

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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*Newtonian Planetary Ephemerides 1800-2000:  
Development Ephemeris Number 28*

*Jay H. Lieske*

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JET PROPULSION LABORATORY  
CALIFORNIA INSTITUTE OF TECHNOLOGY  
PASADENA, CALIFORNIA

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## Contents

<b>I. Introduction</b> . . . . .	1
<b>II. Source Ephemerides</b> . . . . .	2
A. System of Masses . . . . .	2
B. Theoretical Secular Corrections . . . . .	3
C. Constant Corrections . . . . .	3
<b>III. Numerical Integration</b> . . . . .	4
A. Method . . . . .	4
B. Integration Accuracy Tests . . . . .	5
<b>IV. Ephemeris Fitting Procedure</b> . . . . .	6
<b>V. Results</b> . . . . .	7
A. The Fit to Modified Source Ephemerides . . . . .	7
B. Secular Deficiencies in Newtonian Source Ephemerides . . . . .	7
<b>Nomenclature</b> . . . . .	8
<b>References</b> . . . . .	9
<b>Appendix A. Tape Format</b> . . . . .	11
<b>Appendix B. Initial Conditions and Statistics for DE28</b> . . . . .	12
<b>Appendix C. Residual Plots, Modified Source Minus DE28</b> . . . . .	18
<b>Appendix D. Secular Deficiencies of Modified Source Ephemerides</b> . . . . .	58

## Tables

1. Planetary masses . . . . .	2
2. Secular corrections to basic source ephemerides . . . . .	3
3. Constant corrections to source ephemerides . . . . .	3
4. Relativistic corrections to perihelia . . . . .	4
5. Integration statistics . . . . .	6
B-1. The fitted ephemeris of Mercury . . . . .	13
B-2. The fitted ephemeris of Venus . . . . .	13
B-3. The fitted ephemeris of earth-moon barycenter . . . . .	14
B-4. The fitted ephemeris of Mars . . . . .	14

## Contents (contd)

### Tables (contd)

B-5. The fitted ephemeris of Jupiter . . . . .	15
B-6. The fitted ephemeris of Saturn . . . . .	15
B-7. The fitted ephemeris of Uranus . . . . .	16
B-8. The fitted ephemeris of Neptune . . . . .	16
B-9. The fitted ephemeris of Pluto . . . . .	17
D-1. Improved modified source statistics for Venus. . . . .	70
D-2. Improved modified source statistics for earth-moon barycenter . . . . .	70
D-3. Improved modified source statistics for Jupiter . . . . .	71

### Figures

C-1. Relative changes in energy and angular momentum integrals for 1800–2000 . . . . .	18
C-2. Relative changes in angular momentum components for 1800–2000 . . . . .	20
C-3. Mercury rectangular residuals (modified source minus DE28) for 1800–2000 . . . . .	22
C-4. Mercury angular residuals (modified source minus DE28) for 1800–2000 . . . . .	24
C-5. Venus rectangular residuals (modified source minus DE28) for 1800–2000 . . . . .	26
C-6. Venus angular residuals (modified source minus DE28) for 1800–2000 . . . . .	28
C-7. Earth-moon barycenter rectangular residuals (modified source minus DE28) for 1800–2000 . . . . .	30
C-8. Earth-moon barycenter angular residuals (modified source minus DE28) for 1800–2000 . . . . .	32
C-9. Mars rectangular residuals (modified source minus DE28) for 1800–2000 . . . . .	34
C-10. Mars angular residuals (modified source minus DE28) for 1800–2000 . . . . .	36
C-11. Jupiter rectangular residuals (modified source minus DE28) for 1800–2000 . . . . .	38
C-12. Jupiter angular residuals (modified source minus DE28) for 1800–2000 . . . . .	40
C-13. Saturn rectangular residuals (modified source minus DE28) for 1800–2000 . . . . .	42



## Contents (contd)

### Figures (contd)

C-14. Saturn angular residuals (modified source minus DE28) for 1800–2000 . . . . .	44
C-15. Uranus rectangular residuals (modified source minus DE28) for 1800–2000 . . . . .	46
C-16. Uranus angular residuals (modified source minus DE28) for 1800–2000 . . . . .	48
C-17. Neptune rectangular residuals (modified source minus DE28) for 1800–2000 . . . . .	50
C-18. Neptune angular residuals (modified source minus DE28) for 1800–2000 . . . . .	52
C-19. Pluto rectangular residuals (modified source minus DE28) for 1800–2000 . . . . .	54
C-20. Pluto angular residuals (modified source minus DE28) for 1800–2000 . . . . .	56
D-1. Venus rectangular residuals (improved modified source minus DE28) for 1800–2000 . . . . .	58
D-2. Venus angular residuals (improved modified source minus DE28) for 1800–2000 . . . . .	60
D-3. Earth-moon barycenter rectangular residuals (improved modified source minus DE28) for 1800–2000 . . . . .	62
D-4. Earth-moon barycenter angular residuals (improved modified source minus DE28) for 1800–2000 . . . . .	64
D-5. Jupiter rectangular residuals (improved modified source minus DE28) for 1800–2000 . . . . .	66
D-6. Jupiter angular residuals (improved modified source minus DE28) for 1800–2000 . . . . .	68

## **Abstract**

Dynamically consistent Newtonian ephemerides of the nine principal planets have been generated from 1800–2000 with epoch 1899 December 12, 0<sup>h</sup> ET (Julian Ephemeris Date 241 5000.5). The fifty-fourth order set of differential equations determining the motion of the nine planets is solved by numerical methods and the integrated ephemerides are adjusted to make a best fit in the least squares sense to suitably modified classical source ephemerides. Secular deficiencies in the Newtonian modified source ephemerides are determined and analyzed.

# Newtonian Planetary Ephemerides 1800—2000: Development Ephemeris Number 28

## I. Introduction

The classical planetary ephemerides used by astronomers are generated from general theories which usually are only first-order approximations to the solution of the differential equations determining the motion of a planet. Second and higher order perturbation terms have largely been ignored and rather rough approximations have been made for secular terms, while mixed secular terms of the form  $a t \frac{\sin bt}{\cos bt}$  have not been considered at all.

With the advance of space exploration and more precise techniques of observation, the need for more accurate planetary ephemerides has become evident. With present large high-speed digital computers, it has become relatively easy to generate more precise ephemerides than was possible only a few decades ago.

Since numerical integrations can be made over a longer time span than those for which actual observations are readily available, it has become necessary and conventional to fit the numerically integrated positions to one of the classical theories, rather than to actual observations.

The widely distributed Development Ephemeris (DE) series, DE 3 (Ref. 1) and DE 19 (Ref. 2), of the Jet Propulsion Laboratory are two examples. These ephemerides have been generated by assuming that the orbits of (N-1) bodies are known and by numerically integrating the differential equations for a single body (Ref. 3).

One problem which arises when fitting a source theory rather than actual observations is the manner in which to treat known deficiencies of the source theory. A perfect example appears in the usual Newcomb (Ref. 4) source theories of the three inner planets.

The general theory was derived, of necessity, in a fixed coordinate system, but for the sake of convenience, the final results were referred to the mean equator and equinox of date. This implies that some definite value of the speed of general precession in longitude has been employed—in the case of Newcomb, the value is 5024''82 per tropical century at 1900.0. The fact that the internationally adopted value is 5025''64 per tropical century introduces the following dilemma: (1) one may either adopt Newcomb referred to the mean equinox of date (as given by S. Newcomb) as defining the position of the

planet and use the *new* Newcomb general precession in longitude, 5025''64, to determine the position of the planet referred to a fixed equinox; or (2) one may adopt Newcomb referred to the mean equinox of date (as given by S. Newcomb), adjust for the difference in precession, and then use the new Newcomb precession to determine the position referred to a fixed equator and equinox. The two alternatives differ by  $+0.82 T$  in the perihelia and nodes, where  $T$  is measured in centuries from 1900.

From the mathematical point of view, the second alternative is preferred, although there is no guarantee that alternative (2) will be closer to the actual position of the planet than alternative (1). The American Ephemeris adopts the first alternative, and P. Herget adopts the second one (although his ephemeris is strictly consistent with yet a third value of the general precession in longitude) in his work on the sun (Ref. 5).

Another deficiency in the usual Newcomb source theories for the inner planets deals with the so-called empirical terms applied by S. Newcomb. He assumed that the force between two bodies is proportional to  $r^{-(2+\delta)}$ ,  $\delta = 0.000\,000\,1612$ , rather than the Newtonian  $r^{-2}$ . This has the effect that the longitude of perihelion of a planet is advanced by an amount proportional to  $a^{-3/2} T$ , rather than  $a^{-5/2} T$  predicted by general relativity.

Of course, these dilemmas may be circumvented by using the same epoch as Newcomb and fitting Newcomb over equal time spans on either side of the epoch, since in the least squares process, any secular error in the source theory will remain in the final residuals and will not be absorbed by the integration. If, however, one adopts a significantly different epoch, it becomes important to distinguish between the alternatives.

A third problem occurs when one applies, for example, the differential corrections as found by G. M. Clemence (Ref. 6) to the Newcomb theory of Mercury, or those given by R. Duncombe for Venus and the earth (Ref. 7). The corrections usually include secular terms which are primarily due to a different system of planetary masses or another value of the general precession in longitude; hence, these corrections should not be indiscriminately applied. The fact that the observations were analyzed before the introduction of ephemeris time also contributes theoretical reasons for not using the corrections without considerable thought.

In most of the published numerically integrated fits to source theories, the preceding problems have either been

ignored or specific details of what has been done are lacking. While there is little justification in stating that such fitted ephemerides do not represent the real world as well as a set in which the preceding source deficiencies are corrected, there is considerable difficulty in comparing results with other sources.

B. G. Marsden (Ref. 8) has listed the theoretical corrections which should be applied to the classical source theories to render them mathematically consistent. His recommendations have generally been followed in this investigation.

J. Schubart and P. Stumpff (Ref. 9) of the Astronomisches Rechen-Institut in Heidelberg have recently developed an N-body computer program in which the equations of motion of N-bodies are numerically integrated simultaneously. A modification of their program has been employed in this investigation to develop a dynamically consistent set of planetary ephemerides from 1800-2000; this set is referred to as Development Ephemeris (DE) Number 28. A by-product of the fit to the source theories as modified by the theoretical corrections is an ability to determine secular errors in the Newtonian source theories.

## II. Source Ephemerides

### A. System of Masses

The planetary masses adopted by the International Astronomical Union (IAU) (Ref. 10) and used in the following integration are listed in Table 1. The mass ratio of moon/earth is 1/81.30.

These masses are sufficiently close to those employed by S. Newcomb. They are identical with the masses used by G. Clemence in his theory of Mars and by Eckert, Brouwer, and Clemence in the SSEC (selective sequence electronic calculator) integration of the outer planets.

Table 1. Planetary masses

Planet	Mass of sun/mass of planet
Mercury	6,000,000
Venus	408,000
Earth-moon	329,390
Mars	3,093,500
Jupiter	1,047.355
Saturn	3,501.6
Uranus	22,869
Neptune	19,314
Pluto	360,000

## B. Theoretical Secular Corrections

To avoid the philosophical problem of which relativity metric to use in the integration, it was decided to perform a Newtonian integration and, after the integration, to advance the planetary perihelia by an amount

$$\frac{12\pi^2 a^2}{c^2 P^2 (1 - e^2)}$$

per revolution period  $P$ , where  $a$ ,  $e$ , and  $c$  are the semi-major axis, eccentricity, and speed of light, respectively.

To render the basic source ephemerides Newtonian and consistent with the Newcomb general precession in longitude of 5025''64 per tropical century at 1900.0, it is necessary to add the secular corrections listed in Table 2 to compensate for the non-gravitational nature of the source theories.

In Table 2,  $T$  is measured in Julian centuries from 1900. With the exception of the outer planets, the values are identical with those proposed by Marsden in Ref. 8.

The source ephemeris for Mercury is an evaluation of the Newcomb theory of Mercury (Ref. 4) by Duncombe, Tufekcioglu, and Larson (Ref. 11). The ephemeris by Duncombe is a strict evaluation of the Newcomb theory referred to the mean equinox of date and precessed to 1950.0 with the IAU value of the Newcomb general precession in longitude (personal communication). The corrections listed in Table 2 represent removal of the empirical term and adjustment for precession.

The source ephemeris for Venus is P. Herget's evaluation (Ref. 12) of the Newcomb theory of Venus (Ref. 4). The corrections represent removal of the empirical term and adjustment for precession.

**Table 2. Secular corrections to basic source ephemerides**

Planet	Source	Corrections to be applied
Mercury	Newcomb	$\Delta\tilde{\omega} = -42''.55T$ , $\Delta\Omega = +0''.82T$
Venus	Newcomb	$\Delta\tilde{\omega} = -16''.16T$ , $\Delta\Omega = +0''.82T$
Earth-moon	Newcomb	$\Delta\tilde{\omega} = -17''.31T$
Mars	Clemence	$\Delta\tilde{\omega} = -1''.35T$
Jupiter	SSEC	Clemence corrections + $e\Delta\tilde{\omega} = -0''.00301T$
Saturn	SSEC	Clemence corrections + $e\Delta\tilde{\omega} = -0''.00077T$
Uranus	SSEC	Clemence corrections + $e\Delta\tilde{\omega} = -0''.000112T$
Neptune	SSEC	Clemence corrections
Pluto	SSEC	Clemence corrections

The source ephemeris for the earth-moon barycenter is P. Herget's evaluation (Ref. 5) of the Newcomb theory of the sun (Ref. 4). Herget has already applied a correction  $\Delta\tilde{\omega} = -4''.78T$  to the Newcomb longitude of the earth perihelion; hence, the correction  $\Delta\tilde{\omega} = -12''.53T$  is applied to Herget's evaluation. The Newcomb ephemeris for the earth-moon barycenter includes a secular term  $\Delta\tilde{\omega} = (10''.45 + 7''.68)T$ , where the first value is the empirical (relativity) term and the second value represents the secular acceleration of the barycenter perihelion due to the moon.

The source ephemeris for Mars is the preliminary evaluation of the Clemence theory of Mars (Refs. 13 and 14) by Clemence and Duncombe (Refs. 15 and 16). The source ephemeris includes  $\Delta\tilde{\omega} = +1''.35T$  due to relativity, and the Table 2 correction removes this term.

The source ephemerides for the five outer planets are those calculated by Eckert, Brouwer, and Clemence (Ref. 17) on the SSEC. G. M. Clemence (Ref. 18) has published tables of corrections necessary to render the SSEC coordinates heliocentric. These corrections have been applied to the SSEC coordinates (with the exception of one term in the Jupiter y-coordinate in which the typographical error  $-140 \times 10^{-9} T \sin g_s$  was changed to  $-104 \times 10^{-9} T \sin g_s$ ) for the five outer planets. Since the Clemence corrections contain relativistic effects in the coordinates of Jupiter, Saturn, and Uranus, the relativistic contributions were removed in the same manner as they were included in the tables.

## C. Constant Corrections

In addition to the secular corrections to the basic source ephemerides listed in Table 2, the constant corrections of Table 3 were applied.

The constant corrections for Mercury are those determined by Clemence (Ref. 6) reduced to the Newcomb theory of Mercury and consistent with ephemeris time

**Table 3. Constant corrections to source ephemerides**

Planet	Corrections to be applied
Mercury	$\Delta\lambda = +0''.23$ , $\Delta e = -0''.40$ , $\Delta\tilde{\omega} = +0''.92$ , $\Delta l = +0''.06$
Venus	$\Delta e = -0''.12$ , $e\Delta\tilde{\omega} = +0''.01$ , $\sin l \Delta\Omega = +0''.21$ , $\Delta l = +0''.08$
Earth-moon	$\Delta e = -0''.10$ , $e\Delta\tilde{\omega} = -0''.07$

and the IAU general precession in longitude. Clemence lists a correction

$$\Delta\lambda = -0''.58 \pm 0''.08 - (0''.67 \pm 0''.14) T$$

to the mean longitude of Mercury, but the approximation which he employed to remove the effects of the non-uniform nature of universal time (UT) exceeds those due to the adopted definition of ephemeris time (ET) by  $\Delta\lambda = +0''.81 + 0''.75 T$ . Thus the correction found by Clemence to the Newcomb mean longitude of Mercury measured in ET is

$$\Delta\lambda = +0''.23 \pm 0''.08 + (0''.08 \pm 0''.14) T$$

The adopted correction employed in this investigation is  $\Delta\lambda = +0''.23$ .

The constant corrections for Venus are those determined by R. Duncombe (Ref. 7). The correction to the mean longitude of Venus was not employed, since the Duncombe corrections to the mean longitudes of Venus and the earth are in the ratio of their mean motions and, hence, are ascribed to deficiencies in the approximation used to remove effects of the non-uniform nature of UT.

The constant corrections for the earth-moon barycenter are taken from Duncombe's work on Venus (Ref. 7). The correction to the mean longitude was not employed, as noted in the preceding paragraph, to avoid tampering with the definition of ephemeris time. The value of obliquity employed is that of Newcomb for 1950.0,  $\epsilon = 23^\circ 26' 44''.836$ .

The net corrections applied to the basic source ephemerides are the sum of terms listed in Tables 2 and 3. The source ephemerides corrected in this manner will be referred to as modified source ephemerides (MSE).

After the numerical integration was fitted to the modified source ephemerides, the relativistic advance of the longitude of perihelion of each planet was computed and is listed as  $\Delta\pi$  in ephemeris tapes DE 28. The relativistic contributions were calculated from the values in Table 4.

In Table 4,  $T$  is measured in Julian centuries from 241 5000.5. The correction listed for the earth-moon barycenter is a combination of the effects of relativity (3''.839

**Table 4. Relativistic corrections to perihelia**

Planet	$\Delta\omega''$
Mercury	+42''.982 T
Venus	-8''.6250
Earth-moon	+11''.539 T
Mars	+1''.3509 T
Jupiter	+6.2304 $\times 10^{-2}$ T
Saturn	+1.3547 $\times 10^{-2}$ T
Uranus	+2.3468 $\times 10^{-3}$ T
Neptune	+7.8187 $\times 10^{-4}$ T
Pluto	+4.2304 $\times 10^{-4}$ T

$T$ ) and the acceleration of the moon on the barycenter (+7''.700  $T$ ).

The epoch of osculation selected for the integration is Julian Date 241 5000.5 (1899 December 12 0<sup>h</sup> ET). The integrations cover the period 237 8520.5 to 245 1480.5 (1800 January 25 0<sup>h</sup> ET to 1999 October 29 0<sup>h</sup> ET).

In this investigation, it was assumed that the real positions of the planets at 1900 were those given by the modified source ephemerides. Thus, any Newtonian secular error in the modified source ephemerides is not absorbed in a least squares fit from 1800-2000 with epoch 1900, but will appear in the final residuals after the best fit has been made. The final residuals were then examined for such secular errors in the modified source theories.

### III. Numerical Integration

#### A. Method

The Adams-Störmer method of numerical integration using backward differences is employed in the Heidelberg N-body integration program which forms a basis for this investigation. Only a brief outline will be given here and further details may be found in Ref. 9.

The differential equations integrated in a center of mass of the solar system reference frame are given by:

$$\ddot{\mathbf{R}}_i = -k^2 \sum_{\substack{j=1 \\ j \neq i}}^{10} m_j \frac{\mathbf{R}_i - \mathbf{R}_j}{R_{ij}^3} \quad (i = 1, 10) \quad (1)$$

where

$$\begin{aligned} \ddot{\mathbf{R}}_i &= \text{vector from center of mass to } i\text{th body} \\ k^2 &= \text{square of Gaussian gravitational constant} \end{aligned}$$

$$k = 0.017202\ 09895$$

$$m_j = \text{ratio of mass of } j\text{th body to mass of sun}$$

and

$$\mathbf{R}_{ij} = \mathbf{R}_j - \mathbf{R}_i$$

$$R_{ij} = |\mathbf{R}_{ij}|$$

Bodies are numbered in ascending order proceeding outward from the sun, with the sun being body 1 and Pluto body 10.

The integral of the center of mass

$$\sum_{i=1}^{10} m_i \dot{\mathbf{R}}_i = 0$$

is used to eliminate the differential equations determining the motion of the sun about the center of mass of the solar system, and only nine second-order differential equations in equatorial rectangular coordinates  $x, y$ , and  $z$  are integrated.

Heliocentric equatorial rectangular coordinates are then evaluated from

$$\mathbf{r}_i = \mathbf{R}_i - \mathbf{R}_1 \quad (i = 2, 10) \quad (2)$$

where  $\mathbf{r}_i$  is the heliocentric vector to the  $i$ th planet.

## B. Integration Accuracy Tests

To monitor the integration procedure, the integrals of energy

$$E = \frac{1}{2} \sum_{i=1}^N m_i \dot{\mathbf{R}}_i \cdot \dot{\mathbf{R}}_i - k^2 \sum_{i=1}^{N-1} \sum_{j=i+1}^N \frac{m_i m_j}{R_{ij}} \quad (3)$$

and angular momentum about the center of mass

$$\mathbf{L} = \sum_{i=1}^N m_i (\mathbf{R}_i \times \dot{\mathbf{R}}_i) \quad (4)$$

were calculated at each step. The highest order differences retained were also monitored.

An option of using either the predictor method alone or the predictor-corrector method was built into the program; and tests were performed to determine the best step size and method. An integration performed at half-day steps, retaining eleventh differences in the accelerations and using the predictor formula only, proved superior

to using the predictor with one corrector cycle at one-day steps retaining the same order of differences.

The integrations were thus performed with epoch 241 5000.5 at half-day steps retaining eleventh differences and using only the predictor formula. The starting table was iterated until the accelerations retained a relative accuracy of  $10^{-15}$ . The integration was then performed from 1900–2000 and 1900–1800. After the final fit to the modified source ephemerides, the positions and velocities at 1800 and 2000 were used to integrate 1800–1900 and 2000–1900. The coordinates generated in the normal integration (1900–2000 and 1900–1800) were then compared with those of the reverse integration (2000–1900 and 1800–1900).

The Mercury through Pluto portions of Table 5 summarize the results of the reversibility test in the sense of normal minus reverse integration for each planet. The integral portion of the table summarizes the relative deviations of the energy and angular momentum integrals from their epoch values. The integral values at 241 5000.5 are:

$$E = -3.32 \times 10^{-8}$$

$$L = +6.11 \times 10^{-5}$$

$$L_x = +1.68 \times 10^{-6}$$

$$L_y = -2.38 \times 10^{-5}$$

$$L_z = +5.62 \times 10^{-5}$$

where  $L_x, L_y$ , and  $L_z$  are the total angular momentum components of the solar system in an equatorial frame referred to 1950.0. Graphs of the relative changes in energy and angular momentum integrals from their epoch values are given in Figs. C-1 and C-2.

The number of steps required to integrate the solar system from 241 5000.5 to 245 1480.5 or from 241 5000.5 to 237 8520.5 in half-day intervals is 72,960. Examination of the reversibility statistics for Mercury indicates that the accumulation of errors as a function of the number of steps is about a factor of three less than that given by D. Brouwer (Ref. 19). The reversibility statistics for Mercury could be improved by including one corrector cycle in the integration for Mercury.

The integrations were performed on the IBM 7094/7040 DCOS (direct couple operating system) at Yale University. The nine-planet integrations were performed at the rate of 10.5 steps/s, and the 200-yr integration required a total of 232 min of computer time. The five outer planets could be integrated at 40-day steps for 400 yr in about 4 min.

