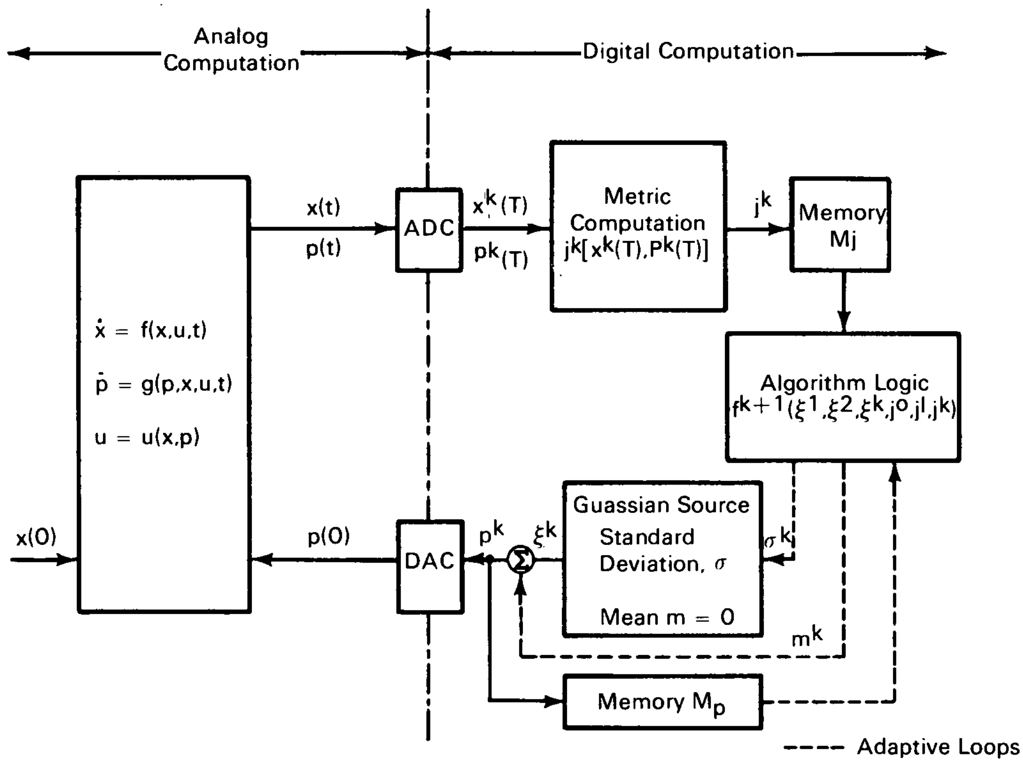
NASA TECH BRIEF

Global Search Algorithm for Optimal Control



The Pontryagin Maximum Principle provides a powerful theory for determining the optimal control of dynamic systems describable by nonlinear differential equations with bounded control. When applied to a high-order system, the Maximum Principle yields much information about the nature of the solution, but the actual solution is seldom obtained, because a difficult, mixed boundary-value problem is invariably encountered. In such instances, the known boundary conditions are divided between initial and terminal values, and the mapping from the initial to terminal boundary values is not, in general, explicitly known. Most current computer algorithms which treat this problem are based on local-seeking methods and the assumption of a single minimum. Such methods are of limited value when the hypersurface of the performance function being studied is multipeaked or discontinuous.

To resolve some of these difficulties, a method based on random techniques has been devised and used successfully. The method is based on an adaptive random-search algorithm employing both local and global properties to solve the two-point boundary-value problem in the Maximum Principle for either

(continued overleaf)

fixed or variable end-time problems. The mixed-boundary-value problem is transformed to an initial-value problem, and the required mapping between initial and terminal values is constructed with the aid of a hybrid computer.

For simplicity, the essence of the method is illustrated for the fixed-time case. On the left-hand side of the figure, the set of equations to be solved which result from an application of the Maximum Principle is shown. The solution of these equations determines the optimal control function which minimizes an arbitrary performance function. The complete set of boundary conditions for these equations is generally not known. On the right-hand side of the figure, the algorithm used to determine the unknown initial boundary conditions is illustrated. The algorithm is based on: (1) introducing a vector metric which measures the vector distance by which the desired terminal boundary conditions are not satisfied; and (2) reducing this vector metric iteratively by searching for the unknown initial boundary conditions from a Gaussian probability density function. However, the pure random search is generally unsatisfactory because of excessively long convergence times. The convergence properties are improved by making the system adaptive by varying the mean M and standard deviation σ as a function of the past performance as indicated by the adaptive loops and the algorithm logic $f(\xi^1, \xi^2, \xi^3, J^0, J^1, \ldots J^k)$. Iteration stops when the vector metric J is small enough to satisfy the problem requirements.

Several variations of the function f of the algorithm were investigated. The most useful characteristic given to the function f was based on the notion of a creeping, expanding and contracting search. The creeping character of the search is provided by varying the mean of the distribution to equal the initial condition of the adjoint vector on the last successful iteration. The expansion and contraction character of the search is provided by varying the standard deviation so that the search is localized when successful, but expanded geometrically when not successful. This type search has desirable local and global search properties; "hanging up" on a local minimum, as with gradient methods, is avoided. Furthermore, it will not matter if the boundary surface is discontinuous or how many peaks or valleys there are.

The hybrid computer is ideally suited to implementing the above algorithm. By comparison, an all-digital implementation is about 1,000 times slower. The requirement for solving the equations of motion a large number of times is fulfilled most efficiently on an analog device, and the memory and computation capability required by the algorithm are best implemented on a digital device. The amount of apparatus required increases linearly with the order of the system.

The feasibility of the method has been demonstrated experimentally on several highly nonlinear systems of an order as large as six.

Notes:

1. The following documentation may be obtained from:

   Clearinghouse for Federal Scientific
   and Technical Information
   Springfield, Virginia 22151
   Single document price $3.00
   (or microfiche $0.65)

   Reference:
   NASA-TN-D-5642 (N70-18083), An Adaptive Random-Search Algorithm for Implementation of the Maximum Principle

2. Requests for further information may be directed to:

   Technology Utilization Officer
   Ames Research Center
   Moffett Field, California 94035
   Reference: TSP70-10637

Patent status:

No patent action is contemplated by NASA.

Source: E. C. Stewart, W. P. Kavanaugh, and D. H. Brocker
Ames Research Center
(ARC-10359)