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RESEARCH IN THE RESTRICTED PROBLEMS OF THREE AND FOUR BODIES

Paul B. Richards, Principal Investigator
Irwin S. Bernstein
Winchung A. Chai
Jane Cronin
Jordan Ellis
William E. Fine
Sheldon Kass
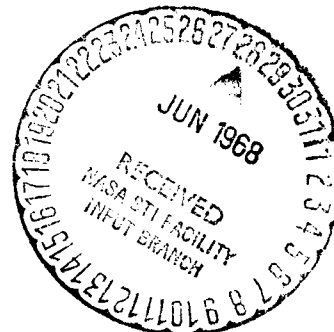Samuel A. Musa
Lawrence H. Russell

Final Scientific Report

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ABSTRACT

Seven studies have been conducted on research in the existence and nature of solutions of the restricted problems of three and four bodies. The details and results of five of these research investigations have already been published, and the latest two studies will be published shortly. A complete bibliography of publications is included in this report.

This research has been primarily qualitative and has yielded new information on the behavior of trajectories near the libration points in the Earth–Moon–Sun and Sun–Jupiter–Saturn systems, and on the existence of periodic trajectories about the libration points of the circular and elliptical restricted four-body models. We have also implemented Birkhoff's normalization process for conservative and nonconservative Hamiltonian systems with equilibrium points. This makes available a technique for analyzing stability properties of certain nonlinear dynamical systems, and we have applied this technique to the circular and elliptical restricted three-body models. A related study was also conducted to determine the feasibility of using cislunar periodic trajectories for various space missions. Preliminary results suggest that this concept is attractive for space flight safety operations in cislunar space.

Results of this research will be of interest to mathematicians, particularly those working in ordinary differential equations, dynamical systems and celestial mechanics; to astronomers; and to space guidance and mission analysts.

INTRODUCTION

The objective of this project has been to extend the knowledge of cislunar trajectories through research in the restricted problems of three and four bodies. Like its predecessor program (Contract No. AF 49(638) 1325) of the same title, the current study has been primarily qualitative, with emphasis on establishing the existence of solutions in these three and four-body models of the n-body problem, and also on determining the nature of these solutions, that is, their stability and boundedness properties.

During this contract period, our research in the restricted problems of three and four bodies was supported in part by the Air Force Office of Scientific Research of the Office of Aerospace Research, under contract AF 49(638) 1466; in part by the NASA Marshall Space Flight Center under contract NAS 8-20062; and in part by the NASA Goddard Space Flight Center under contract NAS 5-9350. Additional support of this and related research on cislunar space flight was provided by General Precision Systems Inc.

Significant accomplishments achieved during this contract period include (1) analysis of the nature and behavior of solutions of the circular restricted four-body problem near the L_1 libration point for the Earth-Moon-Sun system, "Approximate Effect Of The Sun On Satellites Near The Interior Earth-Moon Libration Point"; (2) analysis of the boundedness of solutions of the linearized circular restricted four-body problem near the L_4 libration point for the Earth-Moon-Sun system and the Sun-Jupiter-Saturn system, "On The Boundedness Of Solutions Of A Linearized Restricted Problem Of Four Bodies"; (3) analysis of the existence and nature of periodic solutions about the libration points of the circular and elliptical restricted four-body problems, extended from the libration point solutions in the circular and elliptical restricted three-body problems, "Some Periodic Solutions Of Elliptical And Restricted Four-Body Problems"; (4) analysis of the existence of periodic solutions about the libration points of the circular restricted four-body problem, extended from periodic solutions about the libration points in the circular restricted three-body problem, "Some Periodic Solutions Of A Four-Body Problem-II"; (5) implementation of the Birkhoff normalization technique to analyze stability properties of

conservative Hamiltonian systems near an equilibrium point, with specific application to the circular restricted three-body problem, "Birkhoff Normalization Process Program For Time-Independent Hamiltonian Systems"; (6) implementation of the Birkhoff normalization technique to analyze stability properties of nonconservative Hamiltonian systems near an equilibrium point, with specific application to the elliptical restricted three-body problem, "Birkhoff Normalization Process Program For Time-Dependent Hamiltonian Systems".