Table 5. Integration statistics

Residuals in position	Standard deviation	Maximum difference	Minimum difference	Residuals in position	Standard deviation	Maximum difference	Minimum difference
<b>Mercury</b>				<b>Saturn</b>			
$dx, AU$	1.46 E-08	5.68 E-08	-5.66 E-08	$dx, AU$	9.96 E-14	3.30 E-13	-4.06 E-13
$dy, AU$	1.31 E-08	4.34 E-08	-4.37 E-08	$dy, AU$	9.20 E-14	2.01 E-13	-4.15 E-13
$dz, AU$	7.08 E-09	2.44 E-08	-2.46 E-08	$dz, AU$	3.82 E-14	8.06 E-14	-1.69 E-13
$\cos B \, dl, ''$	1.16 E-02	3.80 E-02	-3.82 E-02	$\cos B \, dl, ''$	2.91 E-09	5.47 E-09	-9.70 E-09
$dB, ''$	1.00 E-03	4.50 E-03	-4.53 E-03	$dB, ''$	1.74 E-10	6.04 E-10	-6.92 E-10
$dr/r, ''$	1.67 E-03	5.56 E-03	-5.53 E-03	$dr/r, ''$	3.43 E-10	5.87 E-10	-1.50 E-09
<b>Venus</b>				<b>Uranus</b>			
$dx, AU$	5.94 E-12	2.70 E-11	-2.70 E-11	$dx, AU$	2.27 E-13	3.67 E-13	-1.01 E-12
$dy, AU$	5.42 E-12	2.43 E-11	-2.48 E-11	$dy, AU$	1.56 E-13	4.81 E-13	-2.22 E-13
$dz, AU$	2.46 E-12	1.10 E-11	-1.12 E-11	$dz, AU$	7.68 E-14	2.36 E-13	-9.95 E-14
$\cos B \, dl, ''$	1.96 E-06	7.81 E-06	-5.14 E-09	$\cos B \, dl, ''$	3.07 E-09	4.50 E-09	-1.07 E-08
$dB, ''$	1.01 E-07	4.58 E-07	-4.56 E-07	$dB, ''$	1.75 E-10	4.95 E-10	-3.83 E-10
$dr/r, ''$	1.27 E-08	5.06 E-08	-8.35 E-08	$dr/r, ''$	6.68 E-10	2.93 E-09	-5.80 E-11
<b>Earth-moon barycenter</b>				<b>Neptune</b>			
$dx, AU$	2.29 E-12	8.65 E-12	-9.00 E-12	$dx, AU$	1.25 E-13	4.46 E-13	-1.47 E-13
$dy, AU$	2.10 E-12	7.96 E-12	-8.16 E-12	$dy, AU$	3.93 E-14	1.78 E-13	-9.06 E-14
$dz, AU$	9.11 E-13	3.45 E-12	-3.54 E-12	$dz, AU$	1.35 E-14	5.15 E-14	-3.38 E-14
$\cos B \, dl, ''$	6.68 E-07	1.89 E-06	-1.51 E-06	$\cos B \, dl, ''$	7.97 E-10	1.06 E-09	-2.99 E-09
$dB, ''$	3.87 E-10	1.08 E-09	-1.08 E-09	$dB, ''$	6.15 E-11	1.27 E-10	-1.42 E-10
$dr/r, ''$	7.56 E-09	3.62 E-08	-2.80 E-08	$dr/r, ''$	4.08 E-10	1.46 E-09	-3.85 E-10
<b>Mars</b>				<b>Pluto</b>			
$dx, AU$	3.32 E-12	1.26 E-11	-1.17 E-11	$dx, AU$	4.34 E-14	8.08 E-14	-1.02 E-13
$dy, AU$	3.02 E-12	1.20 E-11	-1.00 E-11	$dy, AU$	2.31 E-14	1.03 E-13	-2.49 E-14
$dz, AU$	1.39 E-12	5.51 E-12	-4.62 E-12	$dz, AU$	2.57 E-14	7.73 E-14	-2.31 E-14
$\cos B \, dl, ''$	5.44 E-07	1.99 E-06	-5.49 E-07	$\cos B \, dl, ''$	1.99 E-10	5.26 E-10	-3.63 E-10
$dB, ''$	1.49 E-08	5.76 E-08	-5.19 E-08	$dB, ''$	8.09 E-11	2.24 E-10	-6.68 E-11
$dr/r, ''$	4.20 E-08	1.59 E-07	-1.49 E-07	$dr/r, ''$	1.13 E-10	4.27 E-10	-5.02 E-11
<b>Jupiter</b>				<b>Integrals</b>			
$dx, AU$	1.69 E-12	4.83 E-12	-5.20 E-12	$dE/E$	3.40 E-14	1.48 E-13	-5.71 E-14
$dy, AU$	1.57 E-12	4.97 E-12	-4.69 E-12	$dL/L$	2.42 E-14	9.27 E-14	-2.84 E-14
$dz, AU$	6.74 E-13	2.15 E-12	-2.03 E-12	$dL_z/L_z$	2.60 E-13	4.21 E-13	-5.22 E-13
$\cos B \, dl, ''$	5.89 E-08	2.26 E-07	-4.52 E-11	$dL_y/L_y$	2.43 E-14	8.33 E-14	-5.07 E-14
$dB, ''$	1.59 E-09	5.05 E-09	-4.56 E-09	$dL_z/L_z$	2.44 E-14	9.52 E-14	-2.79 E-14
$dr/r, ''$	4.86 E-09	9.91 E-09	-1.68 E-08				

**IV. Ephemeris Fitting Procedure**

The parameters adopted for correction are identical with the Eckert-Brouwer Set III elements (Ref. 20), except that their parameter  $\Delta M_0 + \Delta r$  is replaced by a correction to the mean longitude at epoch

$$\Delta \lambda_0 = \Delta M_0 + \Delta \tilde{\omega}_0$$

where  $\Delta M_0$  and  $\Delta \tilde{\omega}_0$  are the corrections to the mean anomaly and longitude of perihelion at epoch, respectively. The partial derivative matrices were modified accord-

ingly. In evaluating the partial derivative matrices, osculating elements and PQR matrices at epoch were used with the integrated positions and velocities.

Initial positions and numerically differentiated velocities were taken from the modified source ephemerides at Julian Date 241 5000.5 to commence the integration. An integration  $\pm 2000$  days from the epoch was made, and the differential corrections were applied to the initial conditions. The integration was successively made  $\pm 4000$  days,  $\pm 25$  yr,  $\pm 50$  yr, and  $\pm 100$  yr from epoch and corrected until no significant corrections remained.



Integrated coordinates of the four inner planets were compared in 4-day intervals with the modified source ephemerides, while the five outer planets were compared at 40-day intervals (to avoid introducing interpolation errors into the SSEC coordinates).

Difficulties were experienced in fitting the integrated positions of Neptune to the modified source ephemerides. (This difficulty also occurred in JPL DE 3 ephemeris of Neptune.) The difficulty can be traced to the fact that Clemence's corrections to the SSEC coordinates of Neptune and Pluto do not completely include the effect of a change in scale from the SSEC coordinates to heliocentric positions. The effect is that Clemence disregarded a term in the coordinates of Neptune and Pluto with the period of the planet (see Figs. C-17 through C-20). After the integration interval became larger than the linear range of this neglected term, the differential corrections converged nicely.

## V. Results

The ephemerides integrated in this investigation contain Newtonian heliocentric equatorial rectangular coordinates referred to the mean equator and equinox of 1950.0. Differences in position, relativistic minus Newtonian, are also given in the ephemeris tape.

### A. The Fit to Modified Source Ephemerides

Final residuals and statistics of the fit, in the sense of modified source ephemeris minus DE28, along with initial conditions are given in Tables B-1 through B-9. Rectangular coordinates in astronomical units (AU) are heliocentric equatorial and are referred to the mean equator and equinox of 1950.0; velocities are in AU/day. Angular residuals, in seconds of arc, for  $\cos B \, dL$ ,  $dB$ , and  $dr/r$ , where  $L$ ,  $B$ , and  $r$  are celestial longitude, latitude, and heliocentric distance, respectively, are also referred to 1950.0. Epoch values of the osculating equatorial positions and velocities along with ecliptic elements are also given. The osculating **PQR** matrix is listed for reduction from the plane of the orbit to heliocentric equatorial coordinates referred to 1950.0. The value of obliquity used is  $23^{\circ}26'44''.836$  at 1950.0.

Plots of the residuals, MSE minus DE28, in heliocentric equatorial coordinates and celestial longitude, latitude, and radius are given in Figs. C-3 through C-20.

The mixed secular residual with a period of one year, zero at 1850, and growing with time, apparent in the five outer planets, is due to the earth. G. M. Clemence

used the coefficient of time in the earth mean anomaly to compute the corrections of the outer planets due to the earth. For his purposes, this correction was sufficient, but the residuals presented in the plots of Figs. C-11 through C-20 suggest that it would have been better to use the sidereal mean motion of the earth (determined from its mean longitude), which differs from that determined from the mean anomaly by  $1100''0$  per century. The long-period residuals in the Neptune and Pluto plots are attributed to effects of the change in scale from SSEC coordinates to heliocentric coordinates which were below the significant level in Clemence's corrections. Using the data Clemence gives for  $\Delta r$  in his corrections (Ref. 18), these defects in the modified source ephemerides can be removed.

The secular trend of residuals in the Jupiter x-coordinate, shown in Fig. C-11, can be attributed to a secular change in the Jupiter eccentricity,  $-0''.00117 T$ , due to the inner planets—an effect which Clemence did not include in his corrections (Ref. 18).

### B. Secular Deficiencies in Newtonian Source Ephemerides

After making the best fit to the modified source ephemerides, the final residuals were examined in an attempt to detect secular deficiencies of the Newtonian modified source ephemerides in the elements  $e$ ,  $\varpi$ ,  $\Omega$ , and  $I$ . A significant reduction of residuals was found in the ephemerides of Venus, earth-moon, and Jupiter.

The modified source ephemerides corrected by the deduced secular variations will be called improved modified source ephemerides (IMSE). The corrections applied and the new statistics are listed in Tables D-1 through D-3. Plots of the IMSE minus DE28 are given in Figs. D-1 through D-6.

The modified source ephemeris of the earth-moon barycenter was analyzed in an attempt to detect a secular error in the Newcomb obliquity. Observations suggest that a correction to the Newcomb obliquity of approximately  $-0''.3 T$  is needed (Ref. 7). Clemence and Duncombe (Ref. 21) report that P. Herget has determined a correction of  $-0''.47 T$  to the Newcomb obliquity by fitting a numerical integration of the earth-moon barycenter to Newcomb. Such an error would appear in the latitude residuals as a mixed secular term with a period of one year and amplitude of  $-0''.47 T$ . This term is not evident in the 200-yr integration of this investigation (see Fig. D-4), and the deduced correction to  $\Delta(\sin I \cos \Omega)$  of the earth-moon barycenter is  $-(0''.0057 \pm 0''.0009) T$ .

It is known that  $d\epsilon/dt \approx \sin \pi_1 \cos \Pi_1$  (Ref. 22) where  $\pi_1$  is the angle at the node between a fixed ecliptic and the mean ecliptic of date, and  $\Pi_1$  is the longitude of ascending node of the mean ecliptic of date on a fixed ecliptic. Since  $\Omega_{\oplus\epsilon} = \Omega_{\odot} + 180^\circ$ , with  $\Omega_{\odot} = \Pi_1$ , it is seen that

$$\Delta \frac{d\epsilon}{dt} \approx -\Delta(\sin I \cos \Omega)_{\oplus\epsilon}$$

Thus, it appears from this investigation that the secular change in obliquity given by Newcomb is approximately correct, at least as far as Newtonian interactions are concerned.

The correction  $\Delta\tilde{\omega} = +1''.19 \pm 0''.03$  to the longitude of perihelion for the modified source ephemeris of the earth-moon barycenter indicates that Herget's evaluation

of Newcomb plus  $\Delta\tilde{\omega} = -4''.78 T$  is better than the one adopted for the modified source ephemeris, Newcomb plus  $\Delta\tilde{\omega} = -5''.77 T$  ( $-17''.31$  Newtonian +  $11''.539$  for relativity and secular effect of the moon). The difference between Herget's correction to Newcomb and the one adopted is due to a value of the general precession in longitude,  $5026''.65$  per tropical century, recommended by G. M. Clemence (Ref. 23) in 1948. The present investigation suggests that Herget's correction is better, not for the quoted reason of precession, but because of a Newtonian deficiency in the Newcomb theory.

As noted previously, the modified source ephemeris of Jupiter lacks a term  $\Delta e = -0''.00117 T$  (Ref. 18) due to the inner planets. This term would appear in the x-coordinate residuals of Jupiter as  $\Delta x = -43 \times 10^{-9} T$ . It is seen that the correction found,  $\Delta e = -0''.00183 \pm 0''.00001$ , is in good agreement with the known deficiency.

## Nomenclature

$a$	semimajor axis, AU	$\mathbf{P}$	unit vector directed toward perihelion, osculating at epoch
$B$	celestial latitude referred to mean ecliptic and equinox 1950.0	$\mathbf{Q}$	$\mathbf{R} \times \mathbf{P}$
$dB$	residual in ecliptic latitude,"	$\mathbf{R}$	unit vector normal to the orbital plane, osculating at epoch
$dL$	residual in ecliptic longitude,"	$\mathbf{R}_i$	vector from center of mass to $i$ th body
$dx, dy, dz$	residuals in heliocentric equatorial rectangular coordinates, AU	$\mathbf{R}_{ij}$	$\mathbf{R}_j - \mathbf{R}_i$
$e$	eccentricity, rad	$\mathbf{r}_j$	vector from sun to $j$ th body
$E$	energy integral	$r$	heliocentric distance
$I$	inclination to mean ecliptic of 1950.0, rad	$T$	time in Julian centuries from Julian Date 241 5000.5
$k^2$	square of Gaussian constant, $k = 0.017202 09895$	$x, y, z$	heliocentric equatorial rectangular coordinates referred to mean equator and equinox of 1950.0, AU
$\mathbf{L}$	angular momentum integral about center of mass	$\dot{x}, \dot{y}, \dot{z}$	heliocentric equatorial rectangular velocities referred to 1950.0, AU/day
$L$	celestial longitude referred to mean ecliptic and equinox of 1950.0	$\Omega$	longitude of ascending node, osculating at epoch, on ecliptic of 1950.0, rad
$L_x, L_y, L_z$	angular momentum components referred to mean equator and equinox of 1950.0	$\tilde{\omega}$	longitude of perihelion, osculating at epoch, rad
$M_0$	mean anomaly, osculating at epoch, rad	$\omega$	argument of perihelion, osculating at epoch, rad
$m_j$	mass of $j$ th body relative to sun		
$n$	mean motion, osculating at epoch, rad/day		

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## Appendix A

### Tape Format

The set of ephemerides DE28 resulting from the Newtonian fit to the modified source ephemerides is available on tape over the interval of Julian Dates 237 8520.5 to 245 1480.5 (1800 January 25 to 1999 October 29) at four-day intervals in the standard DCOS, 460 words per physical record, format. The first four logical records contain the following reference data:

- (1) Record 1 contains 24 BCD words written in binary and identifying the tape as DE28.
- (2) Record 2 contains the masses used in generating DE28. The masses are in double precision with the sun mass first and Pluto mass last.
- (3) Record 3 contains the osculating elements at 241 5000.5 of the nine planets in double precision. The order of elements is  $a$  (AU),  $n$  (rad/day),  $e$  (rad),

$I$  (rad),  $\Omega$  (rad),  $\omega$  (rad),  $M_0$  (rad). Mercury elements are listed first and Pluto elements are last.

- (4) Record 4 contains the osculating **PQR** matrices at epoch in double precision for reduction to the equator of 1950.0. Mercury is listed first and Pluto is last.

The data records contain the following information: the Julian Date in single precision; the body number, its equatorial Newtonian position and velocity in double precision referred to 1950.0; and the single-precision difference in position between the relativistic and Newtonian coordinates (Table 4). The format is Julian Date (body number  $I$ , position in AU, velocity in AU/day,  $\Delta \mathbf{r}$  in AU) for  $I = 2, 10$ ; where  $I = 2$  indicates Mercury and  $I = 10$  indicates Pluto.

The record immediately following JD 245 1480.5 contains a  $-8$ . in the Julian Date position.

**Appendix B**  
**Initial Conditions and Statistics for DE28**

Table B-1. The fitted ephemeris of Mercury

Statistical summary (modified source minus DE28)					
Residuals in position	Mean square residuals	Mean residuals	Standard deviations	Maximum residuals	Minimum residuals
$dx, AU$	6.50E-14	-7.05E-09	2.55E-07	8.47E-07	-8.69E-07
$dy, AU$	7.03E-14	-1.41E-08	2.65E-07	9.88E-07	-8.59E-07
$dz, AU$	8.58E-14	-2.45E-08	2.92E-07	8.88E-07	-9.87E-07
$\cos\delta \, dL, ''$	1.59E-02	1.92E-04	1.26E-01	4.88E-01	-4.80E-01
$dB, ''$	2.45E-02	-8.02E-03	1.56E-01	4.78E-01	-4.93E-01
$dr/r, ''$	2.19E-02	2.40E-02	1.46E-01	3.50E-01	-3.87E-01
Osculating values at JD 241 5000.5					
$x = -1.10729 \ 98321 \ 32043D -01 \ AU$ $y = 2.56801 \ 39813 \ 66735D -01 \ AU$ $z = 1.48892 \ 38800 \ 73276D -01 \ AU$ $\dot{x} = -3.19700 \ 94618 \ 65669D -02 \ AU/day$ $\dot{y} = -8.90445 \ 12293 \ 25886D -03 \ AU/day$ $\dot{z} = -1.47683 \ 53555 \ 47968D -03 \ AU/day$			$a = 3.87097 \ 75793 \ 17067D -01 \ AU$ $n = 7.14250 \ 40257 \ 46578D -02 \ rad/day$ $e = 2.05625 \ 54122 \ 00547D -01$ $l = 1.22292 \ 71650 \ 06895D -01 \ rad$ $\Omega = 8.34298 \ 97004 \ 37712D -01 \ rad$ $\omega = 5.02591 \ 83313 \ 10970D -01 \ rad$ $M_0 = 3.92282 \ 29482 \ 57636D -01 \ rad$		
Equatorial PQR matrix					
$P = 2.34443 \ 59973 \ 10055D -01$ $Q = -9.67919 \ 99131 \ 51598D -01$ $R = 9.03719 \ 47846 \ 82575D -02$		$8.66856 \ 82375 \ 16508D -01$ $1.66074 \ 33231 \ 72862D -01$ $-4.70083 \ 57050 \ 69548D -01$		$4.39994 \ 82458 \ 36168D -01$ $1.88547 \ 62411 \ 08036D -01$ $8.77983 \ 11360 \ 85723D -01$	

Table B-2. The fitted ephemeris of Venus

Statistical summary (modified source minus DE28)					
Residuals in position	Mean square residuals	Mean residuals	Standard deviations	Maximum residuals	Minimum residuals
$dx, AU$	1.06E-13	7.81E-09	3.26E-07	1.10E-06	-1.10E-06
$dy, AU$	3.79E-14	5.89E-09	1.95E-07	7.17E-07	-8.08E-07
$dz, AU$	1.57E-14	3.67E-09	1.25E-07	4.99E-07	-4.14E-07
$\cos\delta \, dL, ''$	1.03E-02	5.71E-05	1.01E-01	3.19E-01	-3.16E-01
$dB, ''$	8.08E-04	4.56E-04	2.84E-02	1.09E-01	-1.07E-01
$dr/r, ''$	1.92E-03	-6.93E-03	4.33E-02	1.47E-01	-1.46E-01
Osculating values at JD 241 5000.5					
$x = 4.88159 \ 75692 \ 62222D -01 \ AU$ $y = -4.80515 \ 78943 \ 34433D -01 \ AU$ $z = -2.47283 \ 13064 \ 12261D -01 \ AU$ $\dot{x} = 1.48600 \ 98031 \ 27145D -02 \ AU/day$ $\dot{y} = 1.26529 \ 27926 \ 43026D -02 \ AU/day$ $\dot{z} = 4.75939 \ 13961 \ 13112D -03 \ AU/day$			$a = 7.23325 \ 75593 \ 39194D -01 \ AU$ $n = 2.79628 \ 27226 \ 92774D -02 \ rad/day$ $e = 6.80681 \ 87625 \ 15753D -03$ $l = 5.92485 \ 03797 \ 83429D -02 \ rad$ $\Omega = 1.33291 \ 69380 \ 24584D \ 00 \ rad$ $\omega = 9.47022 \ 97082 \ 49603D -01 \ rad$ $M_0 = -3.11381 \ 67358 \ 98620D \ 00 \ rad$		
Equatorial PQR matrix					
$P = -6.49799 \ 92568 \ 24021D -01$ $Q = -7.57923 \ 78984 \ 42415D -01$ $R = 5.75463 \ 75831 \ 03330D -02$		$6.76829 \ 95693 \ 76232D -01$ $-6.11403 \ 37850 \ 69234D -01$ $-4.09984 \ 29011 \ 62613D -01$		$3.45920 \ 89554 \ 54408D -01$ $-2.27458 \ 65017 \ 28645D -01$ $9.10275 \ 39595 \ 80465D -01$	

**Table B-3. The fitted ephemeris of earth-moon barycenter**

Statistical summary (modified source minus DE28)					
Residuals in position	Mean square residuals	Mean residuals	Standard deviations	Maximum residuals	Minimum residuals
$dx, \text{ AU}$	7.89E-14	1.20E-08	2.81E-07	1.23E-06	-1.07E-06
$dy, \text{ AU}$	7.40E-14	-2.71E-08	2.71E-07	1.33E-06	-8.80E-07
$dz, \text{ AU}$	3.28E-14	2.08E-10	1.81E-07	5.92E-07	-6.46E-07
$\cos B \, dl, ''$	5.25E-03	4.57E-05	7.24E-02	2.74E-01	-2.93E-01
$dB, ''$	1.08E-03	2.26E-03	3.27E-02	1.15E-01	-9.79E-02
$dr/r, ''$	1.60E-03	-1.05E-02	3.86E-02	1.62E-01	-1.56E-01
Osculating values at JD 241 5000.5					
$x = 1.62776 \ 19435 \ 35683D \ -01 \ \text{AU}$ $y = 8.90604 \ 17544 \ 57508D \ -01 \ \text{AU}$ $z = 3.86366 \ 38283 \ 98065D \ -01 \ \text{AU}$ $\dot{x} = -1.72499 \ 81320 \ 43152D \ -02 \ \text{AU/day}$ $\dot{y} = 2.55556 \ 00747 \ 39084D \ -03 \ \text{AU/day}$ $\dot{z} = 1.10842 \ 85834 \ 56602D \ -03 \ \text{AU/day}$			$a = 1.00000 \ 25634 \ 13442D \ 00 \ \text{AU}$ $n = 1.72020 \ 58918 \ 00823D \ -02 \ \text{rad/day}$ $e = 1.67690 \ 64295 \ 52822D \ -02$ $l = 1.14128 \ 47135 \ 22253D \ -04 \ \text{rad}$ $\Omega = -1.07891 \ 26039 \ 10112D \ -01 \ \text{rad}$ $\omega = 1.88611 \ 09222 \ 58526D \ 00 \ \text{rad}$ $M_0 = -3.61453 \ 12861 \ 02470D \ -01 \ \text{rad}$		
Equatorial PQR matrix					
$P = -2.05939 \ 15334 \ 21594D \ -01$ $Q = -9.78564 \ 79855 \ 43320D \ -01$ $R = -1.22895 \ 89297 \ 41080D \ -05$		$8.97728 \ 33075 \ 07242D \ -01$ $-1.88922 \ 10466 \ 14020D \ -01$ $-3.97985 \ 27929 \ 78960D \ -01$		$3.89452 \ 06288 \ 86597D \ -01$ $-8.19717 \ 84173 \ 73714D \ -02$ $9.17391 \ 80141 \ 91873D \ -01$	

**Table B-4. The fitted ephemeris of Mars**

Statistical summary (modified source minus DE28)					
Residuals in position	Mean square residuals	Mean residuals	Standard deviations	Maximum residuals	Minimum residuals
$dx, \text{ AU}$	2.28E-13	7.01E-10	4.77E-07	1.85E-06	-1.50E-06
$dy, \text{ AU}$	1.71E-13	3.38E-09	4.14E-07	1.57E-06	-1.28E-06
$dz, \text{ AU}$	3.83E-14	9.99E-10	1.96E-07	7.67E-07	-6.38E-07
$\cos B \, dl, ''$	7.91E-03	-7.99E-05	8.89E-02	2.64E-01	-1.54E-01
$dB, ''$	4.75E-05	-7.56E-05	6.89E-03	2.75E-02	-3.31E-02
$dr/r, ''$	2.61E-04	3.27E-04	1.62E-02	5.29E-02	-5.42E-02
Osculating values at JD 241 5000.5					
$x = 1.33757 \ 33773 \ 62721D \ -01 \ \text{AU}$ $y = -1.30387 \ 78434 \ 87142D \ 00 \ \text{AU}$ $z = -6.02078 \ 22252 \ 04242D \ -01 \ \text{AU}$ $\dot{x} = 1.44852 \ 25377 \ 11578D \ -02 \ \text{AU/day}$ $\dot{y} = 2.40221 \ 52642 \ 67937D \ -03 \ \text{AU/day}$ $\dot{z} = 7.11393 \ 61848 \ 99518D \ -04 \ \text{AU/day}$			$a = 1.52367 \ 13186 \ 55129D \ 00 \ \text{AU}$ $n = 9.14628 \ 16919 \ 77629D \ -03 \ \text{rad/day}$ $e = 9.32101 \ 80192 \ 46872D \ -02$ $l = 3.23610 \ 16727 \ 22402D \ -02 \ \text{rad}$ $\Omega = 8.60806 \ 75094 \ 40483D \ -01 \ \text{rad}$ $\omega = -1.29832 \ 25395 \ 30241D \ 00 \ \text{rad}$ $M_0 = -8.85026 \ 81027 \ 62645D \ -01 \ \text{rad}$		
Equatorial PQR matrix					
$P = 9.05424 \ 58177 \ 23178D \ -01$ $Q = 4.23797 \ 41354 \ 74490D \ -01$ $R = 2.45372 \ 97995 \ 41911D \ -02$		$-3.76009 \ 16384 \ 41312D \ -01$ $8.27472 \ 07447 \ 49755D \ -01$ $-4.17021 \ 67170 \ 22240D \ -01$		$-1.97036 \ 63473 \ 49160D \ -01$ $3.68355 \ 42378 \ 87272D \ -01$ $9.08565 \ 26806 \ 70588D \ -01$	



Table B-5. The fitted ephemeris of Jupiter

Statistical summary (modified source minus DE28)					
Residuals in position	Mean square residuals	Mean residuals	Standard deviations	Maximum residuals	Minimum residuals
$dx$ , AU	2.12E-15	-3.71E-09	4.58E-08	1.31E-07	-1.23E-07
$dy$ , AU	2.64E-16	-9.82E-10	1.62E-08	4.63E-08	-4.71E-08
$dz$ , AU	5.20E-17	-1.11E-09	7.12E-09	1.99E-08	-2.21E-08
$\cos B \, dL$ , "	2.60E-06	1.15E-06	1.61E-03	4.56E-03	-5.14E-03
$dB$ , "	6.46E-09	-3.20E-05	7.37E-05	2.27E-04	-2.99E-04
$dr/r$ , "	1.22E-06	-9.41E-05	1.10E-03	3.46E-03	-4.10E-03
Osculating values at JD 241 5000.5					
$x = -3.19142 \ 51765 \ 06697D \ 00 \text{ AU}$ $y = -4.01920 \ 45093 \ 95551D \ 00 \text{ AU}$ $z = -1.64634 \ 06237 \ 49956D \ 00 \text{ AU}$ $\dot{x} = 6.00193 \ 49645 \ 17663D \ -03 \text{ AU/day}$ $\dot{y} = -3.72897 \ 89854 \ 49904D \ -03 \text{ AU/day}$ $\dot{z} = -1.74686 \ 19631 \ 84758D \ -03 \text{ AU/day}$			$a = 5.20279 \ 66644 \ 36973D \ 00 \text{ AU}$ $n = 1.45021 \ 83365 \ 04639D \ -03 \text{ rad/day}$ $e = 4.87182 \ 94535 \ 83041D \ -02$ $l = 2.28284 \ 56352 \ 71518D \ -02 \text{ rad}$ $\Omega = 1.74267 \ 77821 \ 93899D \ 00 \text{ rad}$ $\omega = -1.51468 \ 05539 \ 87502D \ 00 \text{ rad}$ $M_0 = -2.36749 \ 59377 \ 15825D \ 00 \text{ rad}$		
Equatorial PQR matrix					
$P = 9.73864 \ 71451 \ 63290D \ -01$ $Q = -2.26012 \ 63762 \ 94756D \ -01$ $R = 2.24901 \ 18981 \ 37172D \ -02$		$2.16392 \ 66235 \ 07493D \ -01$ $8.93187 \ 53307 \ 10321D \ -01$ $-3.94195 \ 69562 \ 24135D \ -01$		$6.90053 \ 15018 \ 36209D \ -02$ $3.88759 \ 97530 \ 38516D \ -01$ $9.18751 \ 29828 \ 53226D \ -01$	

