DIRECT STIMULATION OF THE RETINA BY THE METHOD OF VIRTUAL-QUANTA FOR HEAVY COSMIC-RAY NUCLEI*

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The method of virtual-quanta is based on the fact that the electromagnetic field of a fast charged particle approaches the field of a plane wave as the particle speed approaches the speed of light. The electromagnetic field of a particle that passes through the retina appears to a rod in the retina as an electromagnetic pulse in time. The quantization of the components of the Fourier frequency spectrum of this pulse of electromagnetic radiation yields the virtual quanta. This method assumes that the virtual-quanta interact with the retina in the same way as ordinary photons.

We have calculated the contribution to the frequency of visual sensations induced in the dark-adapted eye by the virtual-photon field associated with the heavy-nuclei that exist in the region of space beyond the geomagnetic field. In order to determine the probability that the virtual-photon field induces a light-flash, we utilize only the portion of the virtual-photon spectrum that corresponds to the known frequency dependence of the sensitivity of human rods to visible light. The results of our calculation can be expressed as a curve of the mean frequency of light-flashes induced by the absorption of at least R virtual-photons versus the threshold number R. The contribution to the light-flash frequency from the virtual-photon field of heavy cosmic-ray nuclei is smaller than that from Cerenkov photons. We have used the flux and energy spectra of galactic cosmic-ray nuclei helium to iron measured by Comstock, Fan and Simpson.

INTRODUCTION

In the preceding paper,[1] we calculated the frequency of inducing visual sensations by Cerenkov radiation from heavy cosmic-ray nuclei traversing the retina. The results of that calculation for high detection efficiencies corresponding to states of relatively complete dark-adaption, are in general agreement with the frequencies reported by astronauts on Apollo missions 11-14. However, it has been shown that these sensations can be induced by means other than free or Cerenkov photons. Light-flashes have been induced by a number of stimuli including mechanical pressure, electric currents, X-rays, and energetic neutrons.[2-14] In the case of the energetic neutrons,[13,14] the flashes probably result from recoil protons or heavier nuclei produced in the neighboring media; more particularly, the results of Fremlin[14] seem to rule out any significant contribution from Cerenkov radiation.

In this paper we have modified the well known Method of Virtual-Quanta (MVQ) to provide a theoretical formalism for calculating the frequency of light-flashes to be expected from direct excitation of the photo-receptor elements by charged particles. The results of our calculation agree with the data of Tobias et al[13] and Fremlin[14] in the sense that one can produce visual sensations by non-relativistic particles; however, the frequency of visual sensations calculated by this method for astronauts exposed to the spectrum of galactic cosmic-ray nuclei at solar minimum is at least an order of magnitude lower than the highest rates observed. The MVQ as applied in this paper calculates only the contributions from direct excitation of the rhodopsin molecules through the exchange of virtual-quanta in a narrow energy range corresponding to optical frequencies; it should, therefore, be viewed as a minimum estimate of the

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contribution due to direct excitation.

METHOD OF VIRTUAL-QUANTA

The mechanism for the electromagnetic interaction between two charged particles is the exchange of a virtual-quantum. If the momentum transferred during the collision is small, then the exchange can be looked upon as the absorption by the target particle at rest of a virtual-quantum from the electromagnetic field accompanying the moving particle. The number and configuration of the virtual-quanta change dramatically as the speed of the incident particle approaches the speed of light. Fig.(1) is a schematic diagram showing how the magnitude of the electric field at a given distance from its charged particle source changes as the particle's speed approaches the speed of light. At rest ($\beta = 0$), the electric field at a given distance has the same magnitude in all directions; whereas for $\beta = 0.5$, the field has increased in directions transverse to the particle's motion, and it has decreased slightly in the direction parallel to the motion. This elongation of the electric field in the transverse direction increases as $\beta \rightarrow 1$. In calculating the effect on the target particle of absorbing one of these virtual-quanta, it is possible to ignore the incident particle itself and consider only the cloud of virtual-quanta that accompanies it. Weizsacker[15] and Williams[16] have developed the details of this method for charged particles moving through a vacuum. A quantum of angular frequency ω has an energy k given by the Planck-Einstein formula

