AEC-NASA TECH BRIEF

AEC-NASA Tech Briefs describe innovations resulting from the research and development program of the U.S. AEC or from AEC-NASA interagency efforts. They are issued to encourage commercial application. Tech Briefs are published by NASA and may be purchased, at 15 cents each, from the Clearinghouse for Federal Scientific and Technical Information, Springfield, Virginia 22151.

Structure of the Isotropic Transport Operators in Three Independent Space Variables

Based on the idea of separation of variables, a spectral theory for the three-dimensional, stationary, isotropic transport operator is presented (*1*) in a vector space of complex-valued Borel functions resulting in continuous sets of regular and generalized eigenfunctions.

Because of the non-self-adjoint nature of this operator, the results could not be anticipated intuitively from the known decomposition of this operator in the special case of plane geometry. The results obtained indicate a promising new approach to analytic solution of the linear transport equation in higher space dimensions. Examples are given for slab geometry with and without axial symmetry, spherical symmetry, and cylindrical symmetries.

The use of elementary functions for construction of solutions for more than one space dimension, if only for special geometries, had been previously attempted. In this work (*1*) a decomposition of the transport operator in three space dimensions is carried out. This method is suggested by the principle of separation of variables, which is widely used for the decomposition of linear partial-differential equations of mathematical physics. The solution space for the transport equation is mathematically defined as a set of complex-valued Borel functions. For an attempt at completeness it was necessary to add to the set of regular eigenfunctions, and the corresponding regular expansion modes over a complex continuum, a set of generalized eigenfunctions and the corresponding generalized expansion modes over a complex continuum of even higher dimension. The regular eigenfunctions are characterized by a complex vector called a regular (or proper) eigenvector, Λ. These eigenvectors, however, must satisfy the condition $\Lambda \cdot \Lambda = \lambda_0^2$, where λ_0^2 assumes exactly one value for any given equation; therefore $\pm\lambda_0$ can be called eigenvalues. For yield of the generalized eigenfunctions the improper integral must be evaluated.

A basic result of this research is the general solution to the transport equation in a sense analogous to that of the theory of differential equations. Special representations are obtained by imposition of geometric restrictions on the general solution. These enable the numerical analyst to construct exact test problems for checking of the performance of general multidimensional transport codes, and therefore can be most helpful in error-analysis. The calculation of singular integrals is not difficult if properly executed. There is little doubt that the results of the work can be generalized to anisotropic scattering, multiregion, and multigroup problems. A similar theory can be conceived for the discrete ordinate approximations also; their spectra, modes, and general solutions can then be compared with this continuous case (*1*) for a measure of fitness for the approximate solutions.

Many papers on the spectral representation of the time-dependent equation give a general expansion in exponentials, although little is said regarding the structure of the time-independent coefficient functions. This report fills this gap.

Reference:

1. E. H. Bareiss and I. K. Abu-Shumays, *ANL-7328* (Argonne National Laboratory, May 1967).

Notes:

1. Researchers in mathematical or applied physics may be interested.

(continued overleaf)

2. Inquiries concerning this innovation may be directed to:

   Office of Industrial Cooperation
   Argonne National Laboratory
   9700 South Cass Avenue
   Argonne, Illinois 60439
   Reference: B69-10432

Source: E. H. Bareiss and I. K. Abu-Shumays
Applied Mathematics Division
(ARG-10448)

Patent status:

Inquiries concerning rights for commercial use of this innovation may be made to:

Mr. George H. Lee, Chief
Chicago Patent Group
U.S. Atomic Energy Commission
Chicago Operations Office
9800 South Cass Avenue
Argonne, Illinois 60439