Table B-6. The fitted ephemeris of Saturn

Statistical summary (modified source minus DE28)					
Residuals in position	Mean square residuals	Mean residuals	Standard deviations	Maximum residuals	Minimum residuals
$dx$ , AU	2.21E-15	-4.57E-10	4.70E-08	1.42E-07	-1.16E-07
$dy$ , AU	1.76E-15	-2.42E-09	4.18E-08	1.16E-07	-1.15E-07
$dz$ , AU	3.14E-16	-2.26E-09	1.76E-08	4.57E-08	-5.05E-08
$\cos B \, dL$ , "	1.78E-06	-2.10E-06	1.33E-03	2.77E-03	-3.28E-03
$dB$ , "	3.53E-09	-2.71E-05	5.28E-05	1.48E-04	-2.09E-04
$dr/r$ , "	2.73E-07	2.74E-05	5.22E-04	1.51E-03	-2.04E-03
Osculating values at JD 241 5000.5					
$x = -5.94956 \ 02928 \ 19203D \ -01 \text{ AU}$ $y = -9.29550 \ 93103 \ 32007D \ 00 \text{ AU}$ $z = -3.81681 \ 73382 \ 18680D \ 00 \text{ AU}$ $\dot{x} = 5.26756 \ 59689 \ 84819D \ -03 \text{ AU/day}$ $\dot{y} = -2.40510 \ 33329 \ 67816D \ -04 \text{ AU/day}$ $\dot{z} = -3.26617 \ 76771 \ 81013D \ -04 \text{ AU/day}$			$a = 9.57974 \ 17954 \ 12653D \ 00 \text{ AU}$ $n = 5.80246 \ 78247 \ 81564D \ -04 \text{ rad/day}$ $e = 5.12004 \ 11642 \ 01615D \ -02$ $l = 4.34027 \ 64243 \ 69105D \ -02 \text{ rad}$ $\Omega = 1.97873 \ 81929 \ 09210D \ 00 \text{ rad}$ $\omega = -3.45741 \ 34833 \ 17933D \ -01 \text{ rad}$ $M_0 = 3.00693 \ 80341 \ 14843D \ 00 \text{ rad}$		
Equatorial PQR matrix					
$P = -6.24533 \ 81900 \ 41049D \ -02$ $Q = -9.97252 \ 85568 \ 65767D \ -01$ $R = 3.98285 \ 94177 \ 65996D \ -02$		$9.21397 \ 20167 \ 31201D \ -01$ $-7.29479 \ 13285 \ 67131D \ -02$ $-3.81714 \ 28935 \ 29262D \ -01$		$3.83571 \ 07794 \ 79403D \ -01$ $1.28586 \ 06932 \ 06811D \ -02$ $9.23421 \ 83447 \ 74078D \ -01$	

Table B-7. The fitted ephemeris of Uranus

Statistical summary (modified source minus DE28)					
Residuals in position	Mean square residuals	Mean residuals	Standard deviations	Maximum residuals	Minimum residuals
$dx, AU$	9.49E-17	1.27E-09	9.66E-09	2.94E-08	-3.12E-08
$dy, AU$	8.75E-17	-1.02E-10	9.35E-09	3.11E-08	-2.97E-08
$dz, AU$	1.76E-17	-6.74E-11	4.20E-09	1.28E-08	-1.17E-08
$\cos B \, dl, ''$	1.10E-08	8.42E-08	1.05E-04	3.07E-04	-3.49E-04
$dB, ''$	1.52E-10	1.27E-07	1.23E-05	4.78E-05	-3.90E-05
$dr/r, ''$	1.22E-08	1.12E-06	1.11E-04	3.07E-04	-3.57E-04
Osculating values at JD 241 5000.5					
$x = -6.76950 \ 07781 \ 13280D \ 00 \ AU$ $y = -1.62850 \ 11314 \ 68881D \ 01 \ AU$ $z = -7.03982 \ 80081 \ 07494D \ 00 \ AU$ $\dot{x} = 3.64948 \ 04881 \ 61621D \ -03 \ AU/day$ $\dot{y} = -1.43631 \ 39888 \ 26346D \ -03 \ AU/day$ $\dot{z} = -6.80173 \ 51596 \ 69874D \ -04 \ AU/day$			$a = 1.93133 \ 80668 \ 68614D \ 01 \ AU$ $n = 2.02676 \ 54190 \ 87620D \ -04 \ rad/day$ $e = 4.89964 \ 59177 \ 76881D \ -02$ $l = 1.35150 \ 36183 \ 18392D \ -02 \ rad$ $\Omega = 1.28781 \ 16045 \ 11192D \ 00 \ rad$ $\omega = 1.79256 \ 81297 \ 13942D \ 00 \ rad$ $M_0 = 1.17501 \ 96098 \ 39210D \ 00 \ rad$		
Equatorial PQR matrix					
$P = -9.98041 \ 52757 \ 27898D \ -01$ $Q = -6.11939 \ 87937 \ 02395D \ -02$ $R = 1.29770 \ 98309 \ 56381D \ -02$		$5.08555 \ 84225 \ 01821D \ -02$ $-9.14530 \ 75215 \ 55767D \ -01$ $-4.01306 \ 88122 \ 04531D \ -01$		$3.64255 \ 23926 \ 29153D \ -02$ $-3.99860 \ 97484 \ 26547D \ -01$ $9.15851 \ 72490 \ 12400D \ -01$	

Table B-8. The fitted ephemeris of Neptune

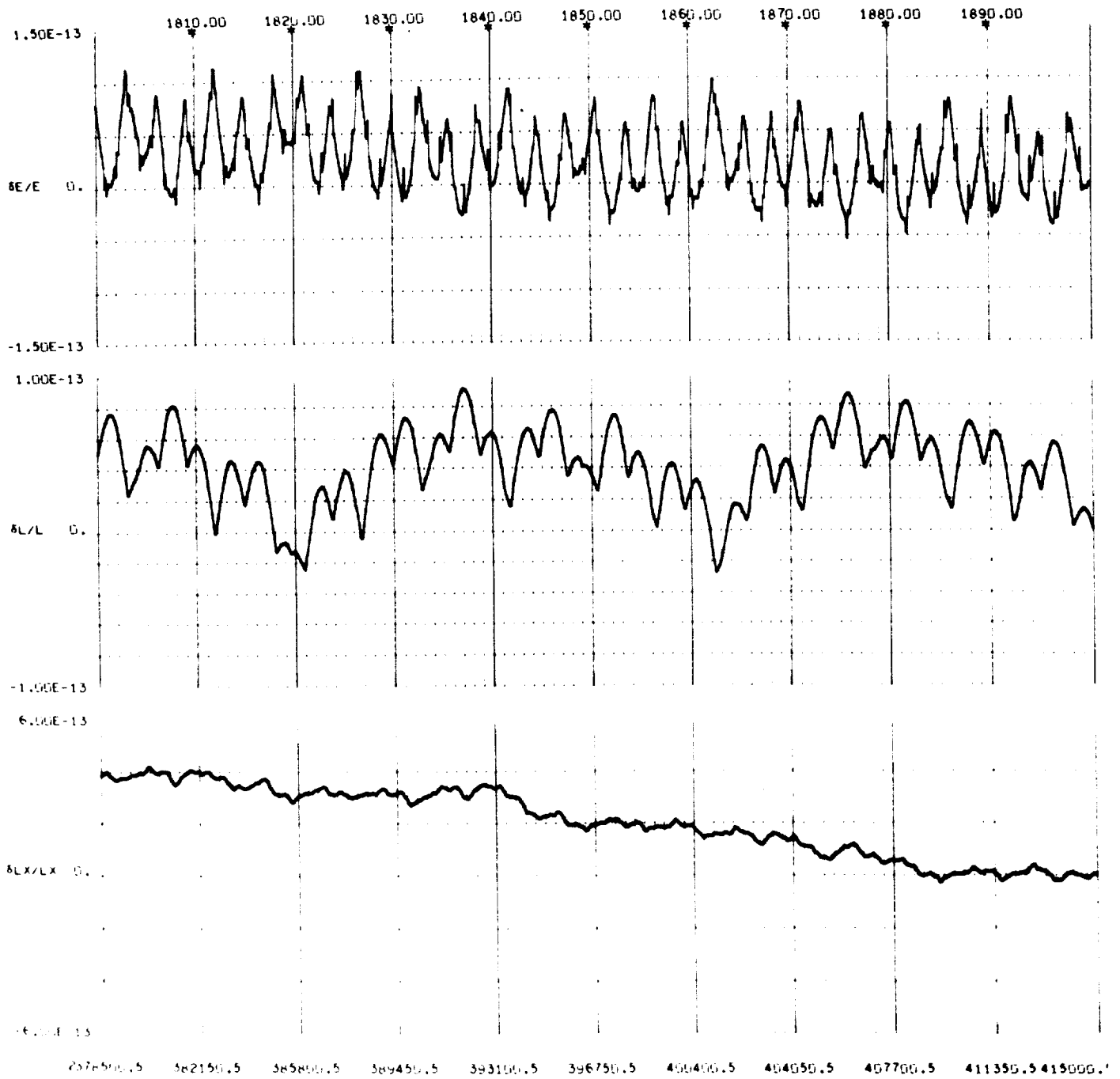
Statistical summary (modified source minus DE28)					
Residuals in position	Mean square residuals	Mean residuals	Standard deviations	Maximum residuals	Minimum residuals
$dx, AU$	9.12E-16	1.40E-10	3.02E-08	6.78E-08	-7.61E-08
$dy, AU$	8.16E-16	-3.99E-09	2.83E-08	5.70E-08	-5.74E-08
$dz, AU$	1.64E-16	-3.96E-09	1.22E-08	2.79E-08	-2.59E-08
$\cos B \, dl, ''$	2.02E-08	3.97E-05	1.36E-04	4.19E-04	-4.22E-04
$dB, ''$	9.32E-10	-1.40E-05	2.71E-05	7.70E-05	-6.13E-05
$dr/r, ''$	6.78E-08	-2.45E-04	8.77E-05	-1.73E-05	-4.71E-04
Osculating values at JD 241 5000.5					
$x = 1.94125 \ 69860 \ 33593D \ 00 \ AU$ $y = 2.76001 \ 39691 \ 69463D \ 01 \ AU$ $z = 1.12571 \ 31503 \ 15774D \ 01 \ AU$ $\dot{x} = -3.14313 \ 20212 \ 10028D \ -03 \ AU/day$ $\dot{y} = 1.75391 \ 80187 \ 76925D \ -04 \ AU/day$ $\dot{z} = 1.50866 \ 52286 \ 66874D \ -04 \ AU/day$			$a = 2.99488 \ 67026 \ 67926D \ 01 \ AU$ $n = 1.04959 \ 58088 \ 43447D \ -04 \ rad/day$ $e = 5.33020 \ 97343 \ 25422D \ -03$ $l = 3.09525 \ 01741 \ 01840D \ -02 \ rad$ $\Omega = 2.29121 \ 13328 \ 21336D \ 00 \ rad$ $\omega = -1.84950 \ 72404 \ 15644D \ 00 \ rad$ $M_0 = 1.05450 \ 12555 \ 94580D \ 00 \ rad$		
Equatorial PQR matrix					
$P = 9.03678 \ 42422 \ 59001D \ -01$ $Q = -4.27579 \ 66183 \ 77455D \ -01$ $R = 2.32580 \ 81849 \ 40044D \ -02$		$4.03746 \ 09728 \ 35746D \ -01$ $8.32693 \ 36959 \ 65117D \ -01$ $-3.78960 \ 21052 \ 12230D \ -01$		$1.42668 \ 82811 \ 90965D \ -01$ $3.51848 \ 52566 \ 51315D \ -01$ $9.25120 \ 65184 \ 51511D \ -01$	

Table B-9. The fitted ephemeris of Pluto

Statistical summary (modified source minus DE28)					
Residuals in position	Mean square residuals	Mean residuals	Standard deviations	Maximum residuals	Minimum residuals
$dx, \text{ AU}$	3.01E -16	-1.49E -09	1.73E -08	4.03E -08	-7.45E -08
$dy, \text{ AU}$	3.65E -16	-5.82E -09	1.82E -08	4.54E -08	-5.42E -08
$dz, \text{ AU}$	3.24E -16	-1.92E -09	1.79E -08	7.45E -08	-3.54E -08
$\cos B \, dl, ''$	7.47E -09	-5.75E -06	8.62E -05	1.85E -04	-4.97E -04
$dB, ''$	1.01E -08	-4.05E -06	1.00E -04	4.71E -04	-2.04E -04
$dr/r, ''$	1.39E -08	-7.57E -05	9.02E -05	4.01E -04	-2.73E -04
Osculating values at JD 241 5000.5					
$x = 1.08898 \ 33456 \ 98867D \ 01 \ \text{AU}$ $y = 4.44014 \ 79876 \ 26816D \ 01 \ \text{AU}$ $z = 1.07217 \ 00090 \ 29059D \ 01 \ \text{AU}$ $\dot{x} = -2.15764 \ 99130 \ 21905D \ -03 \ \text{AU/day}$ $\dot{y} = -2.10708 \ 08193 \ 74032D \ -05 \ \text{AU/day}$ $\dot{z} = 6.48833 \ 61690 \ 99614D \ -04 \ \text{AU/day}$			$a = 3.93162 \ 65690 \ 03125D \ 01 \ \text{AU}$ $n = 6.97788 \ 25104 \ 84292D \ -05 \ \text{rad/day}$ $e = 2.53032 \ 43405 \ 15995D \ -01$ $l = 2.99767 \ 12637 \ 47195D \ -01 \ \text{rad}$ $\Omega = 1.91232 \ 78510 \ 53723D \ 00 \ \text{rad}$ $\omega = 1.99627 \ 89789 \ 48910D \ 00 \ \text{rad}$ $M_0 = -2.28468 \ 75731 \ 26375D \ 00 \ \text{rad}$		
Equatorial PQR matrix					
$P = -6.81713 \ 38551 \ 66848D \ -01$ $Q = 6.76644 \ 78112 \ 95817D \ -01$ $R = 2.78242 \ 16103 \ 50953D \ -01$		$-7.31227 \ 22445 \ 70322D \ -01$ $-6.17701 \ 17309 \ 89745D \ -01$ $-2.89399 \ 39005 \ 63995D \ -01$		$-2.39500 \ 77666 \ 77484D \ -02$ $-4.00745 \ 68110 \ 24311D \ -01$ $9.15876 \ 24319 \ 96500D \ -01$	

## Appendix C

### Residual Plots, Modified Source Minus DE28



**Fig. C-1. Relative changes in energy and angular momentum integrals for 1800–2000**

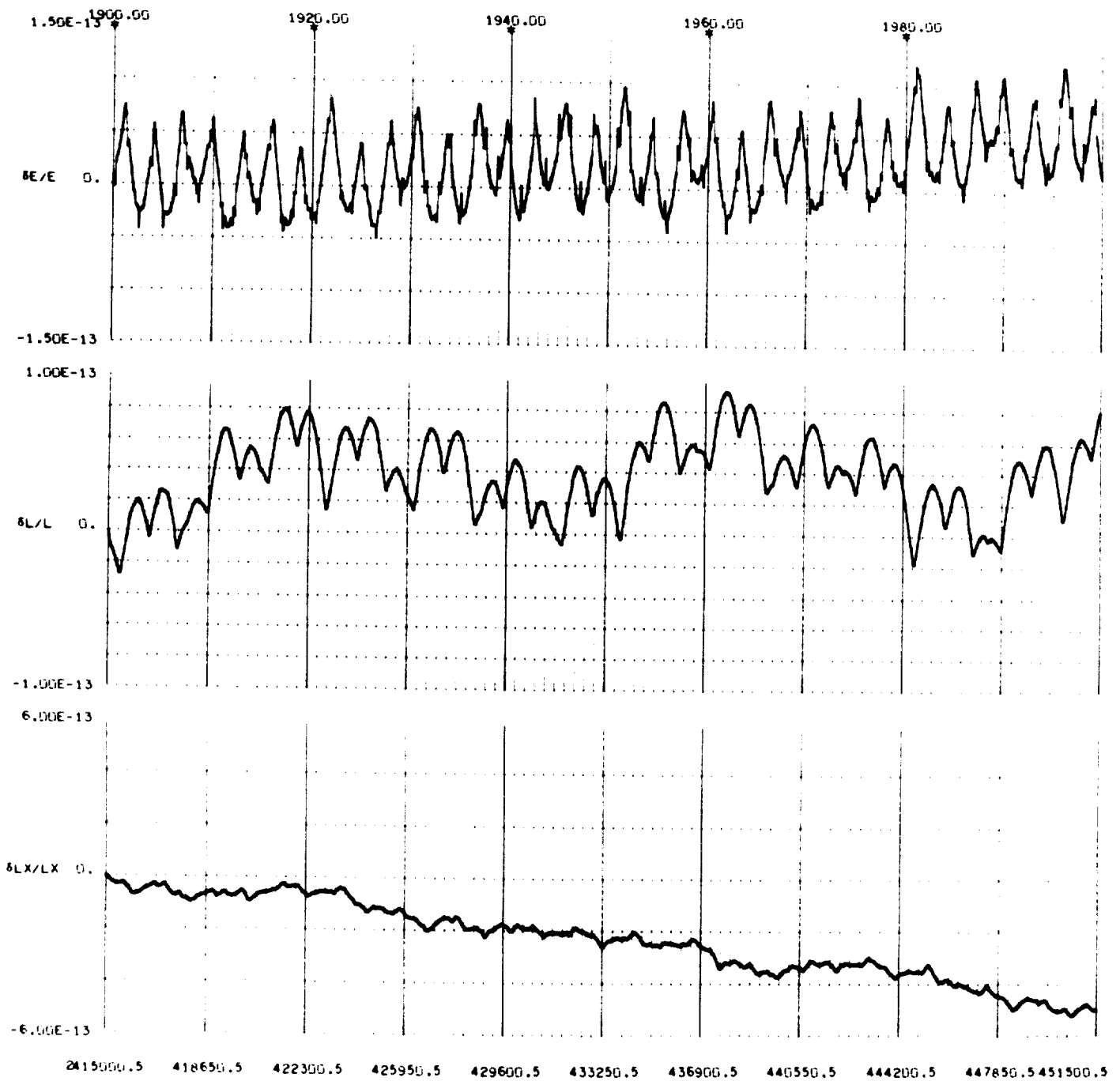


Fig. C-1 (contd)

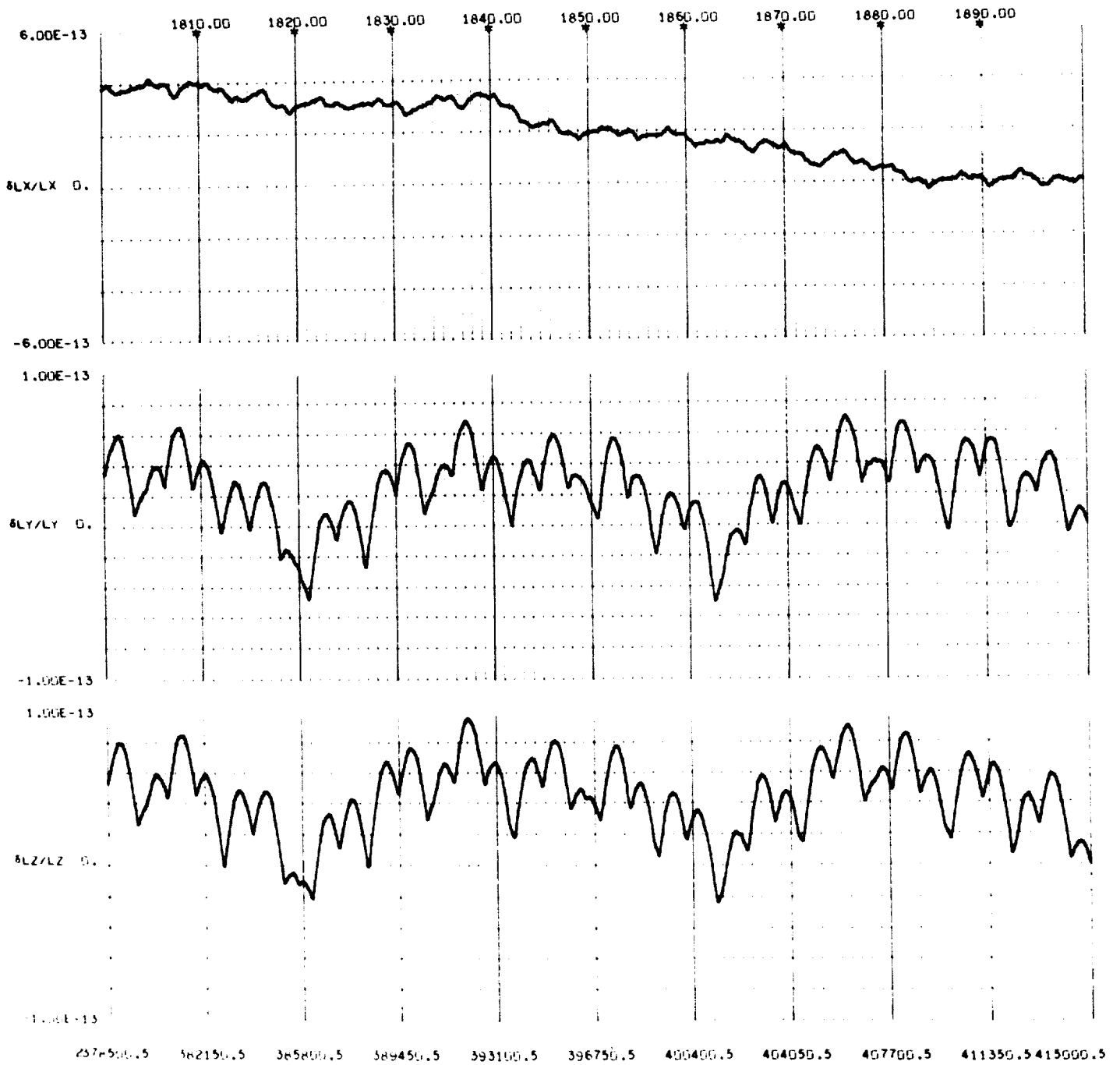


Fig. C-2. Relative changes in angular momentum components for 1800-2000

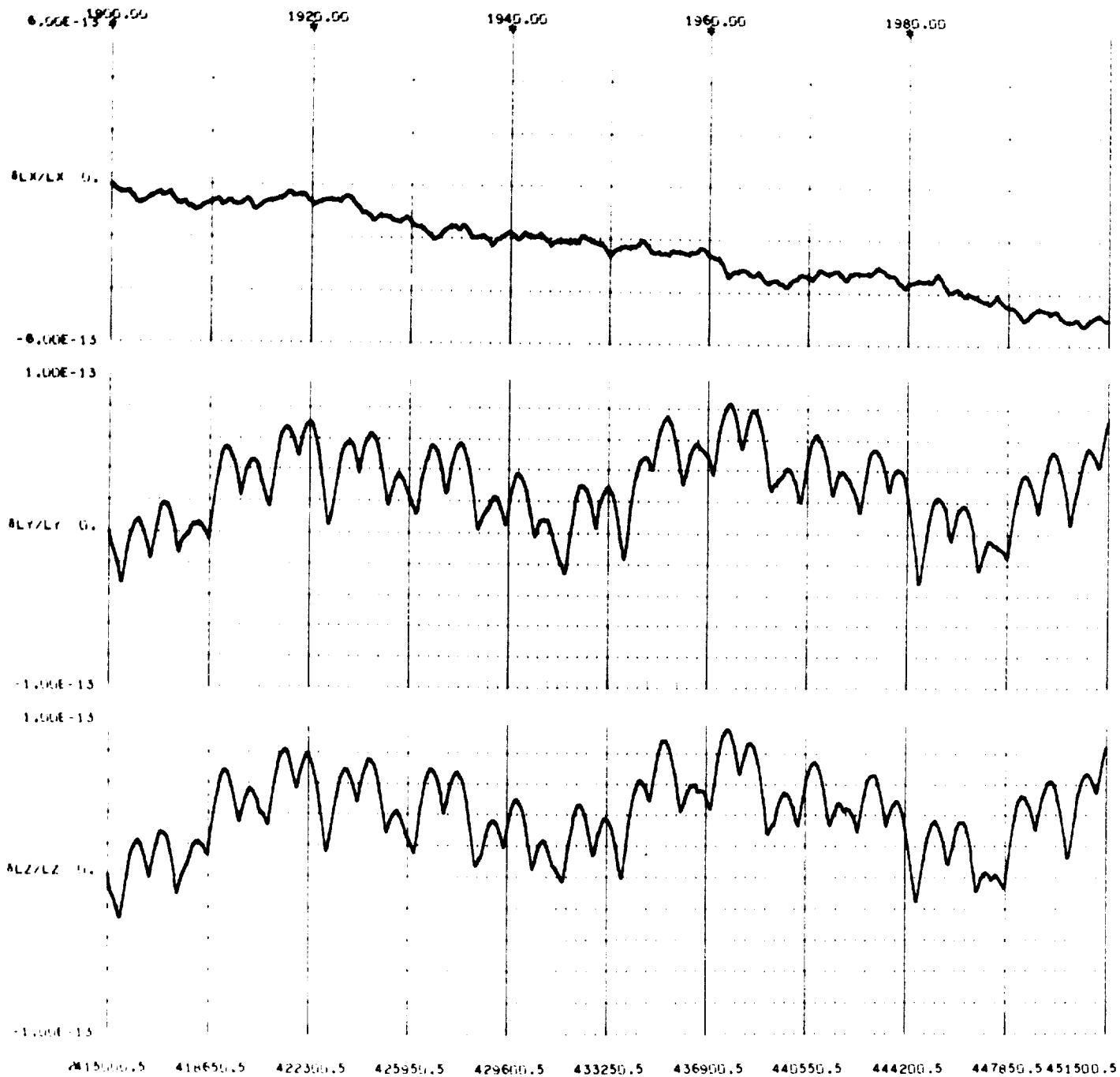


Fig. C-2 (contd)