$$k = \hbar\omega \qquad (1)$$

where $\hbar$ is Planck's constant divided by 2π. The energy spectrum of the photons in the equivalent pulse of virtual-quanta can be obtained directly from the frequency spectrum of the electromagnetic fields in the pulse. The differential cross-section for a given inelastic interaction (e.g. pion production, nuclear excitation, or, as in our case, excitation of a molecule) for events in which a photon of energy between $\hbar\omega$ and $\hbar\omega + d\hbar\omega$ is emitted by the incident particle and absorbed in the target system is given by the MVQ as the product of the number of virtual-quanta in the incident pulse with energies between $\hbar\omega$ and $\hbar\omega + d\hbar\omega$ and the experimental cross-section $d\sigma_Q(\hbar\omega)/d\hbar\omega$ for the analogous photon-induced interaction.

$$d\sigma_P(\hbar\omega)/d\hbar\omega = N(\hbar\omega)\, d\sigma_Q(\hbar\omega)/d\hbar\omega \qquad (2)$$

Visual sensations induced by light incident on the retina result ultimately from the absorption of individual light quanta by rhodopsin molecules that are contained in the outer segments of the rod cells. Although a rod responds to a single absorbed photon, a single absorption does not seem to be sufficient to activate the optic nerve to produce a visual sensation. The threshold for a visual sensation even under optimal physiological conditions requires from two to

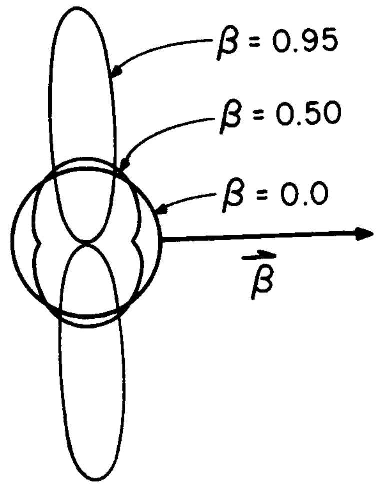

Figure 1. Magnitude of the electric field a given distance from a point source that moves with speed βc. The distance from a point on a curve to the source represents the magnitude of the field at that orientation.

perhaps as high as fourteen absorptions of photons in coincidence by rods in a cluster served by a single optic nerve fiber.[17,18] Previous exposure to light increases the threshold required for the production of light-flashes.[19,20]

The rhodopsin molecules are contained in disk-shaped sacs that are stacked at intervals of a few hundred Angstroms in the outer segments of the rod cells. The rhodopsin molecule is sensitive to light at wave lengths that are longer than distances between successive sacs and much longer than the inter-molecular distances of the medium; therefore, it is necessary to modify the MVQ to take into account the polarizability of the molecules in the rod. The virtual-quanta field polarizes the molecules of the medium which in turn change the field of virtual-quanta experienced by the rhodopsin molecule. For optical wavelengths, we can modify the MVQ to include polarization by using the characteristic macroscopic fields in a dielectric instead of the free space fields.

The expression for the number of virtual-quanta that have energies between $\hbar\omega$ and $\hbar\omega + d\hbar\omega$ has been shown[21] to be of the form

$$N(\hbar\omega) = \frac{2Z^2\alpha}{\pi|\varepsilon(\omega)|^2\beta^2\hbar\omega}\left[\ln\left(\frac{1.123v}{\omega a|1-\beta^2\varepsilon(\omega)|^{1/2}}\right) - \frac{\beta^2\varepsilon'}{2}\right] \qquad (3)$$

