February 1971

brought to you by T CORE

Brief 71-10003

# **NASA TECH BRIEF** *Marshall Space Flight Center*

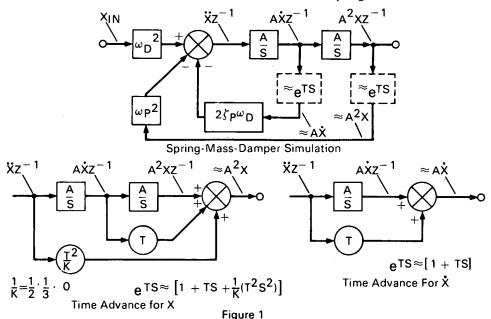


NASA Tech Briefs announce new technology derived from the U.S. space program. They are issued to encourage commercial application. Tech Briefs are available on a subscription basis from the National Technical Information Service, Springfield, Virginia 22151. Requests for individual copies or questions relating to the Tech Brief program may be directed to the Technology Utilization Office, NASA, Code KT, Washington, D.C. 20546.

# **Digital Simulation Error Curves for a Spring-Mass-Damper System**

# The problem:

Reduce the excessive number of samples per cycle and the unnecessary iterations which increase cost and computation time on digital simulations of continuous data systems.  $A = \omega \begin{pmatrix} 1 \\ S \end{pmatrix}$  is the normalized integration gain.  $\omega_D$  is the effective angular resonance frequency.  $\omega_P$  is the programmed angular resonance frequency.  $\zeta_P$  is the programmed damping ratio. T is the sampling interval in seconds.



#### The solution:

Plot the digital simulation errors for a springmass-damper system. Using these error curves, select the type of integration, the feedback update method, and the number of samples per cycle at resonance.

## How it's done:

Block diagrams of a spring-mass-damper simulation and the preferred time advance update method are shown in Figure 1, where:  $Z^{-1}$  is a transfer function for T seconds time delay. e<sup>TS</sup> is a transfer function for T seconds time advance.  $\frac{1}{k}$  is a constant for time advance update.

I is the number of iterations for iteration update.

X,  $\hat{X}$ , and  $\hat{X}$  represent the mass displacement, velocity, and acceleration, respectively.

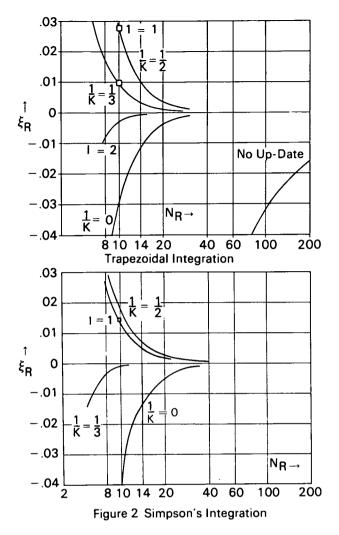
S is the Laplace operator.

 $N_R$  is the number of samples per cycle at resonance.

The effective damping ratio is:

This document was prepared under the sponsorship of the National Aeronautics and Space Administration. Neither the United States Government nor any person acting on behalf of the United States Government assumes any liability resulting from the use of the information contained in this document, or warrants that such use will be free from privately owned rights.

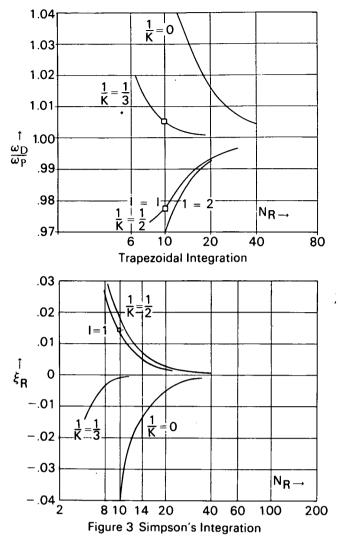
 $<sup>\</sup>zeta_{\rm D} \approx \zeta_{\rm P} + \xi_{\rm R}$ (continued overleaf)



where  $\xi_R$  is the damping bias error. This error is plotted (Figure 2) for trapezoidal integration and Simpson's 1/3 formula, where N<sub>R</sub> is the number of samples per cycle at resonance. Curves are shown for the iteration update method where I = 1, and for the time advance update method where 1/K = 0, 1/3, and 1/2.

The normalized resonance frequency,  $\omega_D / \omega_P$ , shown in Figure 3, is for trapezoidal integration and Simpson's 1/3 formula.

For the curves for trapezoidal integration, the time advance update method with 1/K = 1/3 has smaller errors at 20 samples per cycle than the one-iteration update method at 30 samples per cycle. Thus, for the same accuracy, the time advance update method will solve the simulated problem in half the time of the one-iteration update method. This is based on an added time per iteration of T/4, i.e., one-quarter of the computation time without iteration.



## Note:

Requests for further information may be directed to:

Technology Utilization Officer Marshall Space Flight Center Code A&TS-TU Huntsville, Alabama 35812 Reference: B71-10003

# Patent status:

No patent action is contemplated by NASA.

Source: Lewis Alfred Knox of International Business Machines Corp. under contract to Marshall Space Flight Center (MFS-20770)