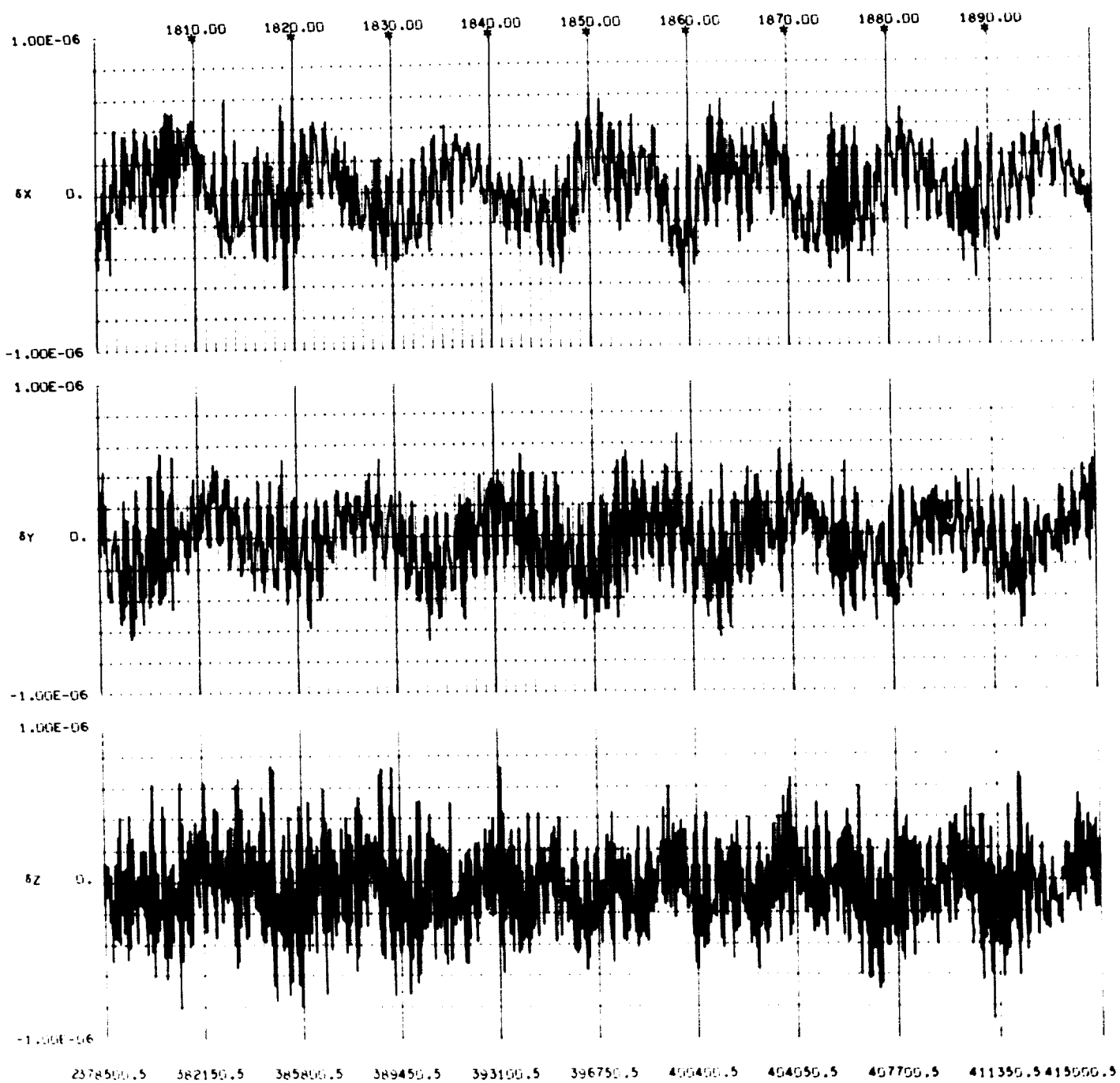


Fig. C-3. Mercury rectangular residuals (modified source minus DE28) for 1800–2000



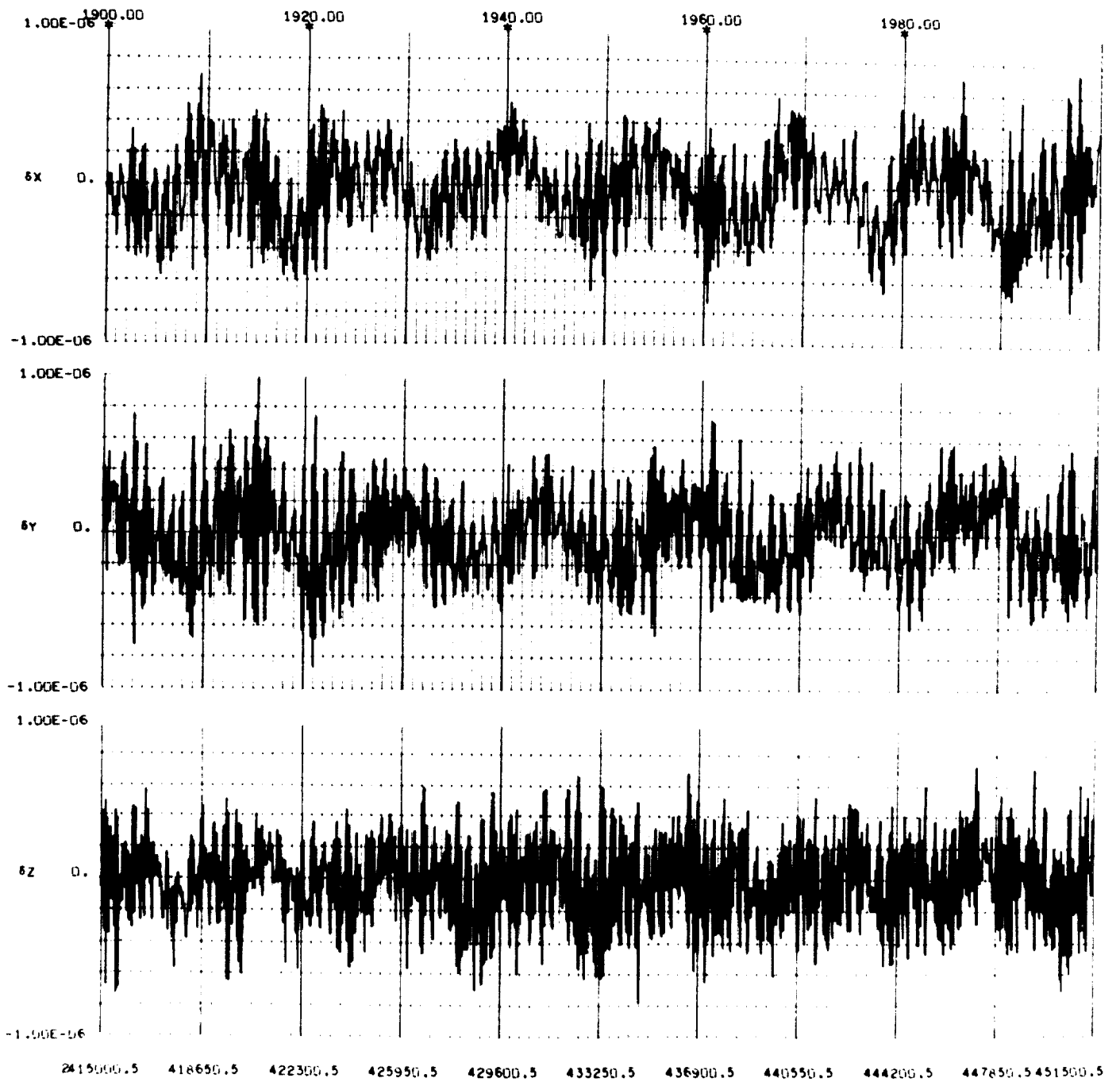


Fig. C-3 (contd)

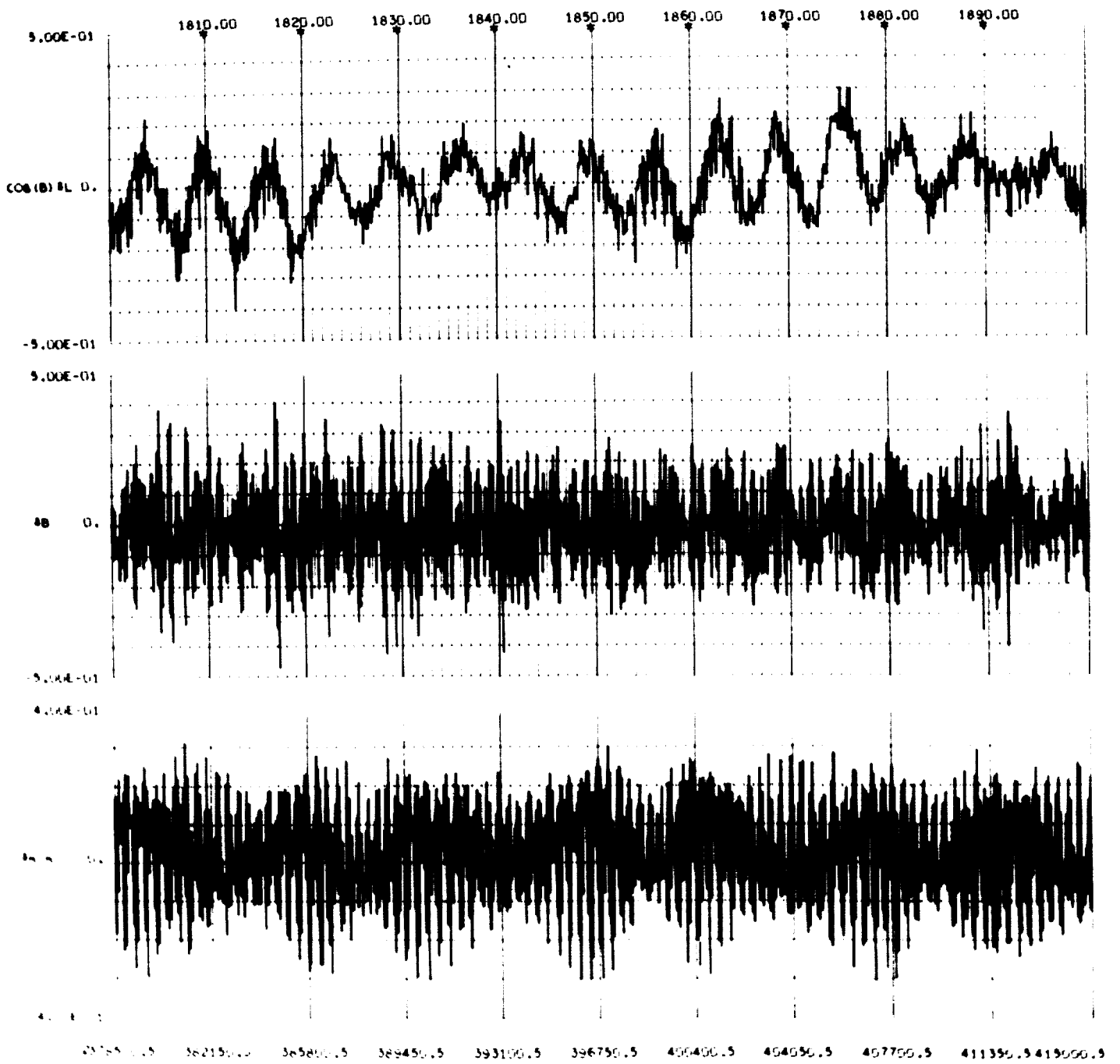


Fig. C-4. Mercury angular residuals (modified source minus DE28) for 1800–2000

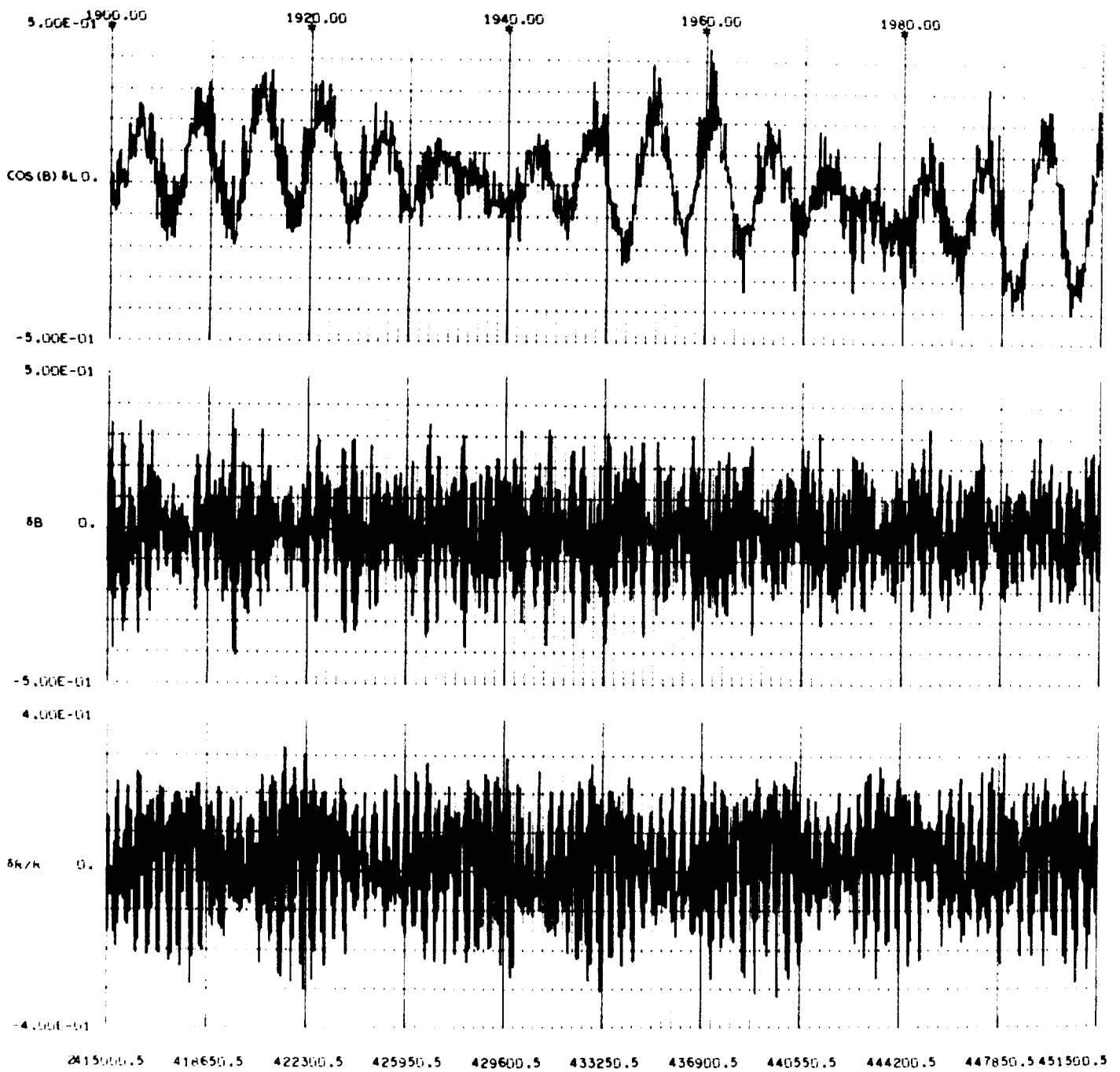


Fig. C-4 (contd)

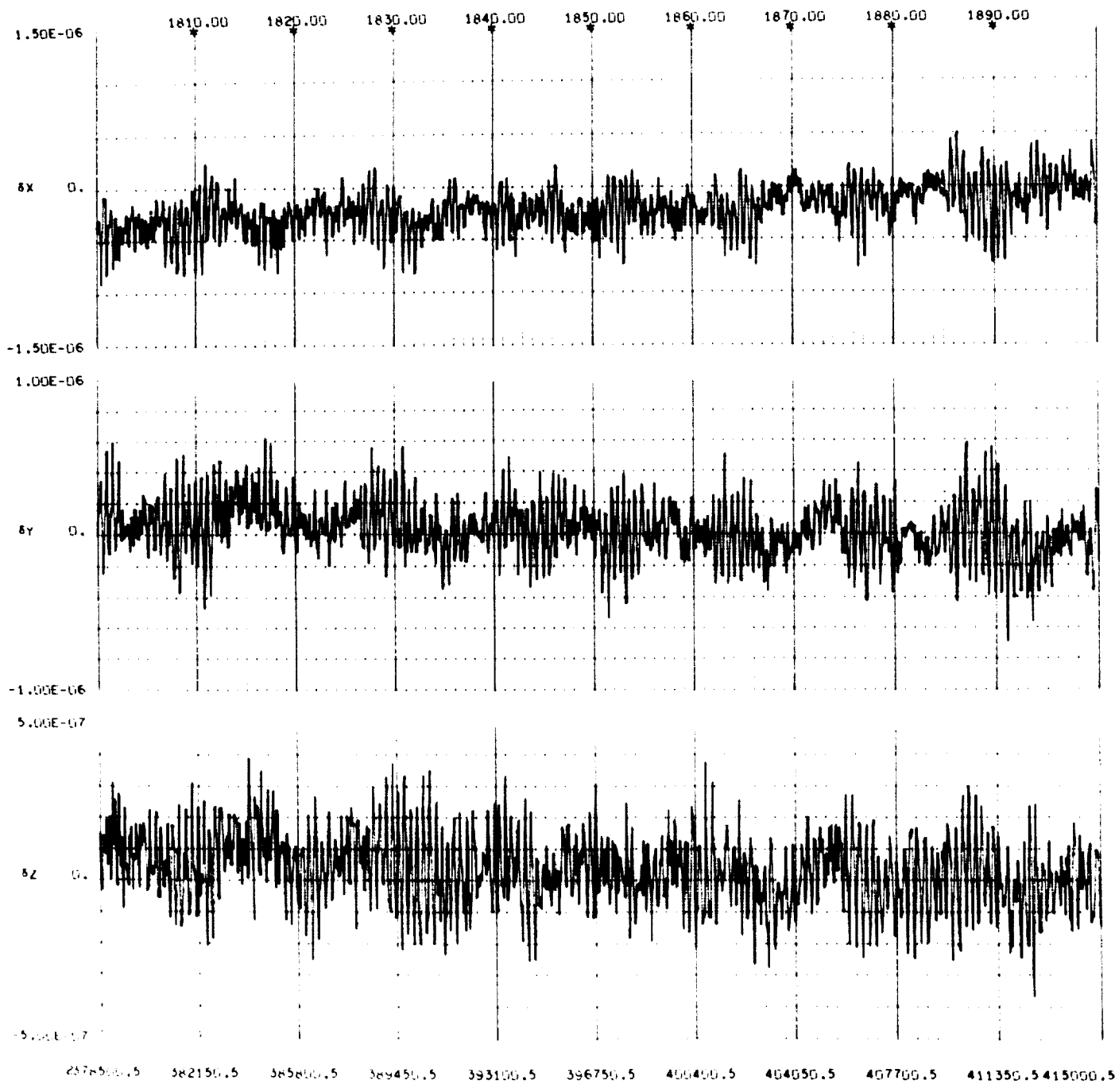


Fig. C-5. Venus rectangular residuals (modified source minus DE28) for 1800-2000

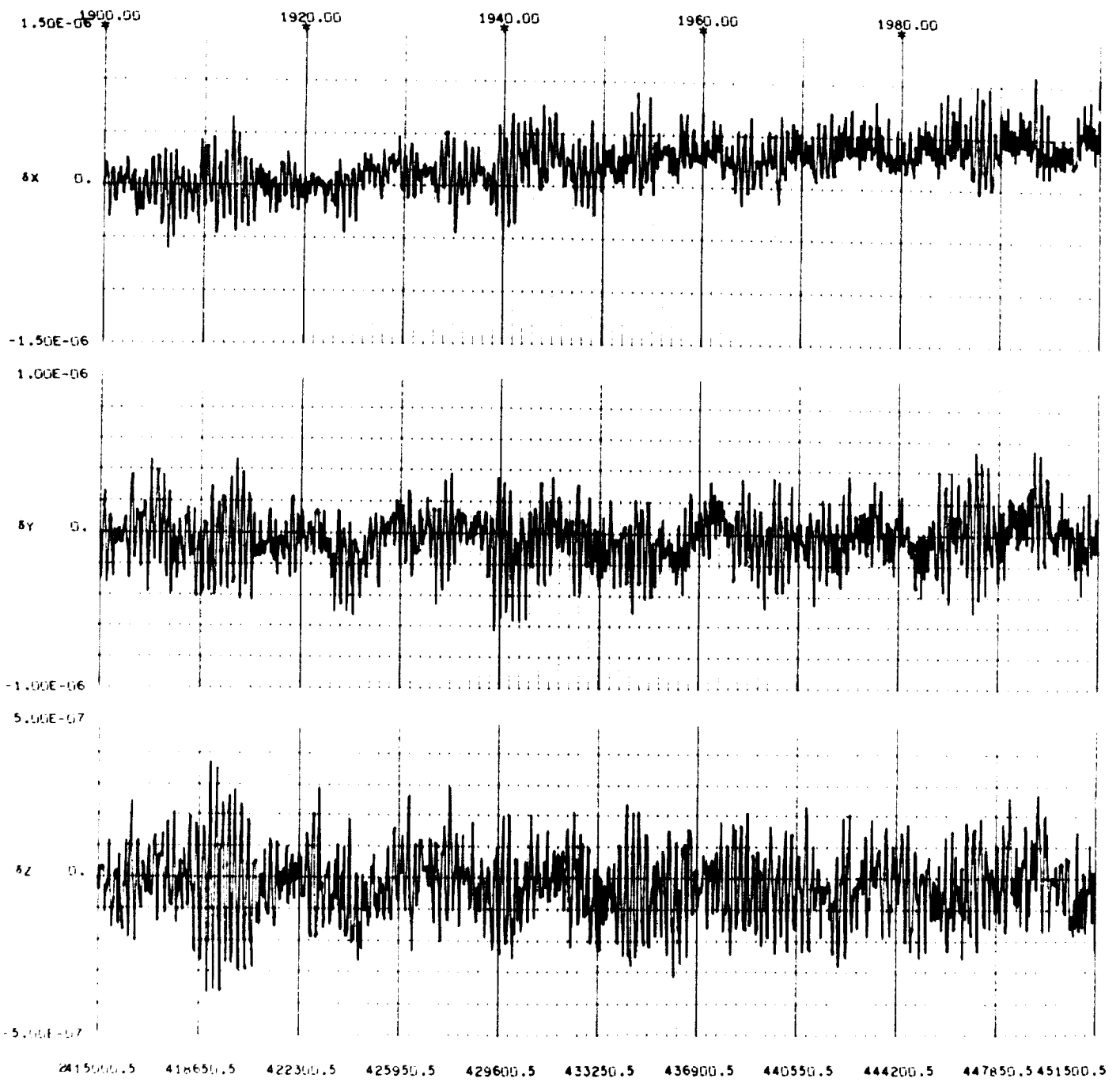


Fig. C-5 (contd)