where α is the fine-structure constant, Z is the atomic number of the incident particle, and $v = \beta c$ is its speed; the function $\varepsilon(\omega)$ represents the dielectric permeability of the medium; the constant a is the minimum value of the impact parameter which we take equal to one Bohr radius; and $\varepsilon'(\omega)$ represents the real part of $\varepsilon(\omega)$. The number of visible virtual-quanta can be obtained by integrating Eq.(3) over the complete optical spectrum. Since the rhodopsin molecules are not equally sensitive to all photons in the visible range, we have taken into account the rhodopsin sensitivity by replacing the actual absorption spectrum by a narrow rectangular distribution of width $\Delta\hbar\omega$ = 0.52 eV centered about the energy of peak sensitivity, $\hbar\omega_0$ = 2.48 eV. All photons in this reduced optical region are assumed to have the maximum probability (20%)[22] of being absorbed by a rhodopsin molecule upon traversing the retina; photons outside of this region are assumed to have zero probability of absorption.

The number of virtual-quanta with energies in this restricted visible region that accompany a charged particle as it passes through the retina is given by

$$M = \frac{2Z^2\alpha\Delta\hbar\omega}{\pi|\varepsilon_0|^2\beta^2\hbar\omega_0}\left[\ln\left(\frac{1.123v}{\omega_0 a|1-\beta^2\varepsilon_0|^{\frac{1}{2}}}\right)-\frac{\beta^2\varepsilon_0'}{2}\right] \quad (4)$$

The dielectric permeability $\varepsilon(\omega)$ is now represented by a complex dielectric constant ε_0 which can be represented in terms of the real quantities ε_0' and ε_0'' as

$$\varepsilon_0 = \varepsilon_0' + i\varepsilon_0'' \quad (5)$$

The real and imaginary parts of the dielectric constant are related to the index of refraction n and the absorption coefficient κ by

$$n^2 - \kappa^2 + 2in\kappa = \varepsilon_0' + i\varepsilon_0'' \quad (6)$$

For transparent materials, therefore, ε_0' can be replaced by the square of the index of refraction. Absorption of visible light passing through the retina occurs primarily in the outer segments of the rod and cone cells. Individual outer segments transmit about 70% of the light incident upon them, resulting in at least 80% transmission for the retina as a whole. Taking advantage of the relative transparency of the components of the retina, the known values of n^2 were substituted for ε_0. Sidman[23] has measured the index of refraction to be 1.41 and 1.39 for the outer segments of rods and cones respectively; the extra cellular fluid which separates the outer segments in the living retina presumably has the same index of refraction as physiological saline, about 1.33.[24]

The term ε_0'' depends upon the amount of absorption in the medium. If the retina were completely transparent, ε_0'' would be zero; since the retina is only slightly opaque, it can be expected to be small. The number of visible virtual-quanta obtained from Eq.(4) is not sensitive to the exact value of ε_0'' used. A crude estimate of ε_0'' can be obtained from the relationship[21]

$$\varepsilon_0'' = \left(\frac{cN}{\omega_0}\right) d\sigma_a(\omega_0)\ /d\omega \quad (7)$$

where N is the number of rhodopsin molecules per unit volume in the medium. Eq.(7) can be rewritten in terms of the ratio of the transmitted light intensity I to the incident intensity I_0 as

$$\varepsilon_0'' = -(c/\omega_0\ x)\ \ln\ (I/I_0) \quad (8)$$

The quantity x in Eq.(8) is the equivalent thickness of the rhodopsin molecules in the retina. Dartnell et al[25] have shown that if the outer segment of the rod is 2.5 μ in diameter, there must be $N_1 = 2\times10^8$ rhodopsin molecules in an outer segment for 30% probability of absorption for photons passing through the outer segment. If we assume a cylindrical outer segment of radius r and with a thickness of rhodopsin x we have

$$N_1/\ \pi r^2 x = N_0\ \rho/A \quad (9)$$

where N_0 is Avogadro's number, A is the molecular weight of rhodopsin ($\sim$ 40,000), and ρ is its density ($\sim$ 1.27 gm/cm^3). The value of x obtained from Eq.(9) is 2.13 μ. Substituting this value into Eq.(8) for 70% transmission through an outer segment yields ε_0'' = 0.0133. This value was used in Eq.(4) for our calculations.