The results of all of these studies have been published, or will be shortly. It seems appropriate, therefore, to include in this report abstracts of these papers, together with a list of the publications, to which the interested reader may refer for complete details.

We also include here an abstract of a related study to exploit periodic trajectories in cislunar space, "A Concept For Space Flight Safety". This conceptual study was undertaken as a result of our mathematical interests in cislunar trajectories, and because these periodic trajectories appear to have characteristics suitable for certain space missions, such as supply depots for manned vehicles, experimental stations, observation stations, communication stations, and space flight safety vehicles. Our work to date in this area, conducted under a company financed R&D program, has established the feasibility of using cislunar periodic trajectories for space flight safety. The article abstracted here has been published and is included in our list of publications for further reference.

ABSTRACTS OF PUBLICATIONS

"Approximate Effect Of The Sun On Satellites Near The Interior Earth-Moon Libration Point", Paul B. Richards.

A restricted four-body problem which is a model for the motion of a vehicle in the Sun-Earth-Moon system is obtained by rotating the barycenter of the two massive bodies in a restricted three-body configuration about a third massive body. The presence of this third body (Sun) introduces nonautonomous terms into the equations of motion of the vehicle, and the libration points of the embedded restricted three-body problem no longer represent point solutions. Approximate analytical expressions for the motion in the neighborhood of the collinear libration point lying between the Earth and the Moon (L_1) are obtained and analyzed. In contrast to the motion near L_1 in the restricted three-body problem, the present model indicates that arbitrarily small neighborhoods of L_1 do not contain periodic or bounded trajectories. General solutions in the neighborhood of L_1 do indicate, however, that the rate at which a vehicle leaves the area can be controlled to some extent by the initial conditions. An important role is played by the initial configuration of the Sun-Earth-Moon geometry, which is equivalent to the launch date. The effect of this factor is not visible in the restricted three-body analysis. The present study indicates that L_1 is an attractive parking and rendezvous site for manned lunar missions of several days duration, without the application of thrust. Longer missions will require maneuvering thrust.

"On The Boundedness Of Solutions Of A Linearized Restricted Problem Of Four Bodies", Irwin S. Bernstein and Jordan Ellis

A four-body problem which is a perturbation of the restricted three-body problem is linearized about a triangular libration point of the three-body problem resulting in a linear nonhomogeneous system with periodic coefficients. Using an analysis similar to that of Danby and Bennett in their study of the elliptic three-body problem, necessary and sufficient conditions are found to determine whether all solutions of this system will be bounded. As this four-body problem is a model for the motion of a vehicle in the

Earth–Moon–Sun and Sun–Jupiter–Saturn systems,the constants relevant to each system are inserted into the model to ascertain whether the linearized model gives bounded solutions. For the Earth–Moon–Sun system there are solutions which are unbounded. For the Sun–Jupiter–Saturn system all solutions are bounded.

"Some Periodic Solutions Of Elliptical And Restricted Four–Body Problems"
Irwin S. Bernstein and Samuel A. Musa

The existence of periodic solutions in elliptical and restricted four–body problems about the libration points of the restricted three–body problem is proved for sufficiently small values of the parameter μ , the ratio of the mass of the third body to the cube of its distance from the barycenter of the other two massive bodies. In the case of the elliptical four–body problem, an additional assumption is required, namely that the periods of the motion of this barycenter about the third body and the motion of the other two massive bodies about each other are commensurable.

It is shown that the elliptical and restricted four–body problems can be regarded as perturbations of the elliptical and restricted three–body problems respectively, where μ is the perturbation parameter. Accordingly, the Poincaré small–parameter method is employed to show that the particle or space vehicle of insignificant mass has a periodic motion in the neighborhood of each of the five libration points of the restricted three–body problem provided μ is small and the period of the nonautonomous terms (produced as a result of the third massive body) is not an integral multiple of the fundamental period of the unperturbed linear system.