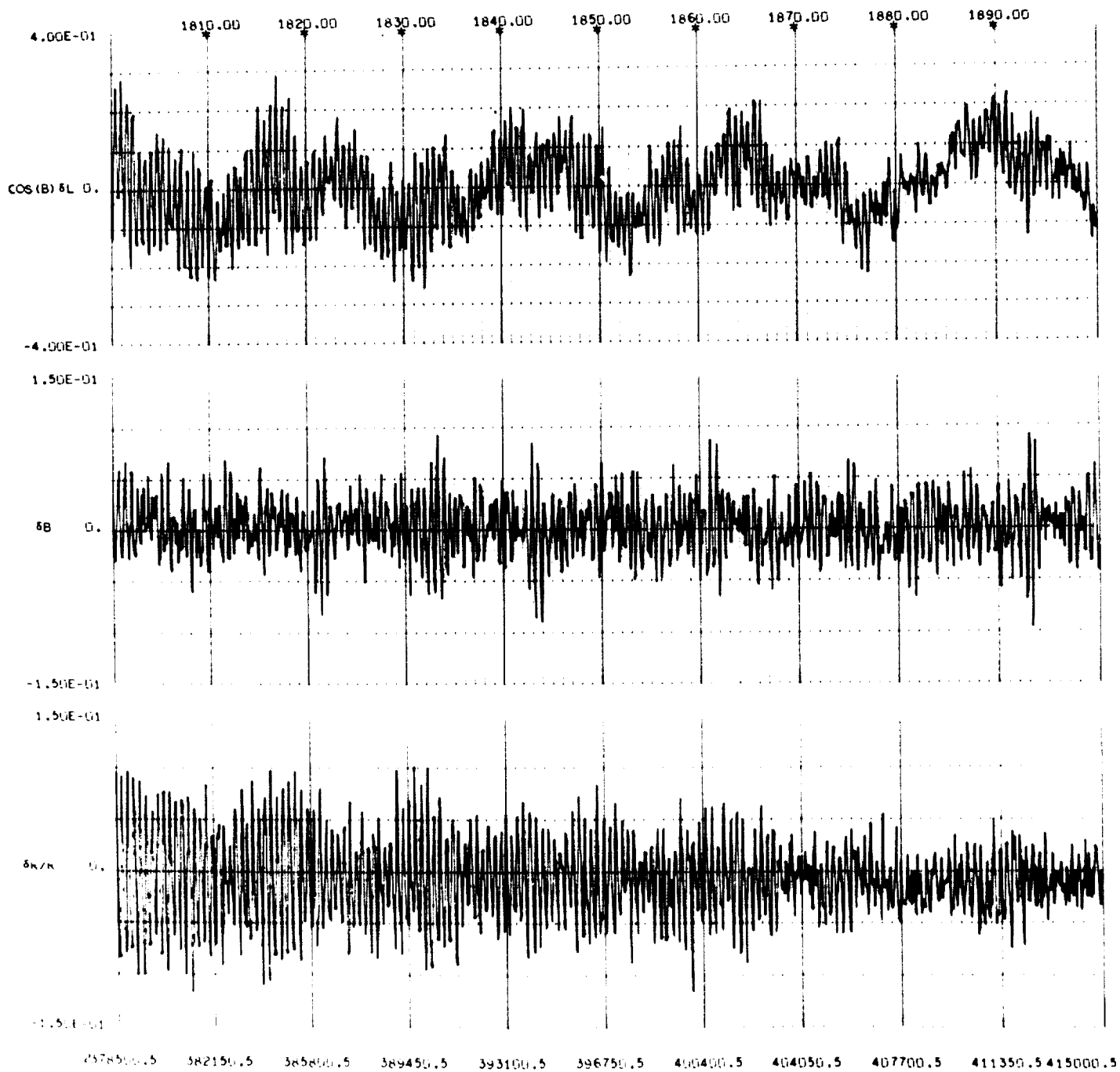


Fig. C-6. Venus angular residuals (modified source minus DE28) for 1800-2000

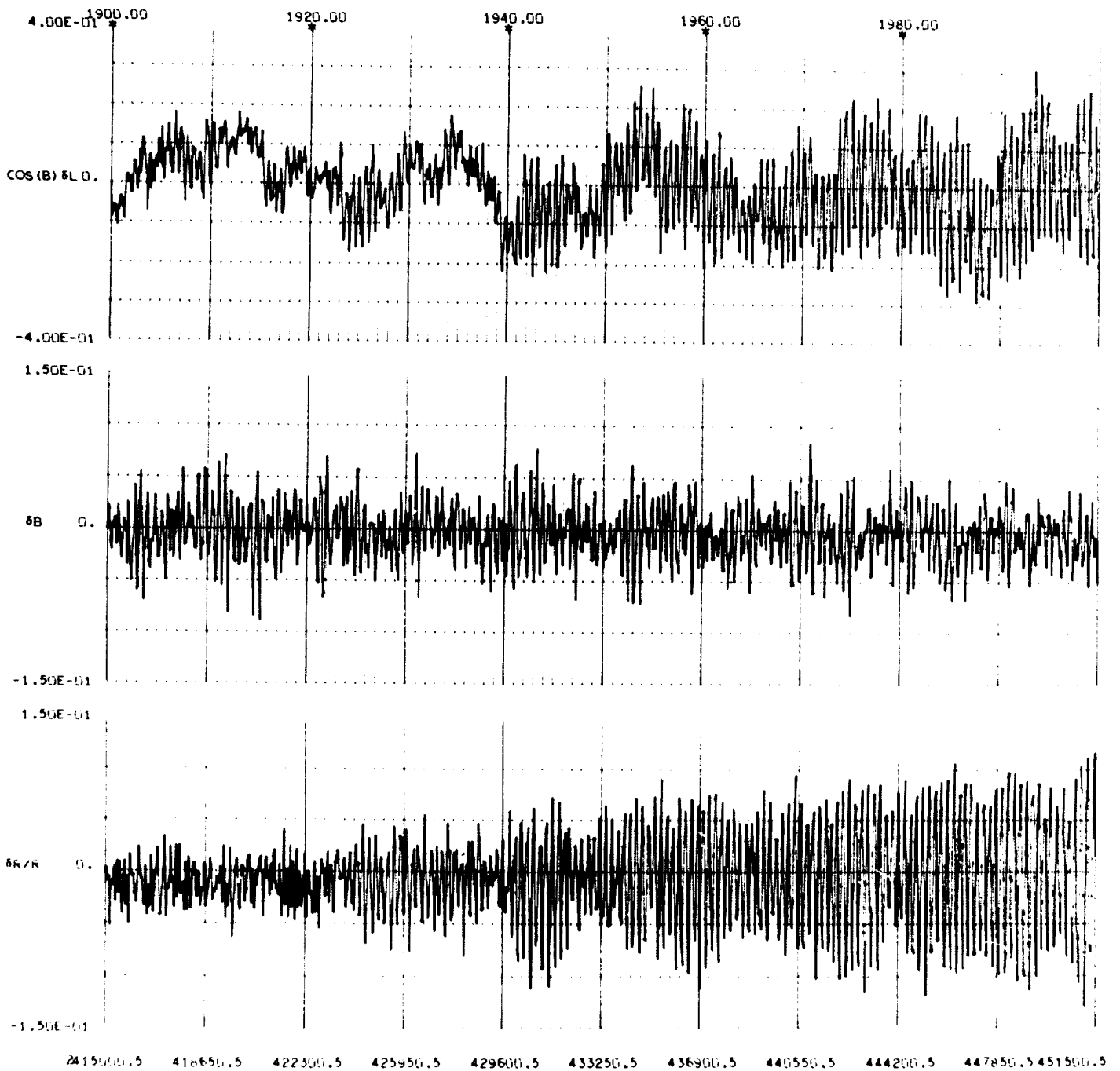


Fig. C-6 (contd)

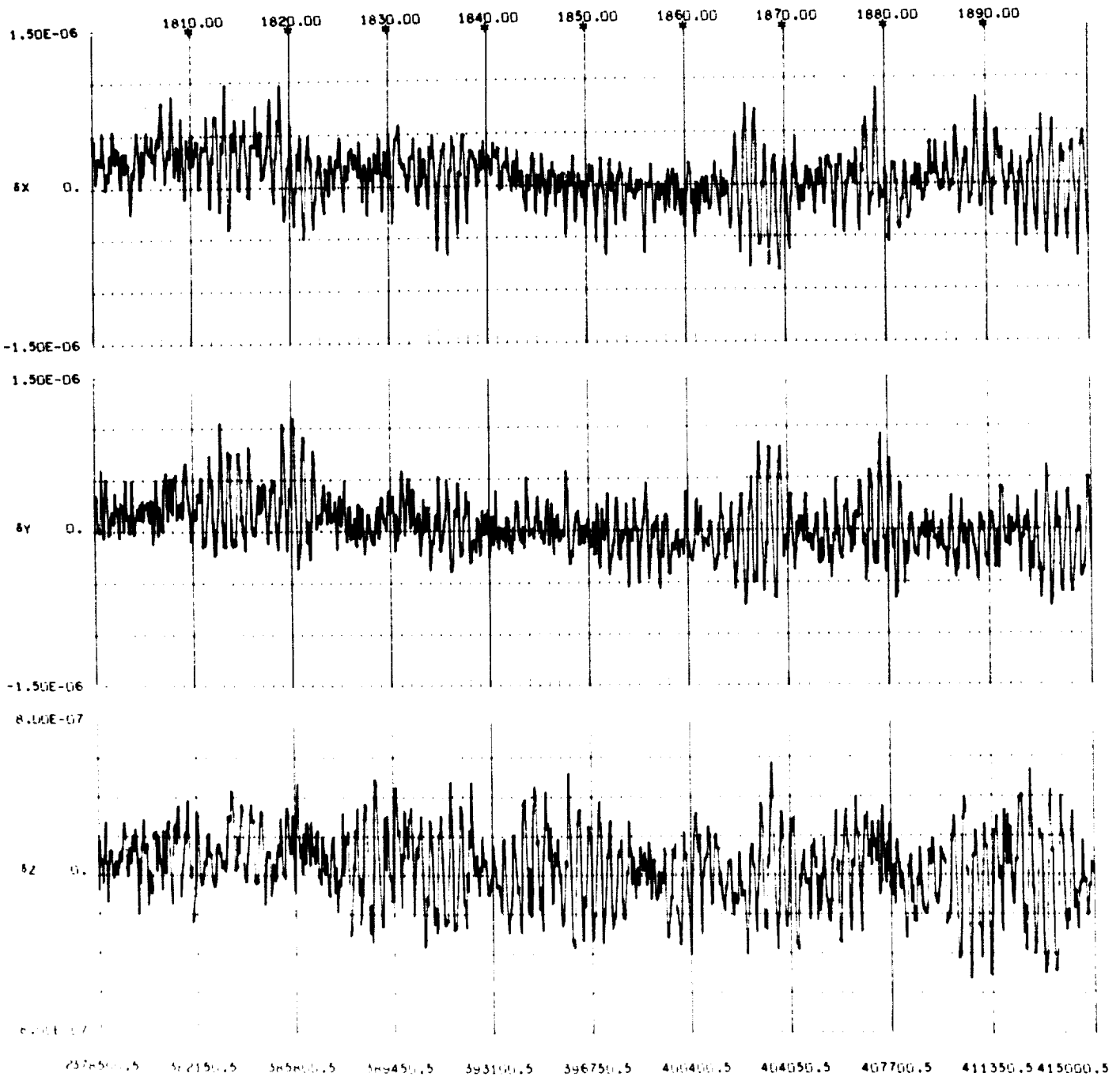


Fig. C-7. Earth-moon barycenter rectangular residuals (modified source minus DE28) for 1800–2000



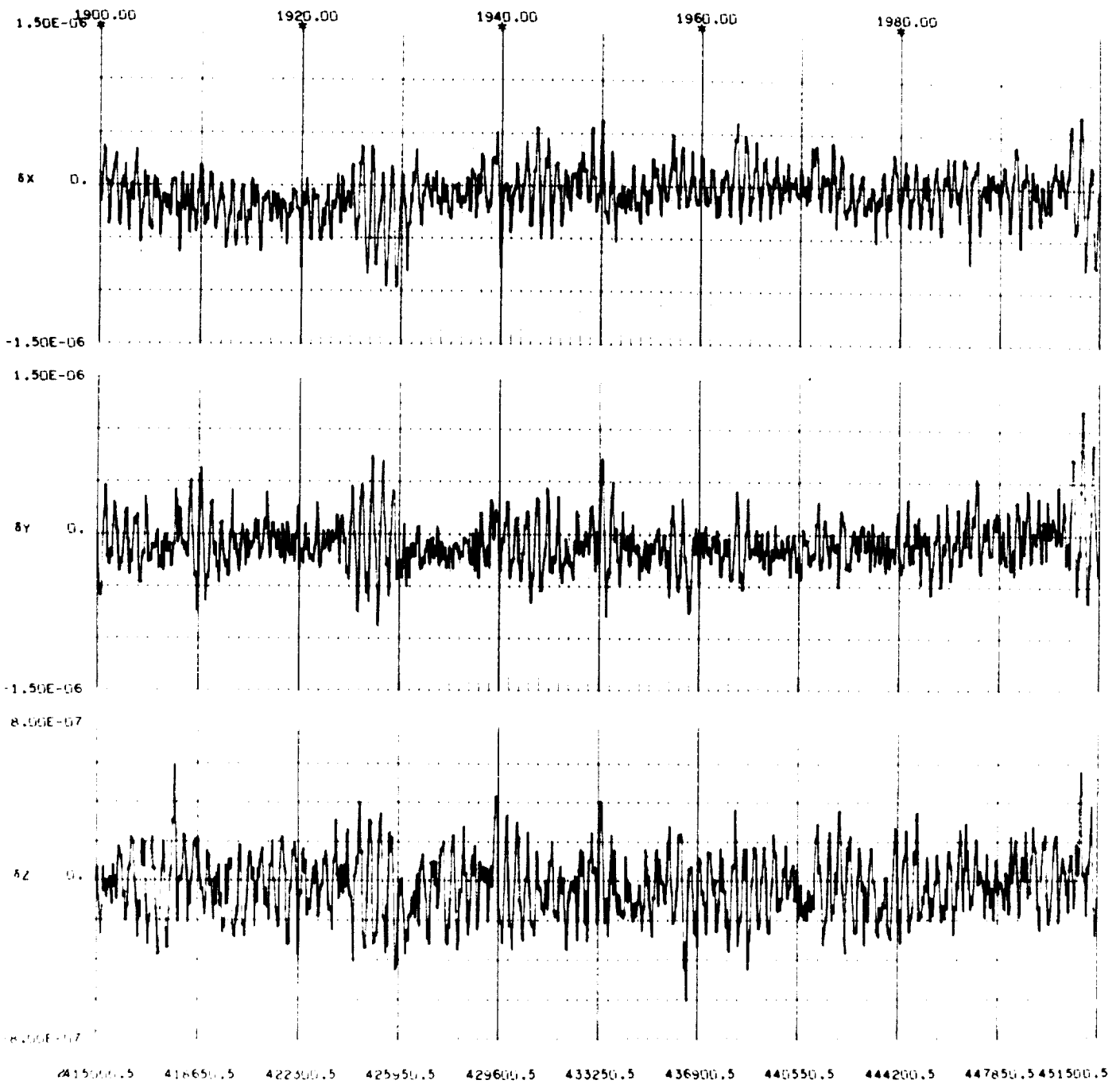


Fig. C-7 (contd)

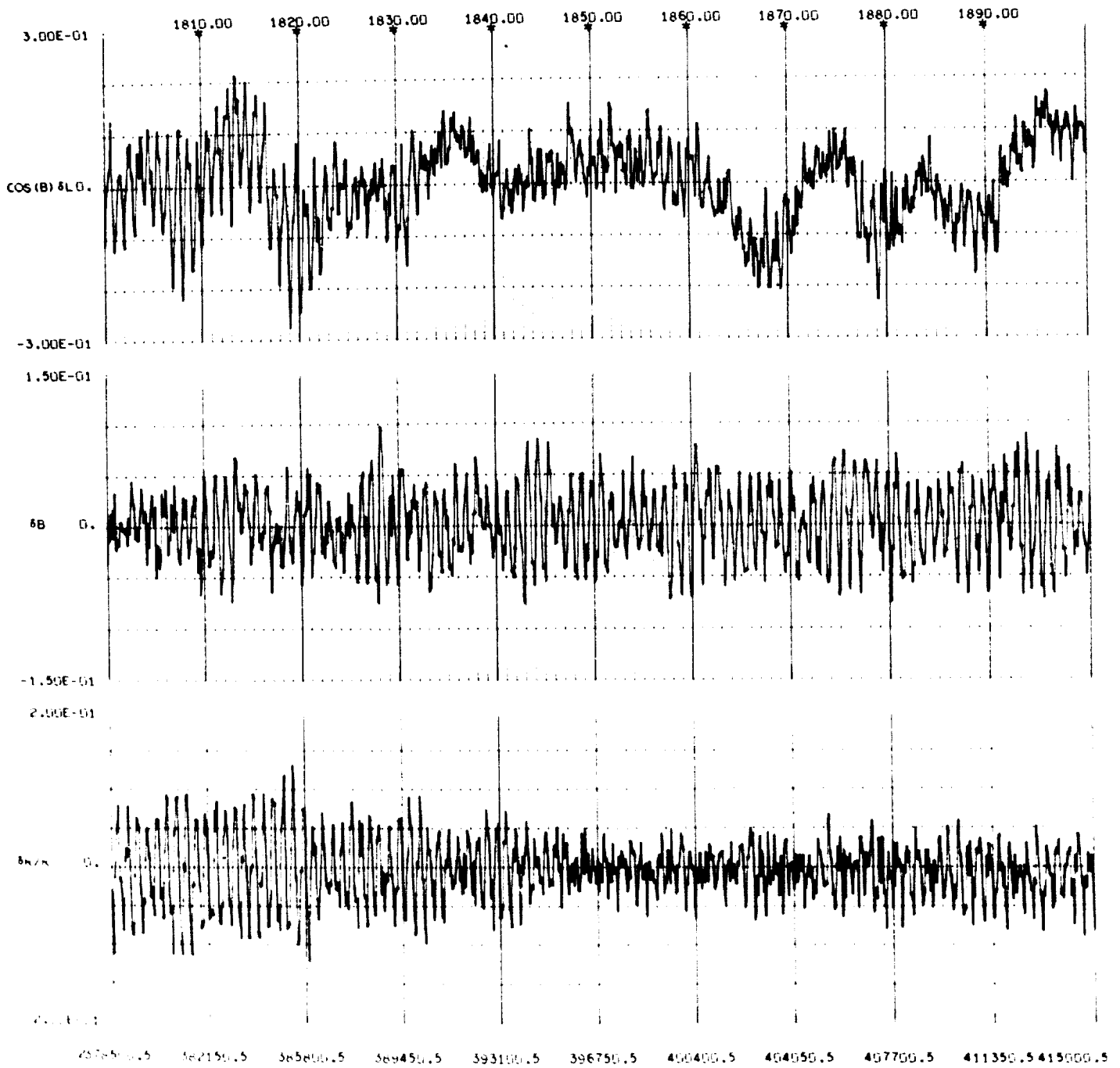


Fig. C-8. Earth-moon barycenter angular residuals (modified source minus DE28) for 1800-2000

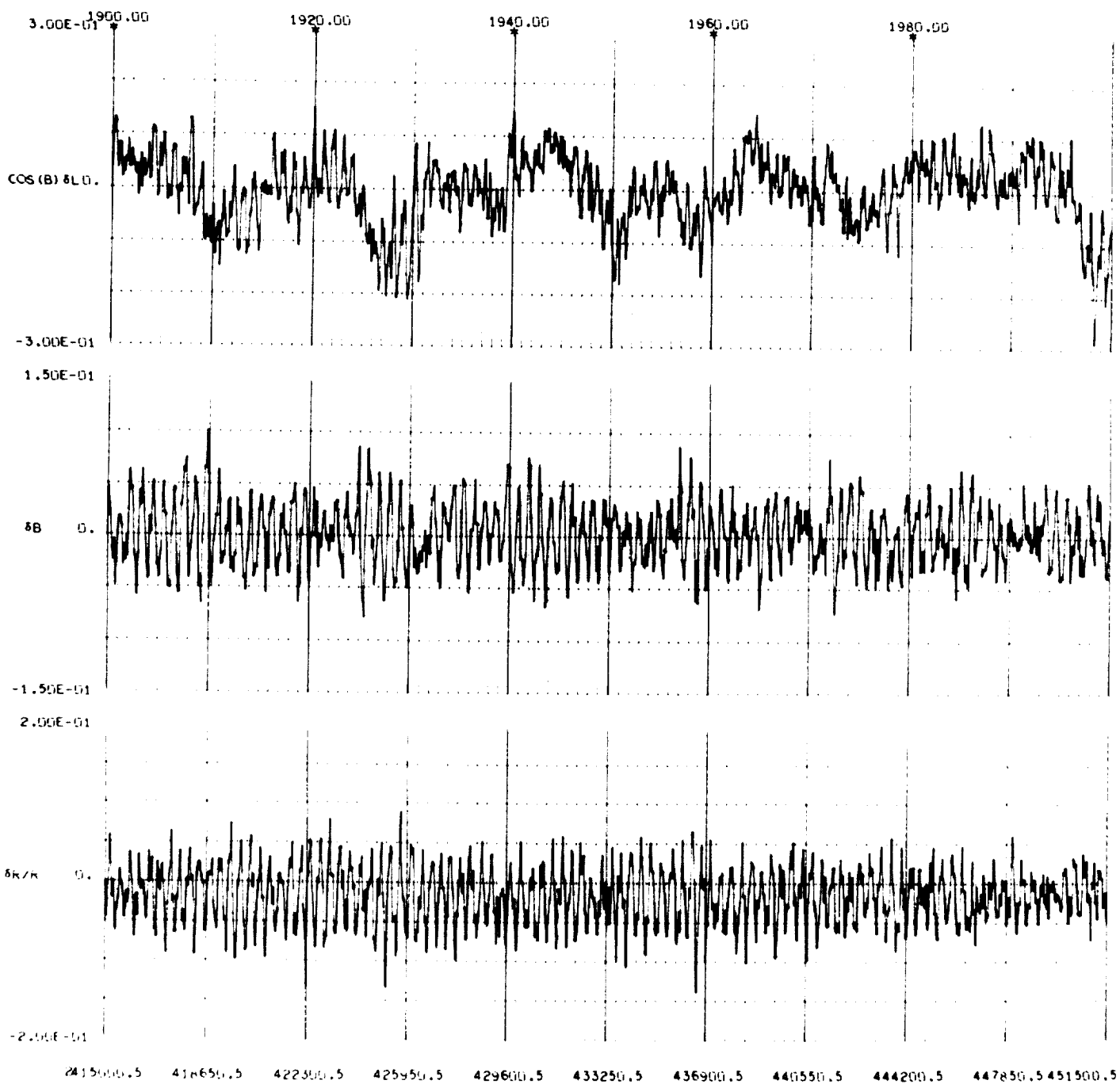


Fig. C-8 (contd)

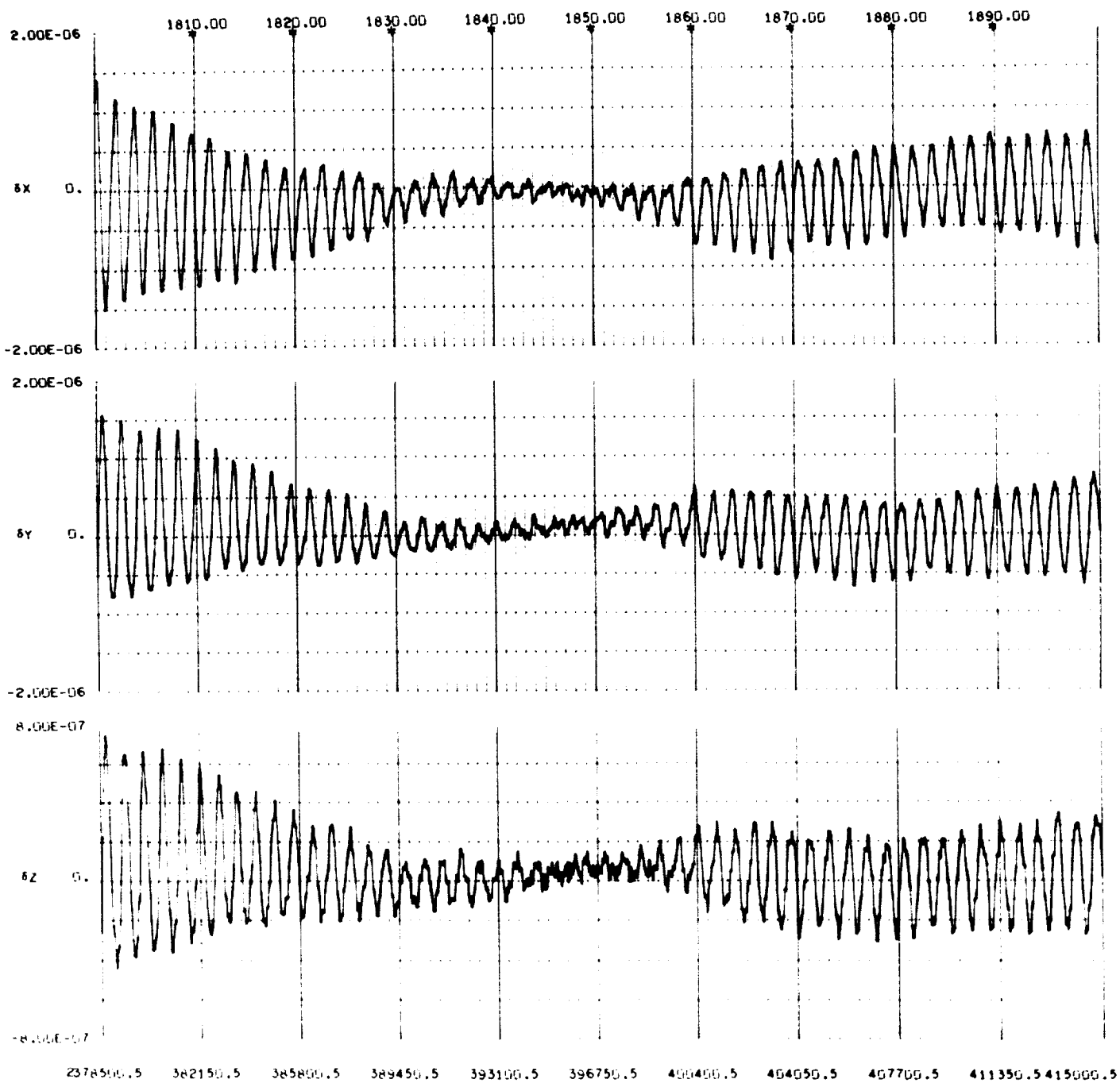


Fig. C-9. Mars rectangular residuals (modified source minus DE28) for 1800–2000

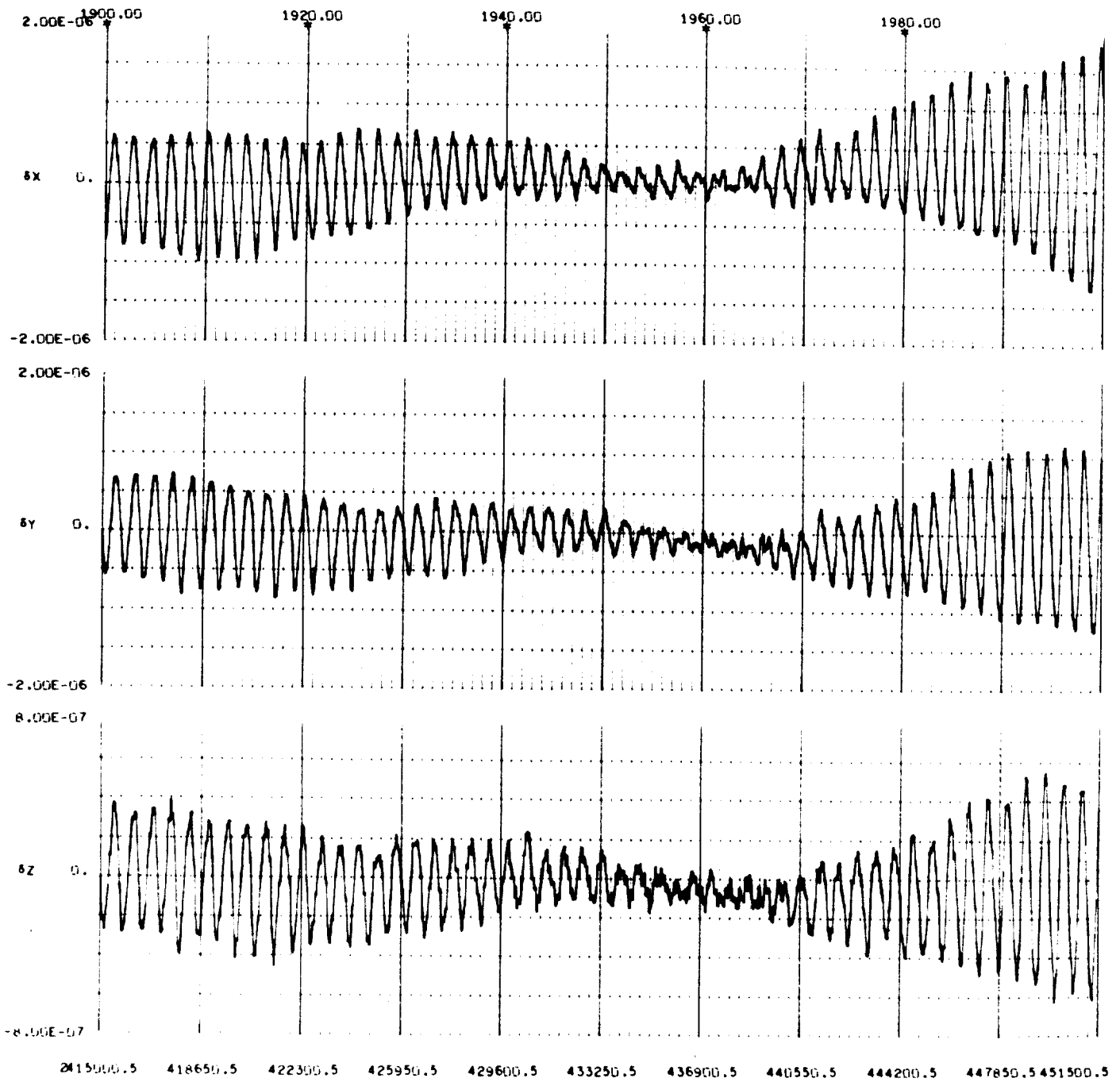


Fig. C-9 (contd)