The number of virtual-quanta M accompanying a nucleus of atomic number Z is plotted in Fig.(2) as M/Z^2 versus the velocity of the nucleus. There is a maximum value of the velocity above which less than M virtual-quanta accompany the nucleus through the layer of outer segments. The values of this maximum velocity are plotted in Fig.(3) for M = 2, 10, 20, 50 and 100. If the threshold number of photons R required for a visual sensation is greater than or equal to M then, ignoring fluctuations in M, particles moving faster than β_{max} will not produce visual sensations. When $M \geq R$, the probability that at least R out of the M incident virtual-quanta will be absorbed is given by

$$Q_M\ (\geq R) = \sum_{T=0}^{T=M-R} \frac{M!\ P^{(M-T)}(1-P)^T}{T!\ (M-T)!} \quad (10)$$

For the retina the probability P of absorption by a rhodopsin molecule for photons in the "visible" wavelength region defined earlier is 0.2.

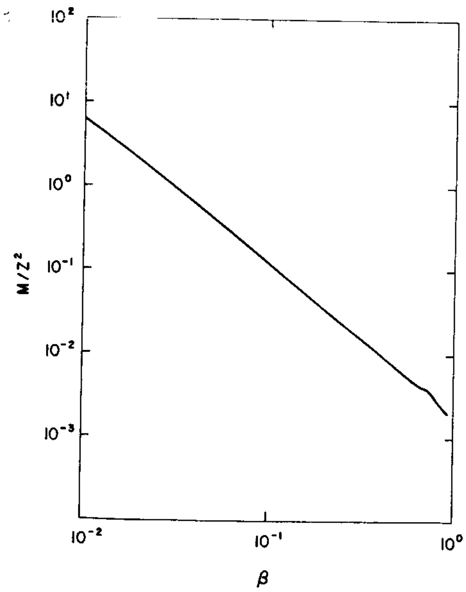

Figure 2. Plot of M/Z^2 as a function of speed at the retina.

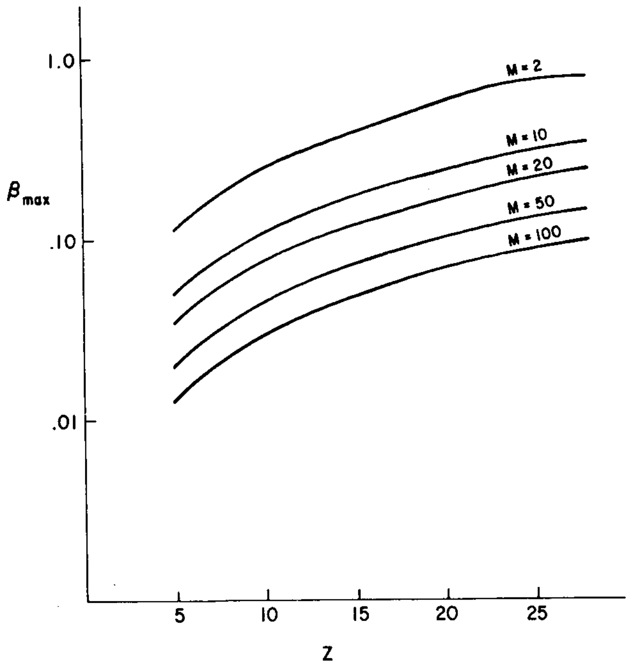

Figure 3. Maximum velocity at the retina for at least M virtual-quanta.