Symmetry arguments are then used to prove the existence of a symmetric periodic solution in the neighborhood of each of the three collinear libration points of the restricted three–body problem for both elliptical and restricted four–body models. However, in the former case, more stringent conditions on the initial configuration of the three bodies are required. Such conditions imply that the three bodies must lie along the same axis and at their least or greatest separations at some arbitrary time.

"Some Periodic Solutions Of A Four-Body Problem-II", Jane Cronin, Paul B. Richards, Irwin S. Bernstein

Further results are obtained for the four-body problem studied in a previous paper. It is shown that the problem can be formulated in two ways as a perturbation of the restricted three-body problem. According to the Lyapunov Center Theorem, there is a family of periodic solutions near a libration point of the restricted three-body problem. Here we prove that corresponding to each periodic solution in an infinite subset of this family, there is a periodic motion of the particle of negligible mass in our four-body problem provided that a single bifurcation equation, which can be formulated explicitly, has a solution.

"Birkhoff Normalization Process Program For Time-Independent Hamiltonian Systems", W. A. Chai and S. Kass

This is a complete documentation (program description) of a computer package for analyzing finite time stability properties of equilibrium points of time-independent Hamiltonian systems. Using this package, approximate analytic solutions which contain nonlinear effects can be constructed near these equilibrium points.

Three main programs constitute the complete package. The first program normalizes and computes the generating functions for a time-independent Hamiltonian in the neighborhood of an equilibrium point, the second expresses coordinate transformations as truncated power series, and the third program computes point-by-point coordinate transformations.

To demonstrate the procedure, this report describes in detail the application of the computer package to the construction of solutions of the planar restricted three-body problem near the L_4 equilibrium point. Numerical results compare very favorably in precision with values obtained by numerical integration. The computer time required for the entire normalization process is only a small fraction of the integration time, however.

"Birkhoff Normalization Process Program For Time-Dependent Hamiltonian Systems",
W. Fine and S. Kass

This is a complete documentation of a computer program for analyzing finite time stability properties of equilibrium points of time-dependent (nonconservative) Hamiltonian systems. Using this package, approximate analytic solutions which contain nonlinear effects can be constructed near these equilibrium points. Qualitative rate-of-growth estimates can also be obtained. This program description is a sequel to "Birkhoff Normalization Process Program For Time-Independent Hamiltonian Systems", AFOSR Scientific Report No. AFOSR 67-0123 (October, 1966) which contained the program description for analyzing time-independent Hamiltonian systems.

Eleven programs constitute the complete package. The first six programs perform the second order normalization and the next three apply the Birkhoff technique for higher order normalizations. These nine programs compute the generating functions and coordinate transformations for a time-dependent Hamiltonian in the neighborhood of an equilibrium point. The remaining two programs evaluate actual trajectories for the particular application under consideration. The first of these evaluates the trajectory for the normalized Hamiltonian and the second performs a point by point integration of the equations of motion of the given dynamical system.

The computer program developed here has been used to generate an algebraic solution for the motion of a particle near an equilibrium point of the planar elliptical restricted three-body problem. Sufficient time-dependent and nonlinear terms have been retained to produce trajectories which match solutions obtained by numerical integration of the equations of motion for 30 days or longer. Computation time for the normalized trajectory is insignificant compared to the time required for numerical integration. Sources of truncation and round-off errors in the normalization process are identified.

"A Concept For Space Flight Safety", Paul B. Richards

        Manned or unmanned space vehicles performing routine missions while coasting on periodic orbits that encompass the earth and the moon may be used to save astronauts whose space vehicles encounter emergency situations on the way to the moon. The launching of a manned space vehicle, such as an Apollo, would be timed to insure that the Apollo trajectory between the earth and the moon would always be "close enough" to a rescue vehicle's orbit in case the Apollo needs help. Preliminary calculations indicate that a rescue vehicle can be so prepositioned in a periodic orbit encompassing the earth and the moon that upon receipt of the Apollo distress signal, the rescue vehicle can leave its orbit and rendezvous with the Apollo in three to ten hours or less. It is possible that this rescue operation can be achieved with boosters and capsules that are currently operational or under test.

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