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Some Numerical Methods for Integrating Systems of First-Order Ordinary Differential Equations

Study of some numerical methods of integration of systems of coupled first-order ordinary differential equations has been reported (*1*). The report describes the extrapolation methods of Bulirsch-Stoer and Neville, presents the results of tests comparing these extrapolation methods with the Runge-Kutta and Adams-Moulton methods, and shows some circumstances under which the extrapolation method may be preferred. Results of double-precision tests of the Bulirsch-Stoer method show the efficiency of various orders of this method as a function of desired accuracy.

The speed and accuracy of the methods were compared by programming the methods on the same computer (a CDC-3600) and applying each to the same sets of equations. The numbers of times that the functions to be integrated had to be evaluated, for a desired accuracy, were used as a computer-independent and program-independent measure of time. The accuracies were compared by use of each method to integrate from given initial conditions to some final point at which the relative error was computed.

The results of cases run by the four methods tested indicate that the efficiency of the solution is quite dependent on proper matching of the order of the method of solution to the desired accuracy. For example, in tests on the negative-exponental equation, for a relative error of 10^{-16} the Neville method with $M = 4$ takes twice as long as the method with $M = 6$ and about 2.5 times as long as the shortest time with $M = 10$. The Runge-Kutta fourth-order method and the Adams-Moulton fourth-order method take about 160 times as long as the Neville method with $M = 10$, or about 250 times as long as the Bulirsch-Stoer method with $M = 10$. Even at relative errors of 10^{-9} the Adams-Moulton and Runge-Kutta methods take about 9 times as long as the Neville and Bulirsch-Stoer methods with $M = 6$. At the other extreme, if a rough estimate is desired and errors up to 10^{-2} can be tolerated, Neville's method with $M = 10$ takes almost 7 times as long as the fourth-order Adams-Moulton methods, and the methods of lower order are preferable.

The double-precision tests of the Bulirsch-Stoer method show it to be quite powerful for work of great accuracy; it seemed especially capable of coping with changing difficulty of solution of the equations presented to it.

Reference:

1. N. W. Clark, *ANL-7428* (Argonne National Laboratory, March 1968); available from CFSTI, Springfield, Va. 22151, at $3.00 (microfiche, $0.65).

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Patent status:

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Category 02