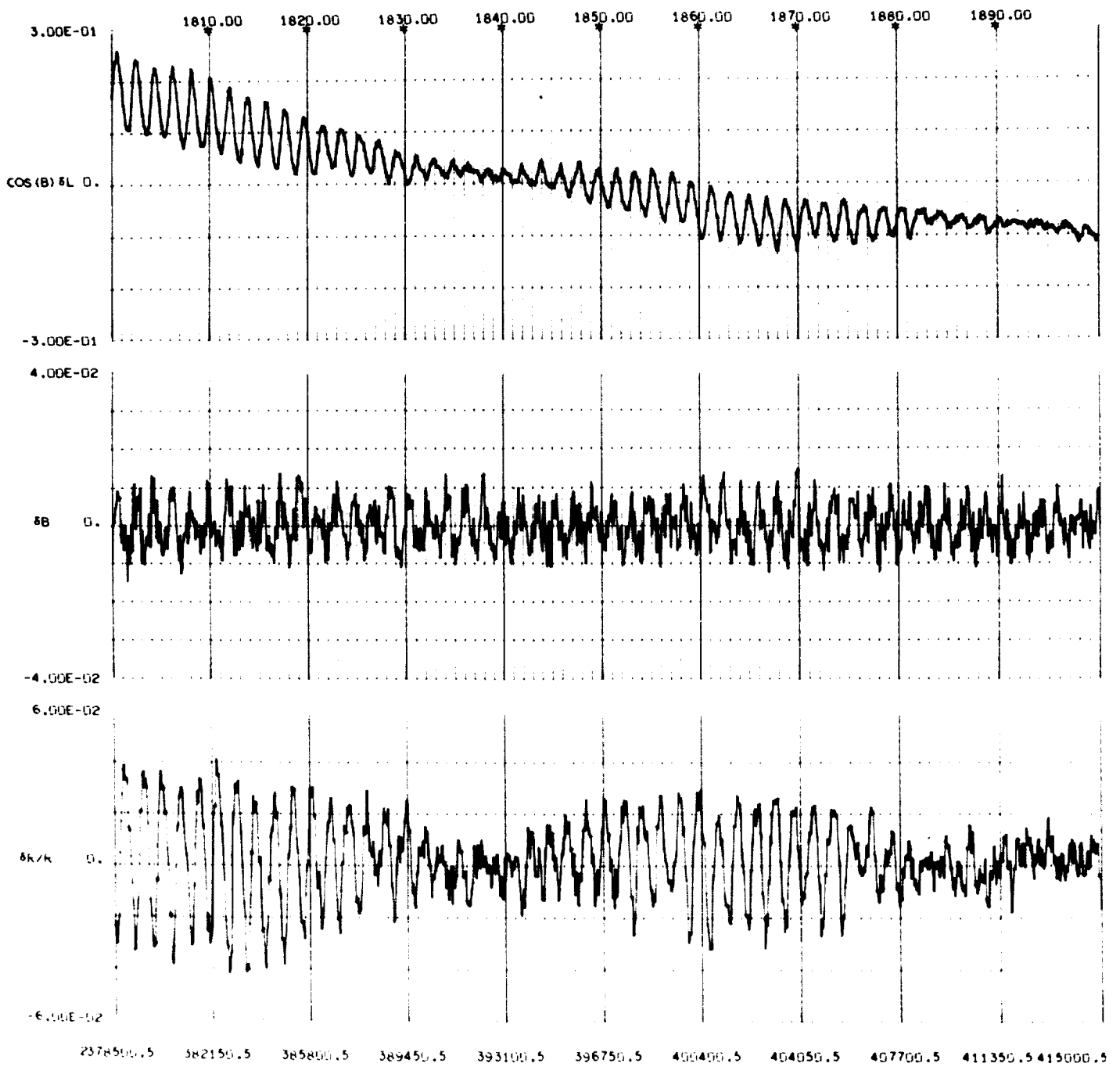


Fig. C-10. Mars angular residuals (modified source minus DE28) for 1800–2000

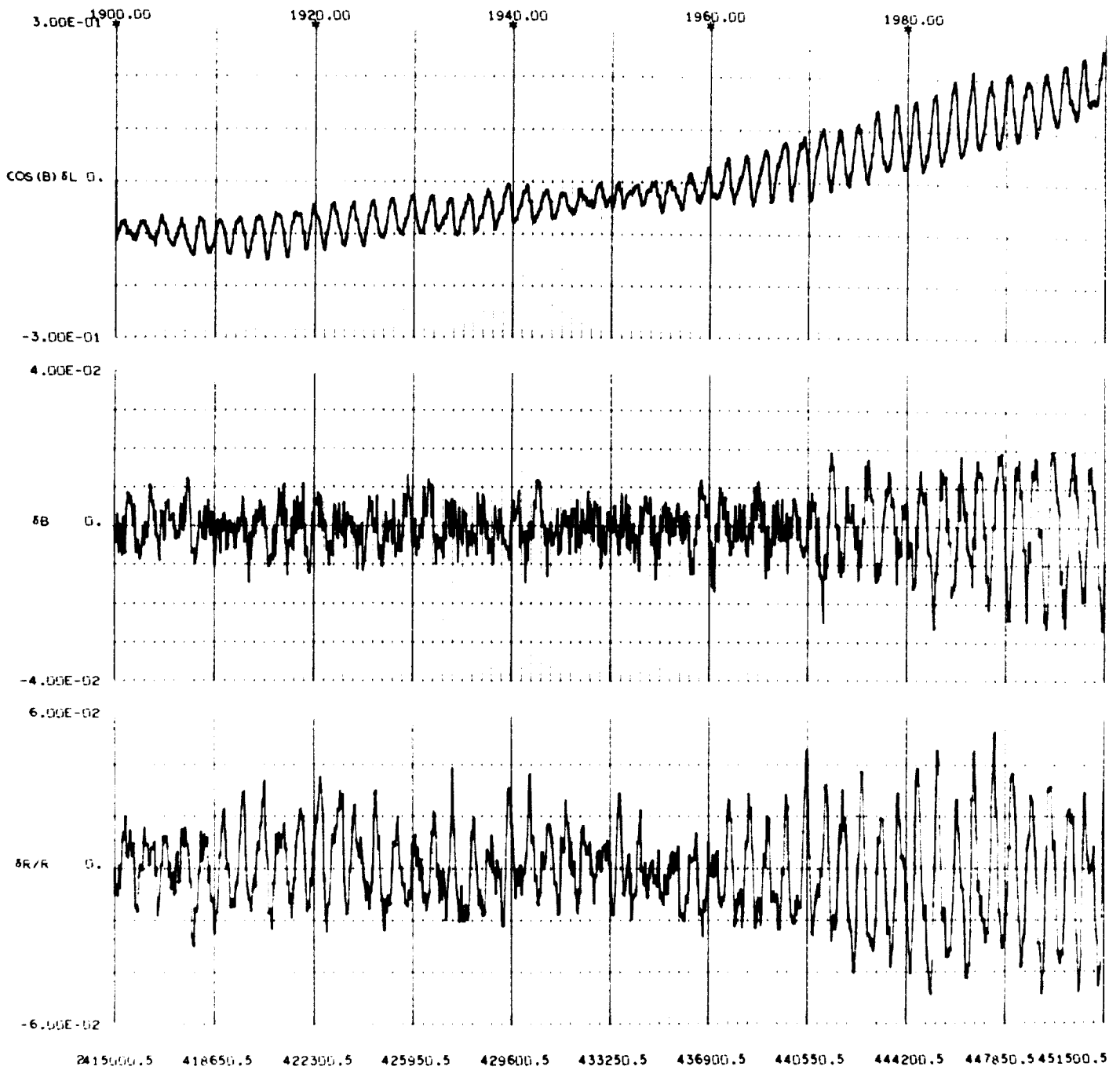


Fig. C-10 (contd)

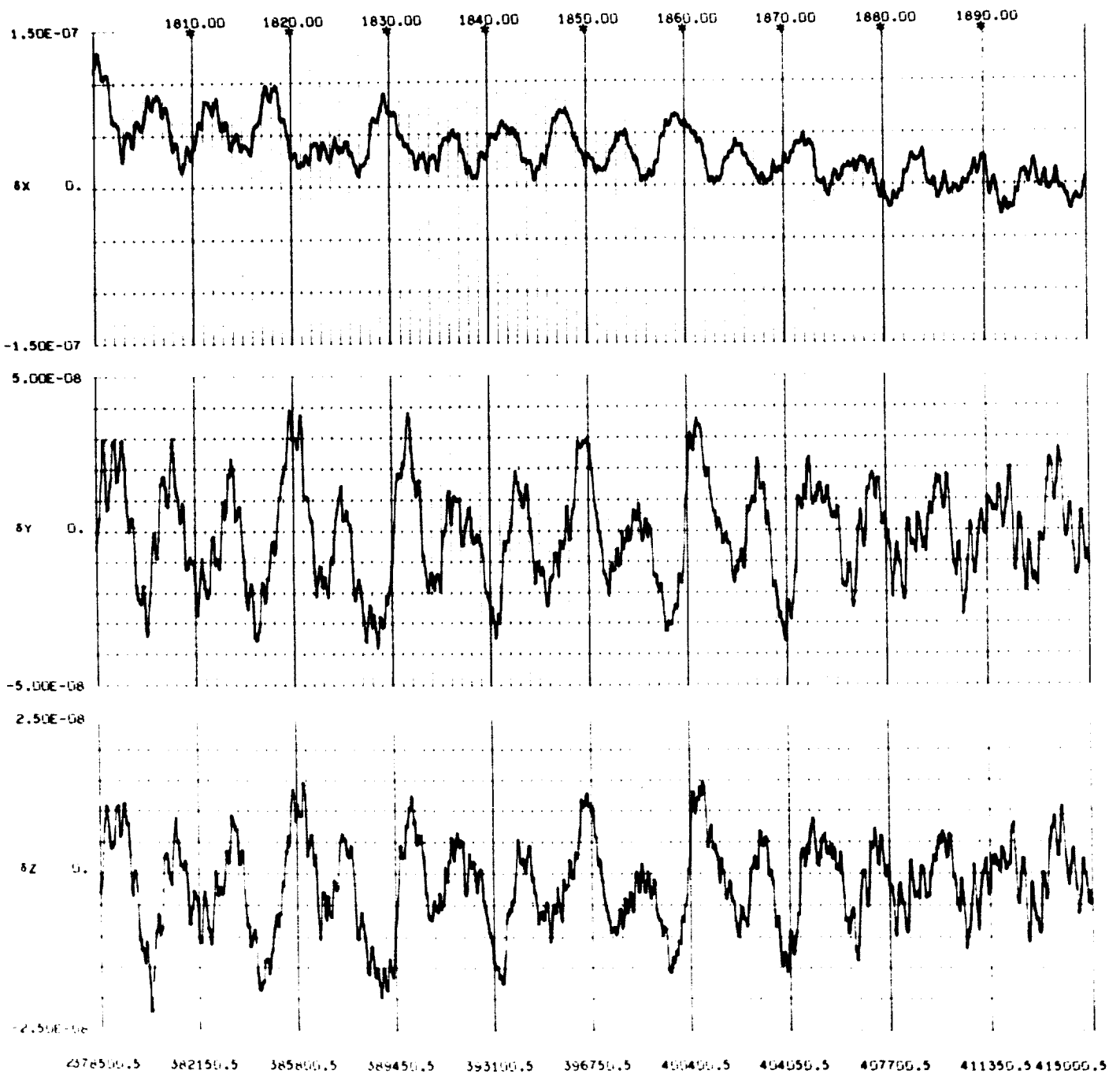


Fig. C-11. Jupiter rectangular residuals (modified source minus DE28) for 1800–2000



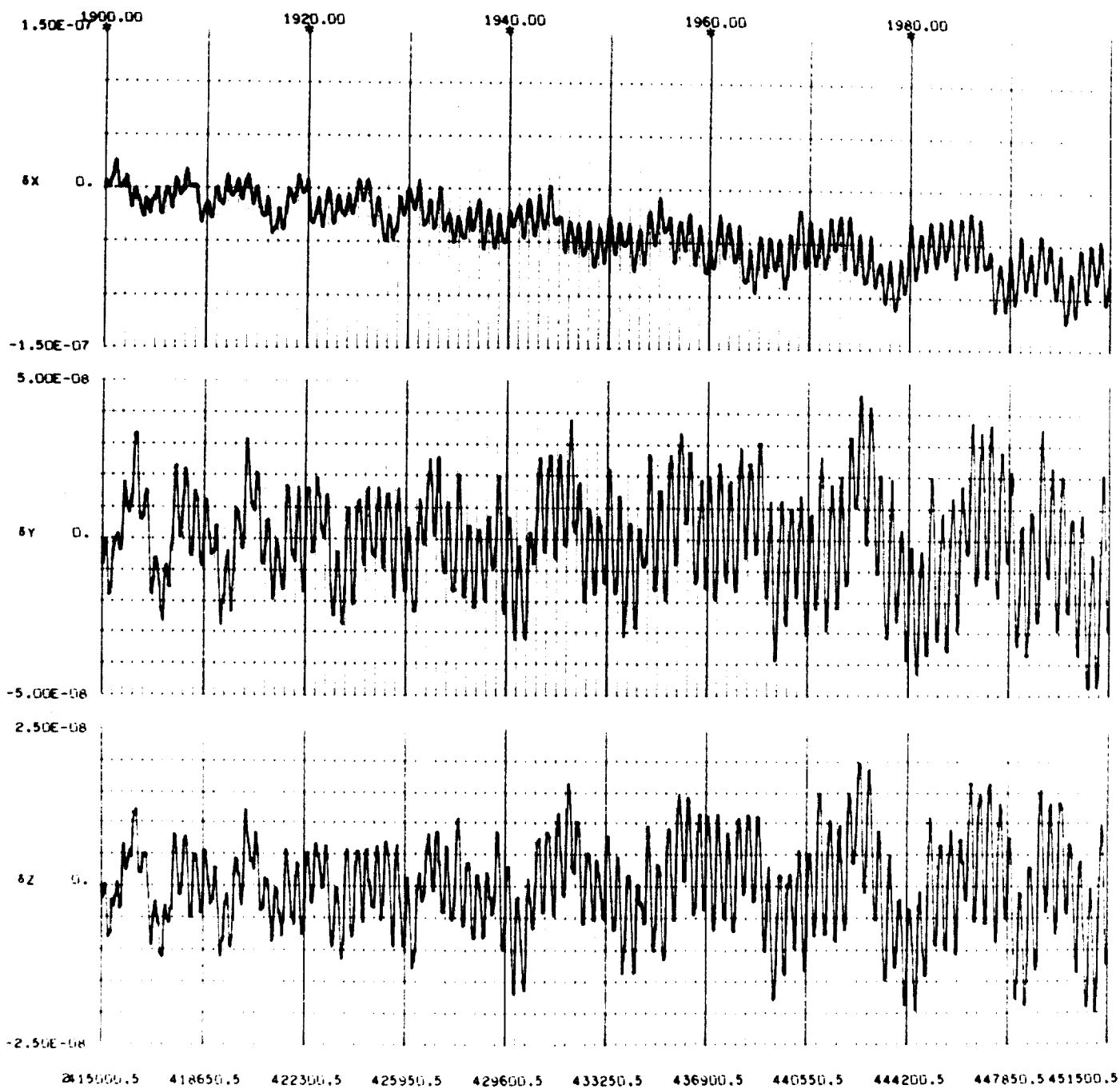


Fig. C-11 (contd)

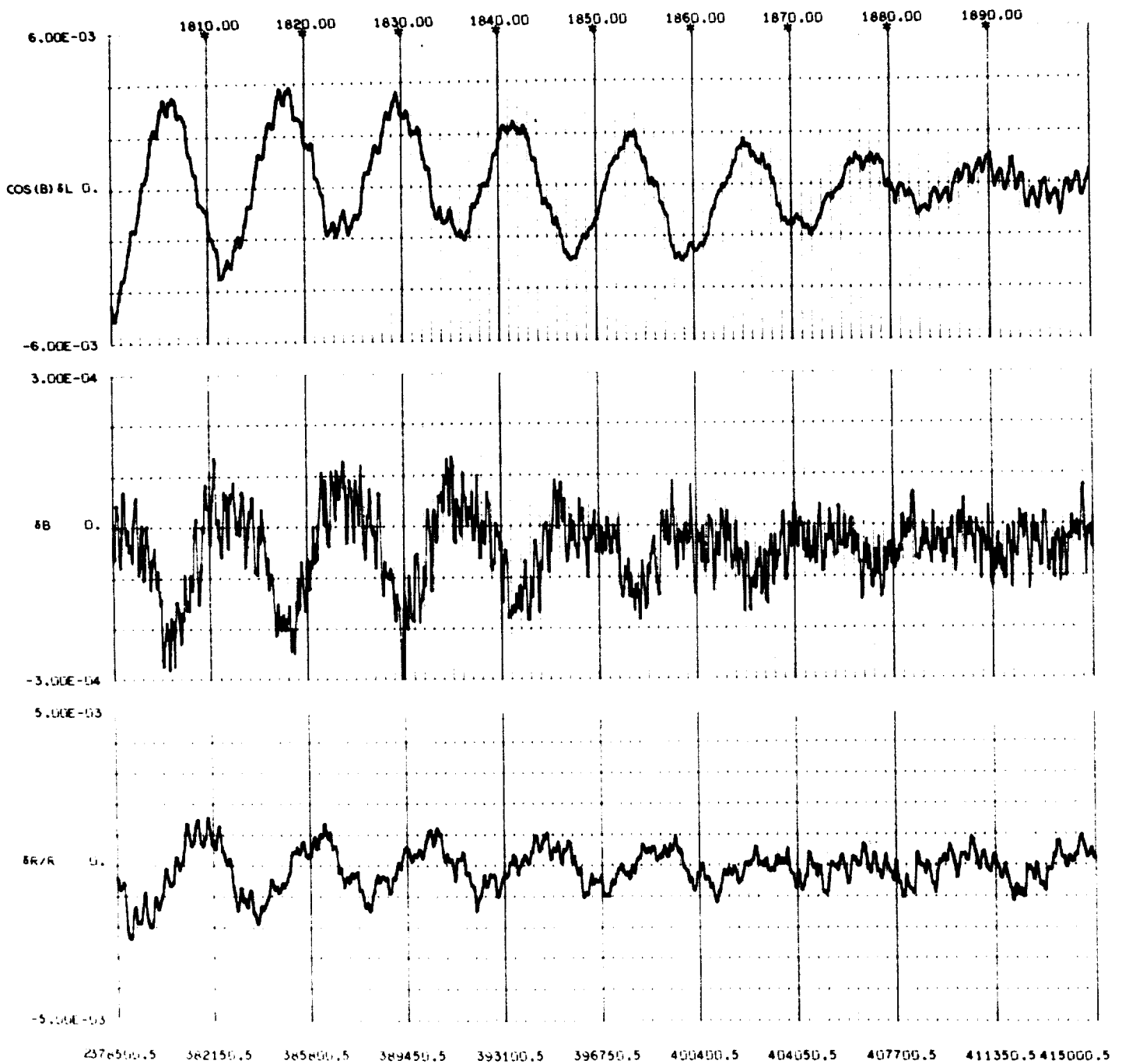


Fig. C-12. Jupiter angular residuals (modified source minus DE28) for 1800–2000

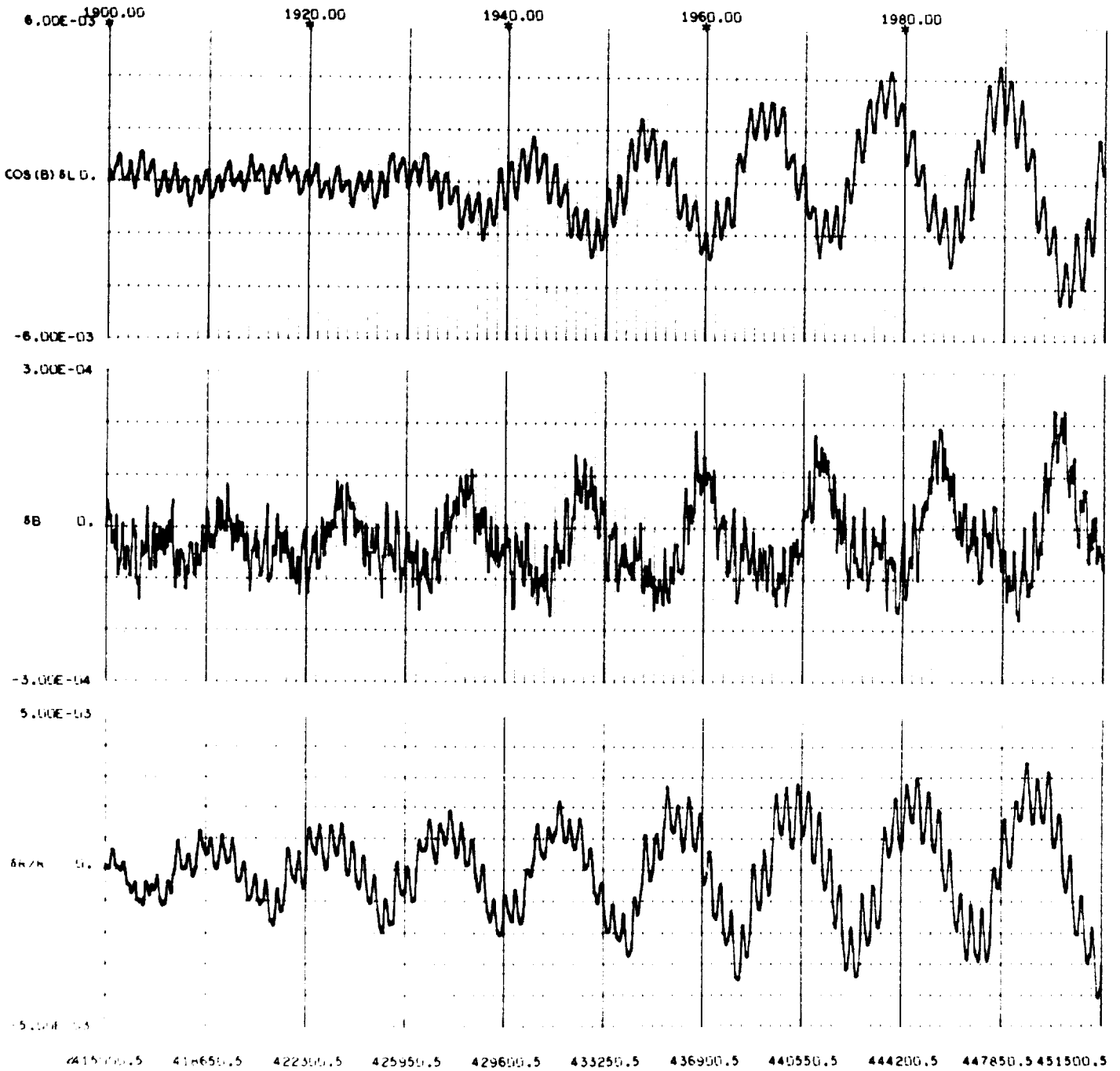


Fig. C-12 (contd)