EXPECTED FREQUENCIES OF LIGHT FLASHES

The flux and energy spectra of the cosmic-ray nuclei from boron to nickel measured by Comstock, Fan and Simpson[26] were used in calculations. Each energy spectrum was separated into kinetic energies per nucleon above and below a particular value T_1; the spectrum for that atomic number Z then being represented as

$$n_Z\ (T) = C_1 \qquad (0 < T < T_1) \qquad (11)$$

$$n_Z\ (T) = C\ (T + mc^2)^{-\gamma} \qquad (T > T_1) \qquad (12)$$

where mc^2 is the proton rest mass. The values of T_1, C_1 and C used for the nuclei included in our calculations are listed in Table 1. The value of γ used for all nuclei was 2.5. The integral flux spectra corresponding to Eqs.(11-12) can be written

$$N\ (> T) = \frac{C(T + mc^2)^{(1-\gamma)}}{(\gamma - 1)} \qquad (T > T_1) \qquad (13)$$

and

$$N\ (> T) = C_1\ (T_1 - T) + C\ (T_1 + mc^2)^{(1-\gamma)}/(\gamma - 1) \qquad (0 < T < T_1)\ (14)$$

To compute the mean frequency of light-flashes to be expected from direct excitations in the optical region defined earlier, we have prepared a program that calculates the contribution to the the light-flash frequency from each nucleus of atomic number Z. We sketch the steps in the program:

TABLE 1

Parameters for Representing the Differential Flux Spectra of Galactic Cosmic-Rays at a Time of Minimum Solar Modulation

Nucleus	C_1 p/m^2 -s-sr-(BeV-nuc^{-1})	C p/m^2 -s-sr-(BeV-nuc^{-1}) - + 1	T_1 (BeV/nuc)
C^{12}, O^{16}	5.4	12	.437
B^{11}	1.7	4.3	.513
Ne,Mg,Si	1.0	1.4	.207
P-Mn	1.0	1.4	.207
Fe-Ni	1.0	1.4	.207

a. Find from Eq.(4) the maximum kinetic energy per nucleon T_M' at the retina that each cosmic ray nucleus can have and still be accompanied by M visible virtual-quanta.

b. Calculate the range in tissue R_p' of a proton with this kinetic energy. For this purpose we used the power-law[1]

$$R_p' = k\ T_m'^{\alpha} \qquad (15)$$

The values $k = 4.25 \times 10^{-3}$ gm/cm^2 H_2O - MeV $^{\alpha}$ and $\alpha = 1.627$ were used.

c. The maximum kinetic energy per nucleon T_M in the incident spectrum that a particle can have and be accompanied by M virtual-quanta after it has traversed a shielding thickness ℓ is given by

$$T_M = ((R_p' + Z^2\ \ell/A)/k)^{1/\alpha} \qquad (16)$$

d. The unidirectional flux density $N_{Z,M}(>T_M)$ of nuclei of atomic number Z with kinetic energies per nucleon greater than T_M can be obtained from Eqs.(13-14); the differential unidirectional flux of nuclei of atomic number Z that are accompanied by M visible virtual-quanta can then be written:

$$\Delta N_{Z,M} = N_{Z,M+1/2}(> T_{M+1/2}) - N_{Z,M-1/2}\ (> T_{M-1/2}) \qquad (17)$$

The number of nuclei of charge Z to pass through each eye per minute is obtained by multiplying $\Delta N_{Z,M}$ by

$$Y = 120\ G\ (\cos\beta_1 - \cos\beta_2) \qquad (18)$$

where β_1, β_2 and G are given in Ref. 1.

e. The number of light-flashes per minute in both eyes caused by direct excitation by cosmic-ray nuclei of atomic number Z is given by

$$C_Z\ (\geq R) = \sum_{M=R} 2\ Y\ \ \Delta N_{Z,M}\ Q_M\ (\geq R) \qquad (19)$$

In Fig.(4) the relative contributions to the light-flash frequency from the most abundant of the heavy cosmic-ray nuclei are given for a threshold of 10 absorbed "visible" virtual-quanta. Similar results were obtained for R = 2 and 20.

f. The total frequency of light flashes induced by at least R direct excitations of rhodopsin molecules is given by

$$C(\geq R) = \sum_{Z} \sum_{M=R} 2\, Y\, \Delta N_{Z,M}\, Q_M\, (\geq R) \qquad (20)$$

Fig.(5) plots the light-flash frequency as a function of the threshold R.

