of loading tests. As a matter of fact, the standard Miner sums as characteristics of a material, together with the σ -N curve, should contain enough data to predict the fatigue

life of a material with sufficient accuracy even in irregular stress sequences. The main point obviously is 'What is meant by "sufficient accuracy"?' Of course, the M.I.S.S. method is more expensive than the determination of a number of σ -N curves, although not impossibly so. In return, it refines the results obtained from Miner's rule (which are derived only from σ -N curves). Only systematic tests with different materials can give a definite answer. Such tests should prove worthwhile, for even Miner's rule in its present form is better than it is usually believed to be. Thus, it should usually be possible to supplement it by introducing additional parameters (mean stress, interaction, and sequence) so that the result can satisfy the demands of practice.

It must be stressed in this connection that the testing equipment available should be improved in the sense of cheaper and faster execution of large numbers of irregular fatigue tests. Only such a development will guarantee in reasonable time the acquisition of the information necessary for an efficient verification of the methods proposed here or elsewhere.

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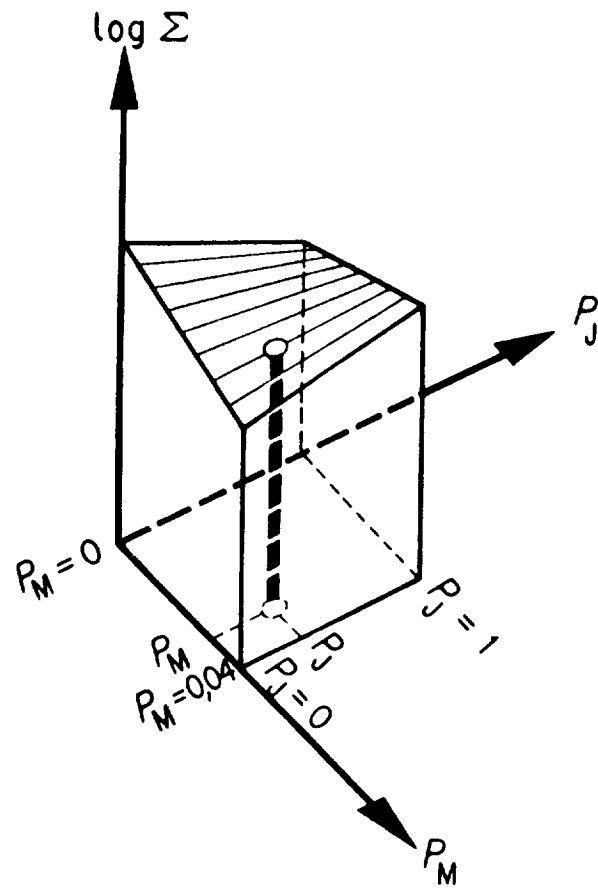


Figure 1.- Interpolation for P_M and P_J .

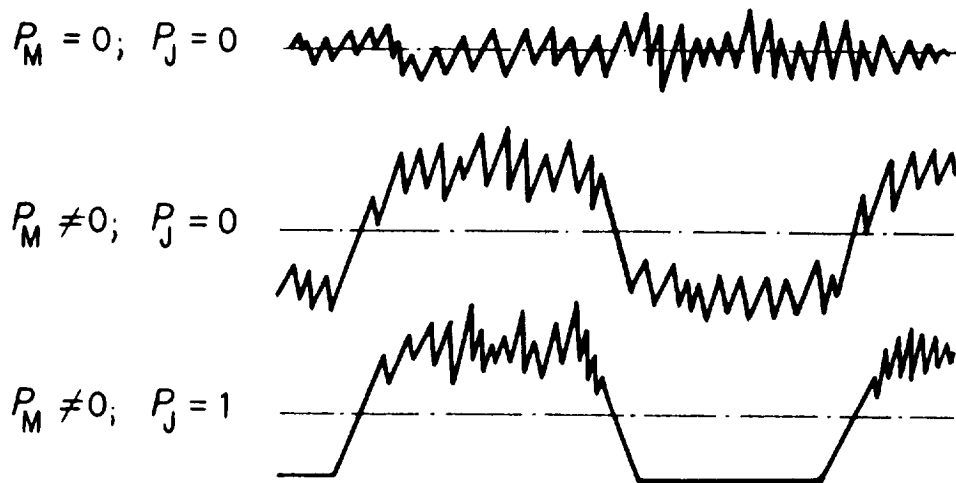


Figure 2.- Characteristic stress curves for various values of P_M and P_J .

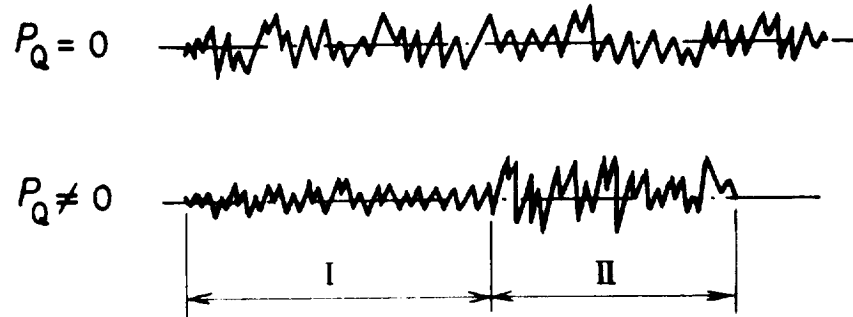


Figure 3.- Characteristic loading histories for $P_Q = 0$ and $P_Q \neq 0$ and $P_M = P_J = 0$.

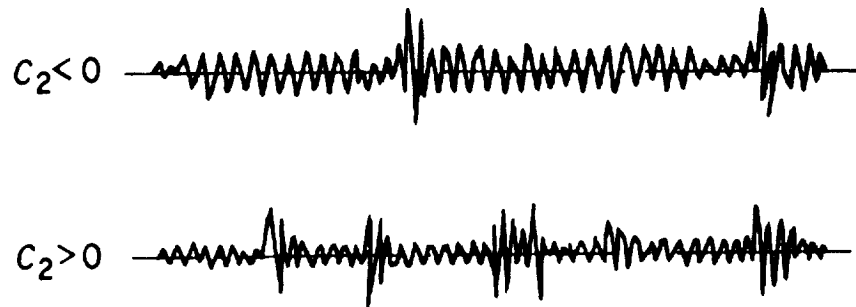


Figure 4.- Characteristic stress curves for various values of the spectrum curvature parameter c_2 .