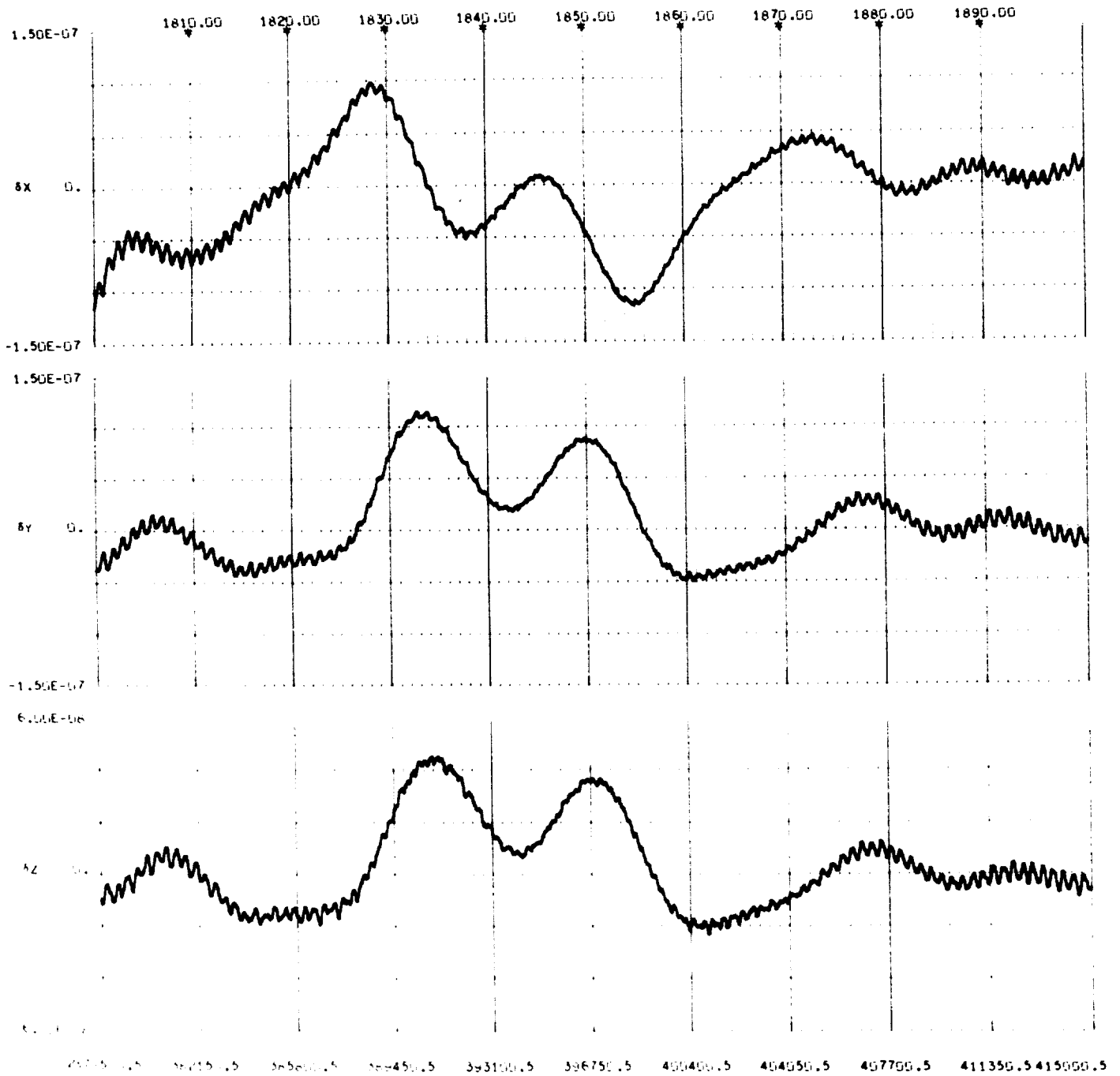


Fig. C-13. Saturn rectangular residuals (modified source minus DE28) for 1800-2000

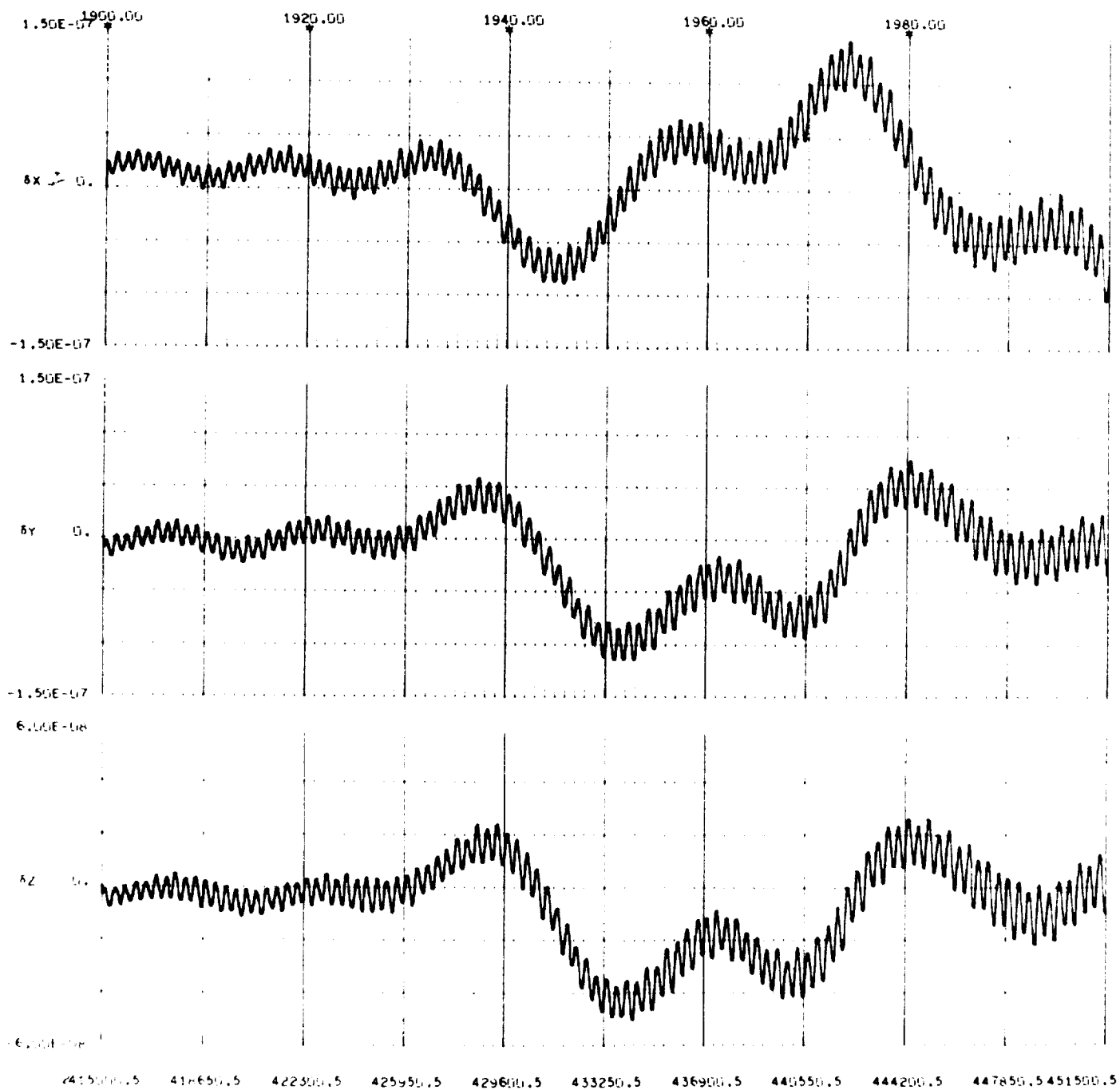


Fig. C-13 (contd)

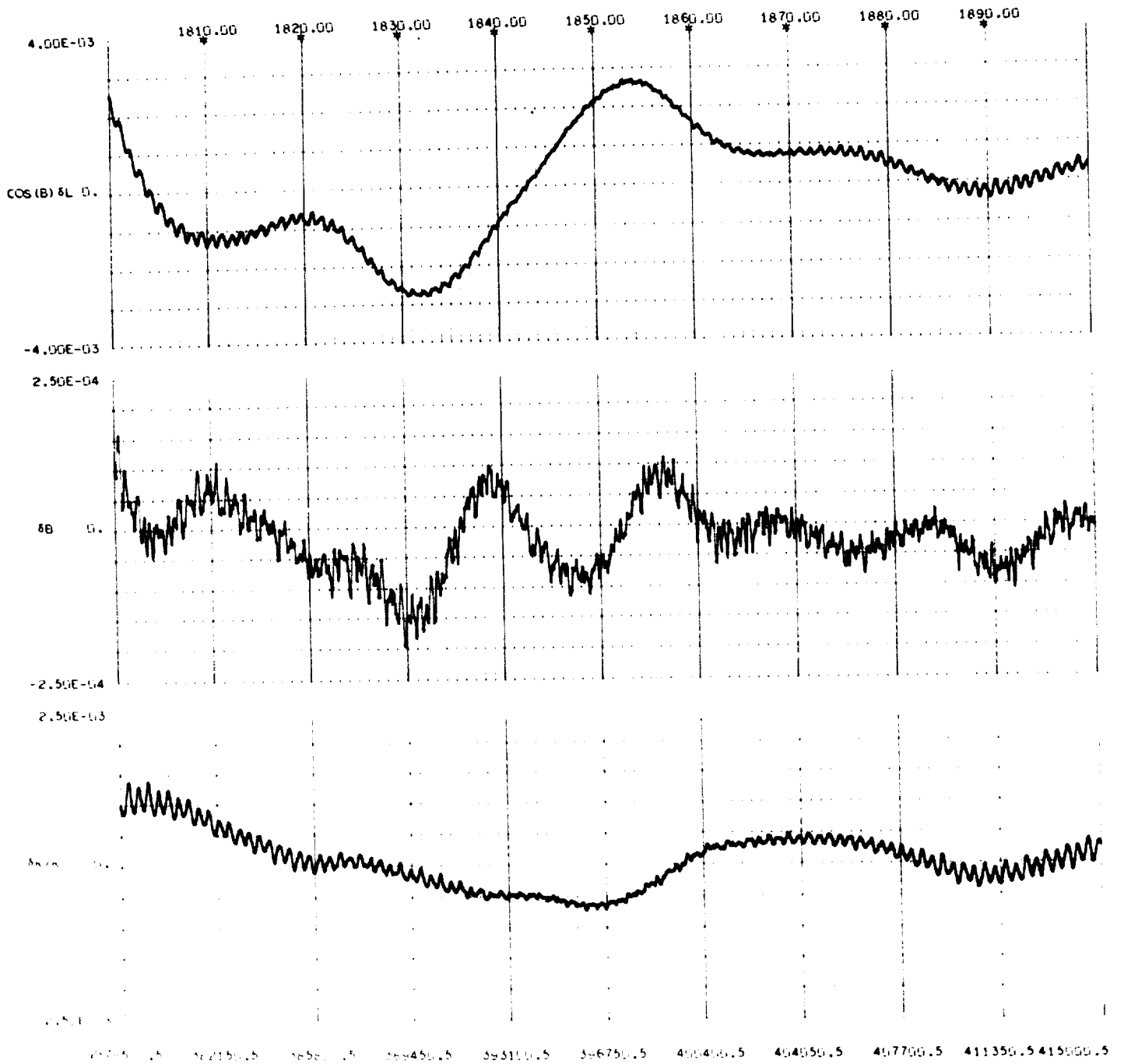


Fig. C-14. Saturn angular residuals (modified source minus DE28) for 1800-2000

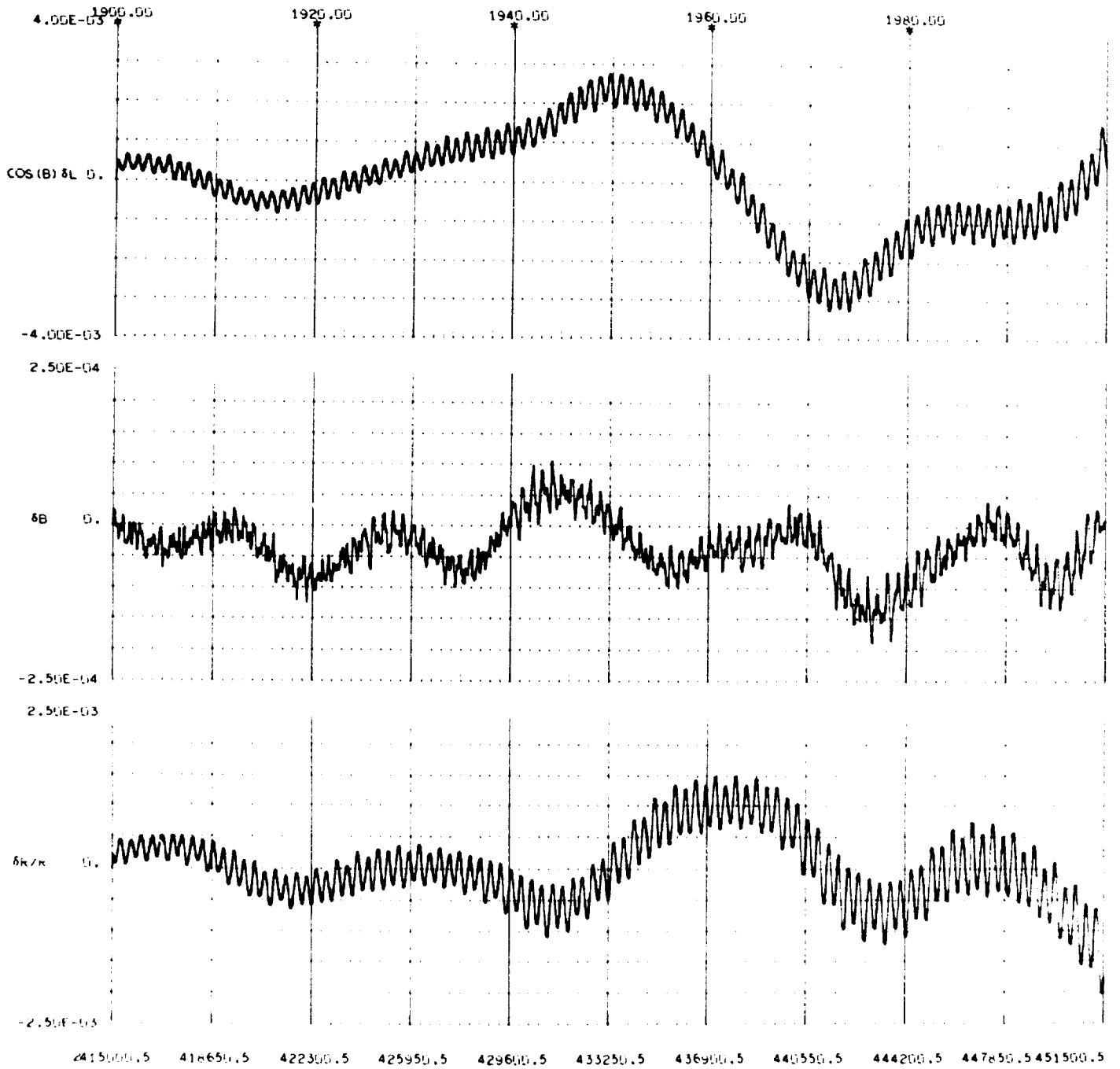


Fig. C-14 (contd)

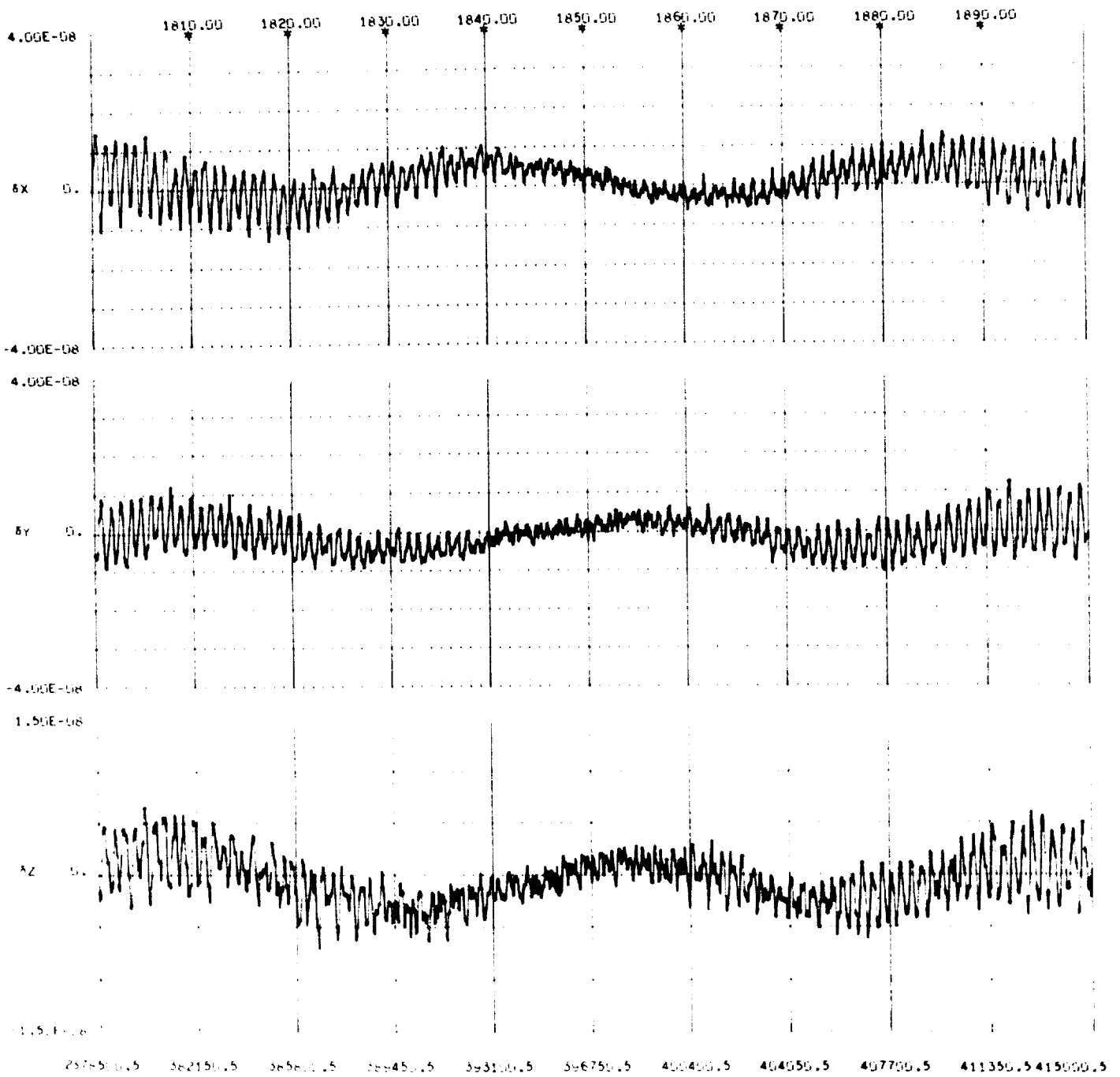


Fig. C-15. Uranus rectangular residuals (modified source minus DE28) for 1800–2000



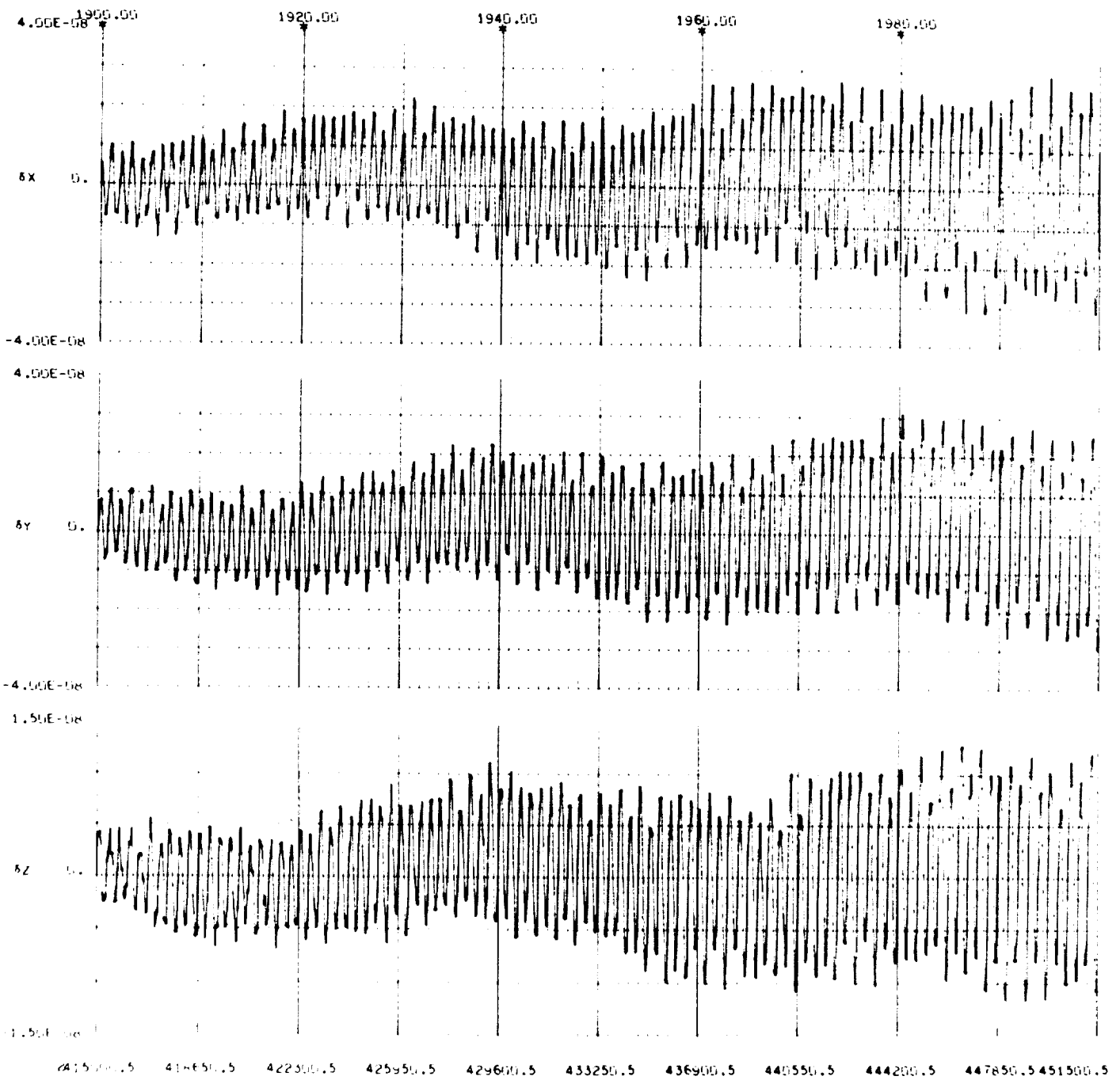


Fig. C-15 (contd)

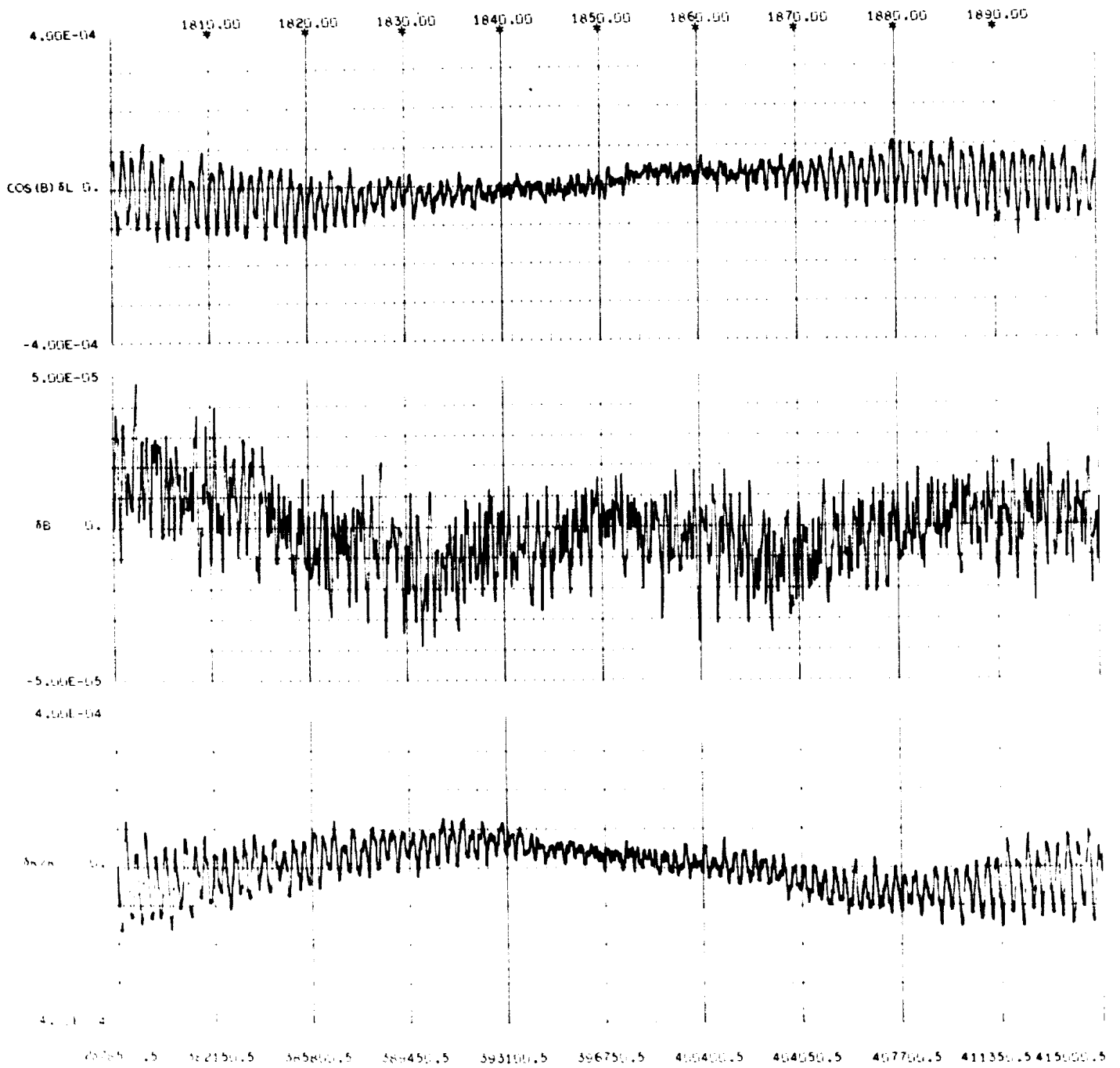


Fig. C-16. Uranus angular residuals (modified source minus DE28) for 1800–2000

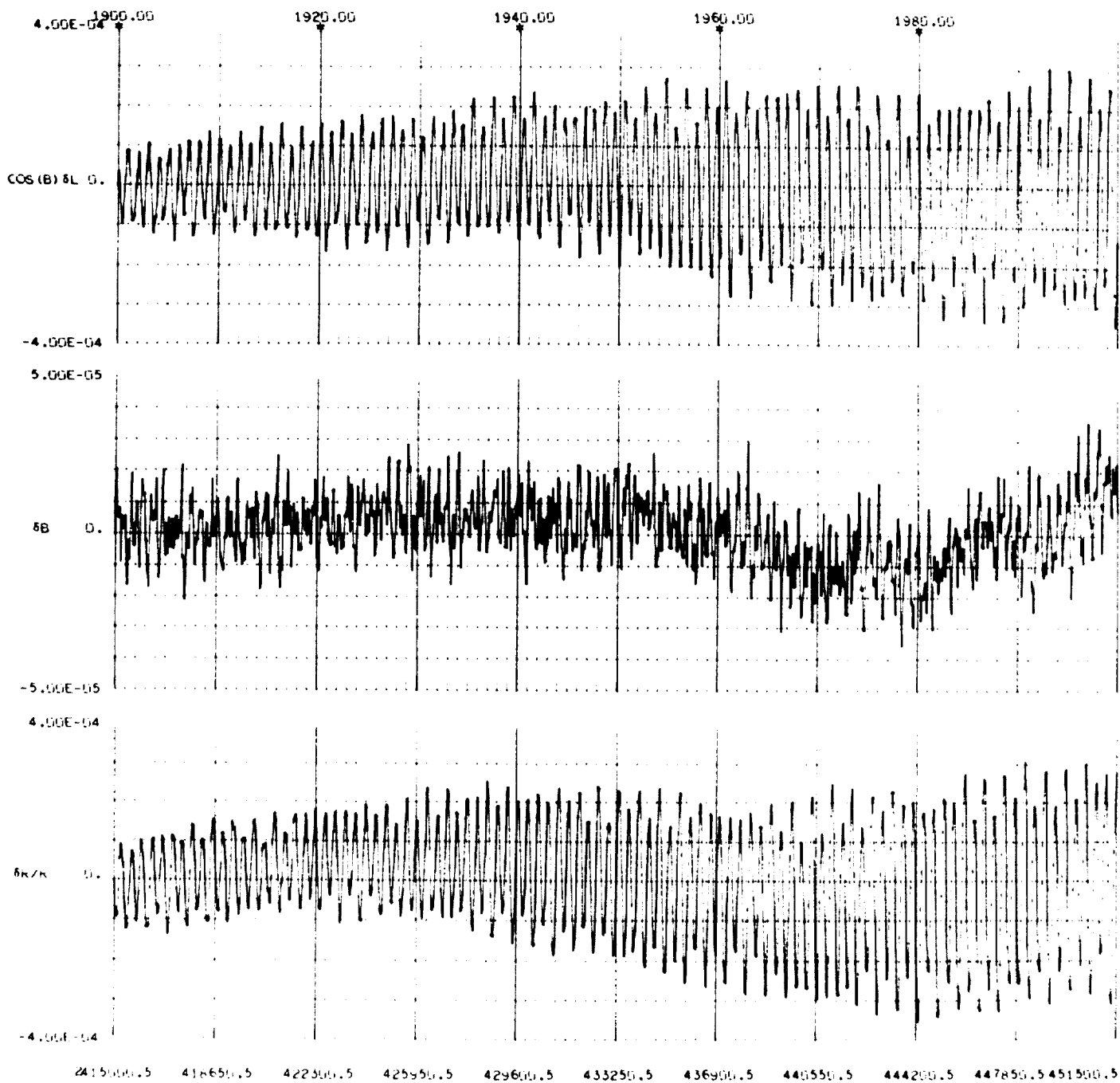


Fig. C-16 (contd)