RESULTS AND DISCUSSION

The curve in Fig.(5) shows that the frequency of light flashes to be expected from the direct excitation of the rhodopsin molecule by the exchange of "visible" virtual-quanta is much lower than the frequencies to be expected from the absorption of Cerenkov quanta that were calculated in Ref.(1). Even if one assumes that the astronauts were completely dark-adapted and, therefore, had thresholds for visual sensations somewhere between 2 and 14 quanta, examination of Fig.(5) shows the frequency expected to be much less than the rates observed by astronauts on Apollo missions 11, 12, 13 and 14.

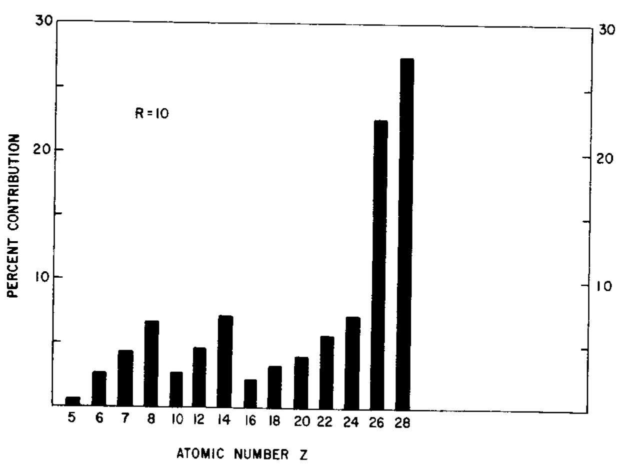

Figure 4. Relative contributions to the observed light-flash frequencies for threshold R=10.

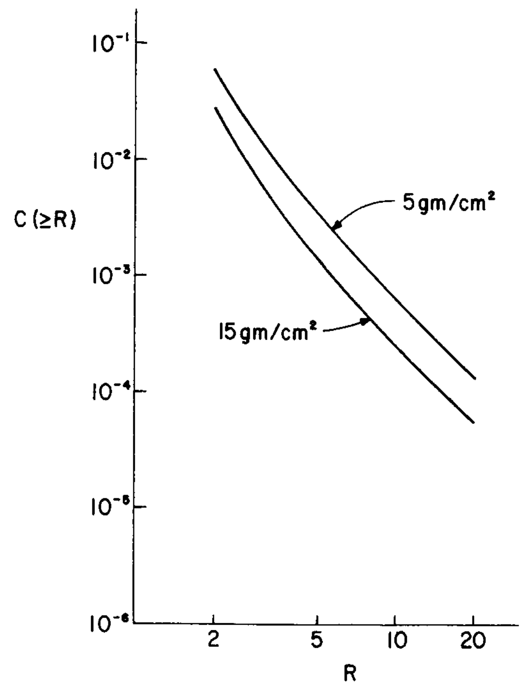

Figure 5. Counting rate versus threshold R.

The importance of the MVQ is that given the cross-section for a photon of frequency ω exciting a molecule producing an effect of interest, one can calculate the cross-section for a direct excitation of the molecule by a particle of charge Z and speed βc in which the incident particle loses energy $\hbar\omega$. For the retina we know the geometrical and coincidence requirements for visual sensations as well as the probability that each photon will excite a rhodopsin molecule; this knowledge extends, however, only over a very narrow band of photon energies. The contribution from direct excitation calculated in this paper is thereby limited to those collisions with rhodopsin molecules in which the energy lost by the incident particle corresponds to optical frequencies. The potential contribution from more energetic collisions is, therefore, ignored but could be calculated if the cross section for the analogous photo-excitations and photo-ionizations leading to visual sensations were known.

ACKNOWLEDGEMENTS

We would like to thank Mr. John B. McLaughlin for his help in writing the computer programs.

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