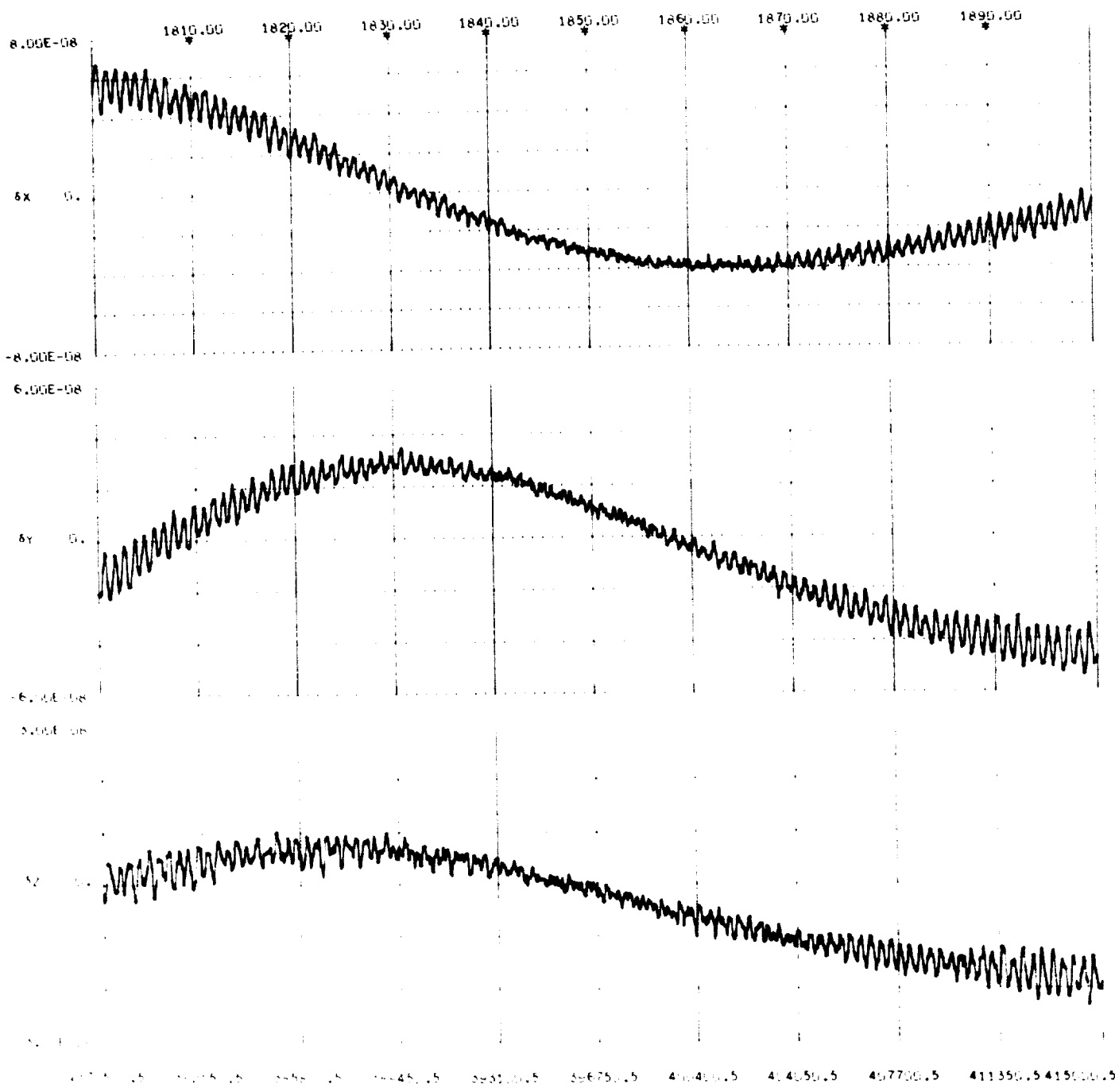


Fig. C-17. Neptune rectangular residuals (modified source minus DE28) for 1800-2000

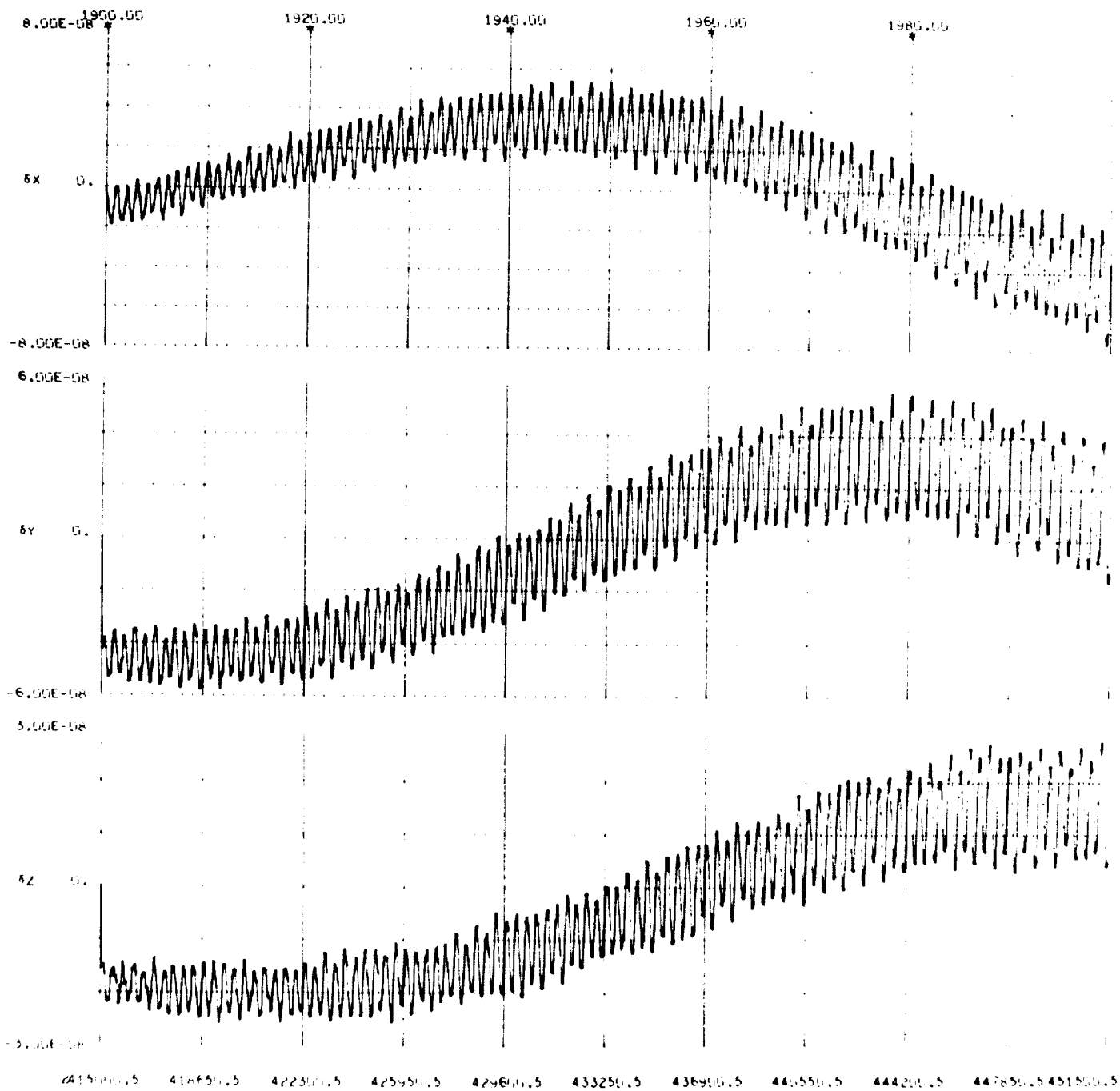


Fig. C-17 (contd)

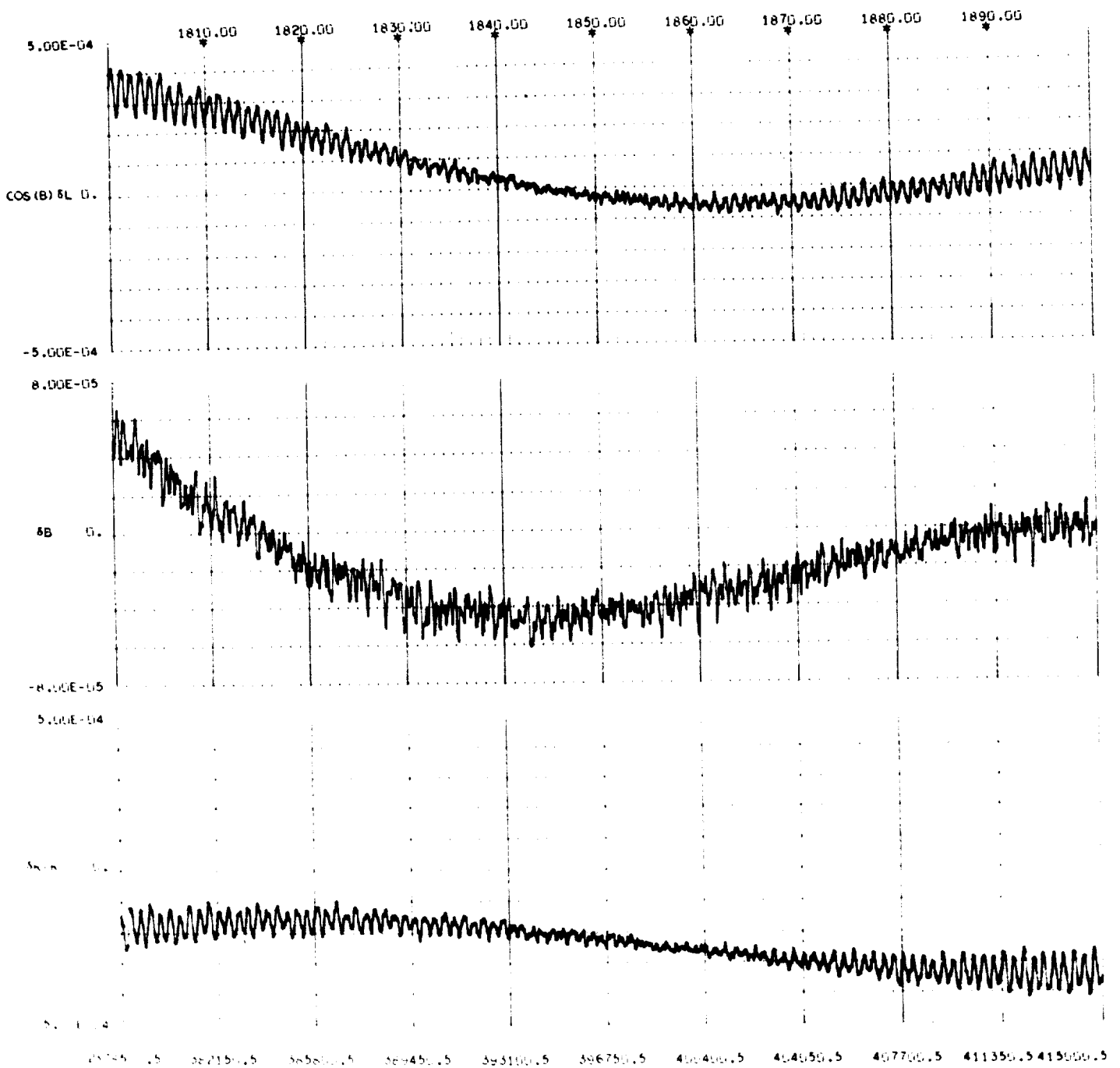


Fig. C-18. Neptune angular residuals (modified source minus DE28) for 1800-2000

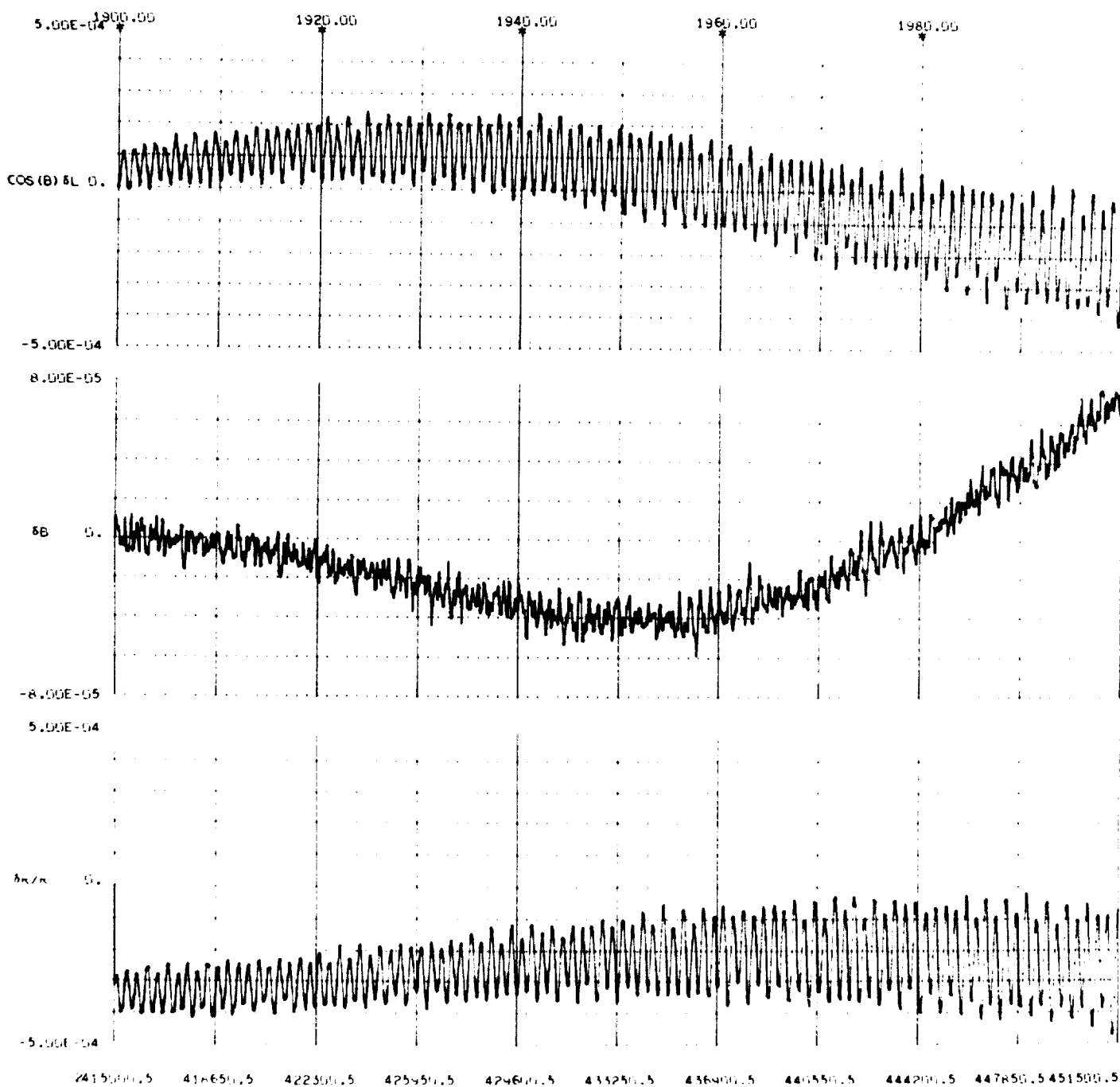


Fig. C-18 (contd)

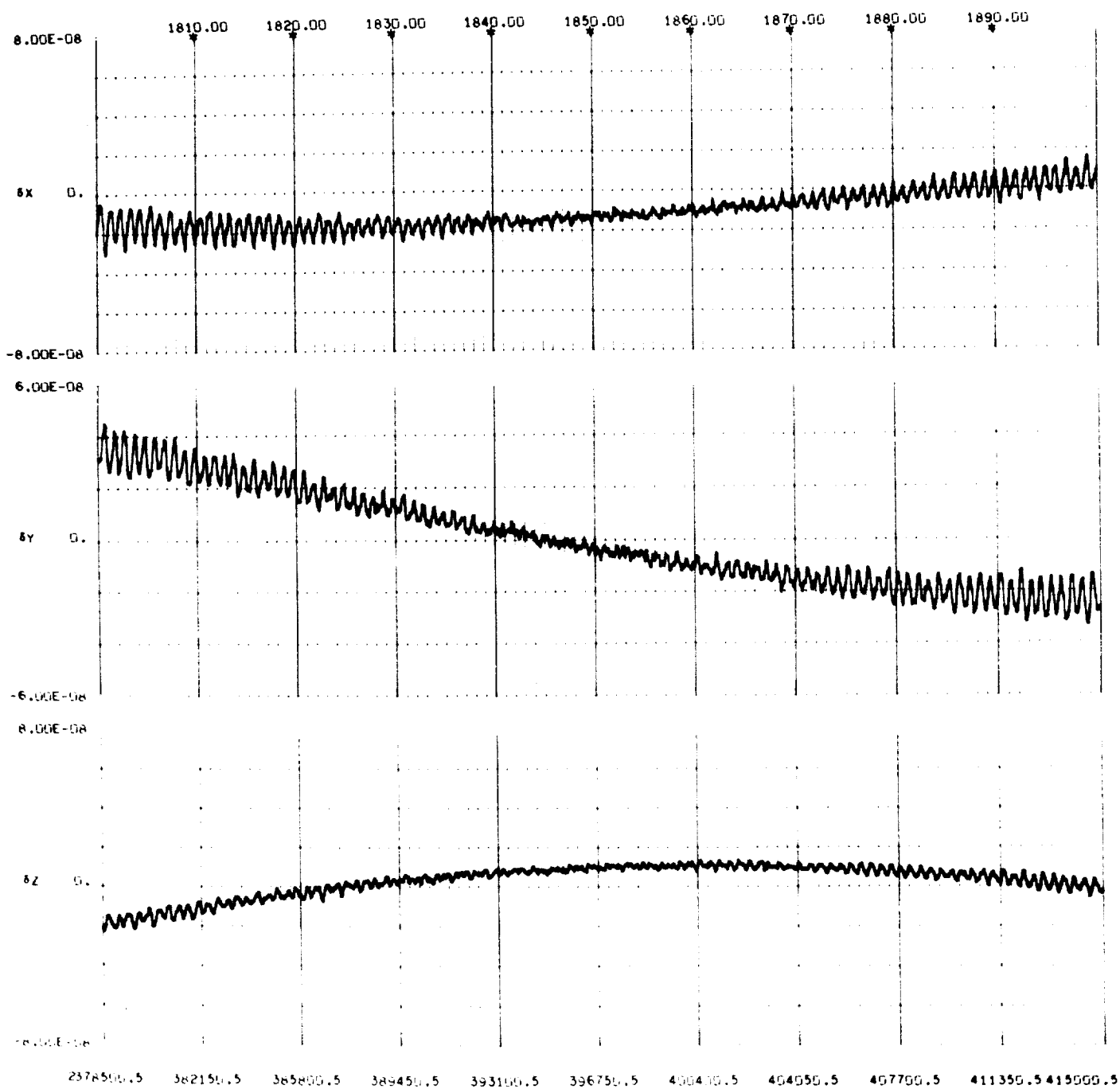


Fig. C-19. Pluto rectangular residuals (modified source minus DE28) for 1800–2000



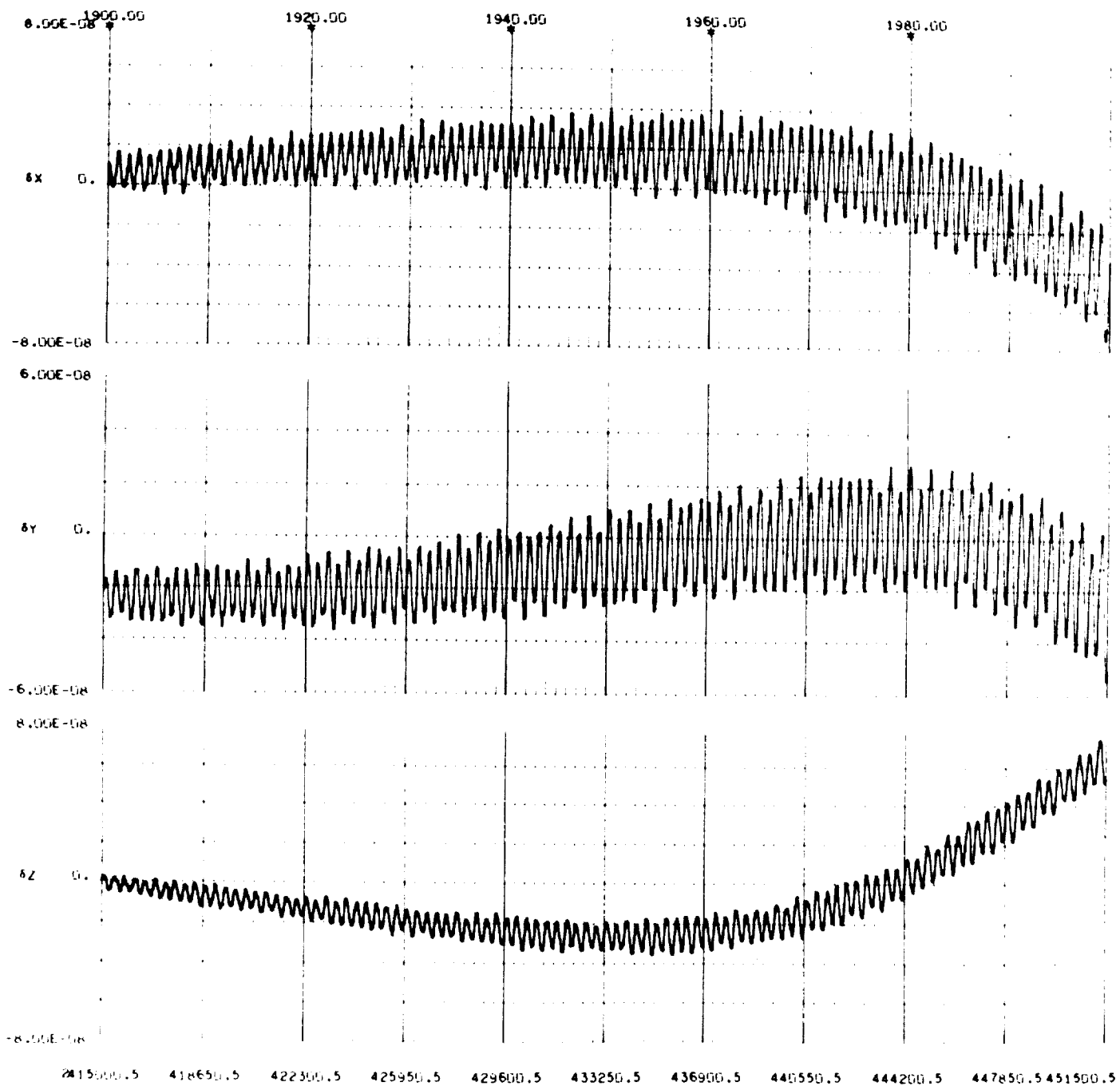


Fig. C-19 (contd)

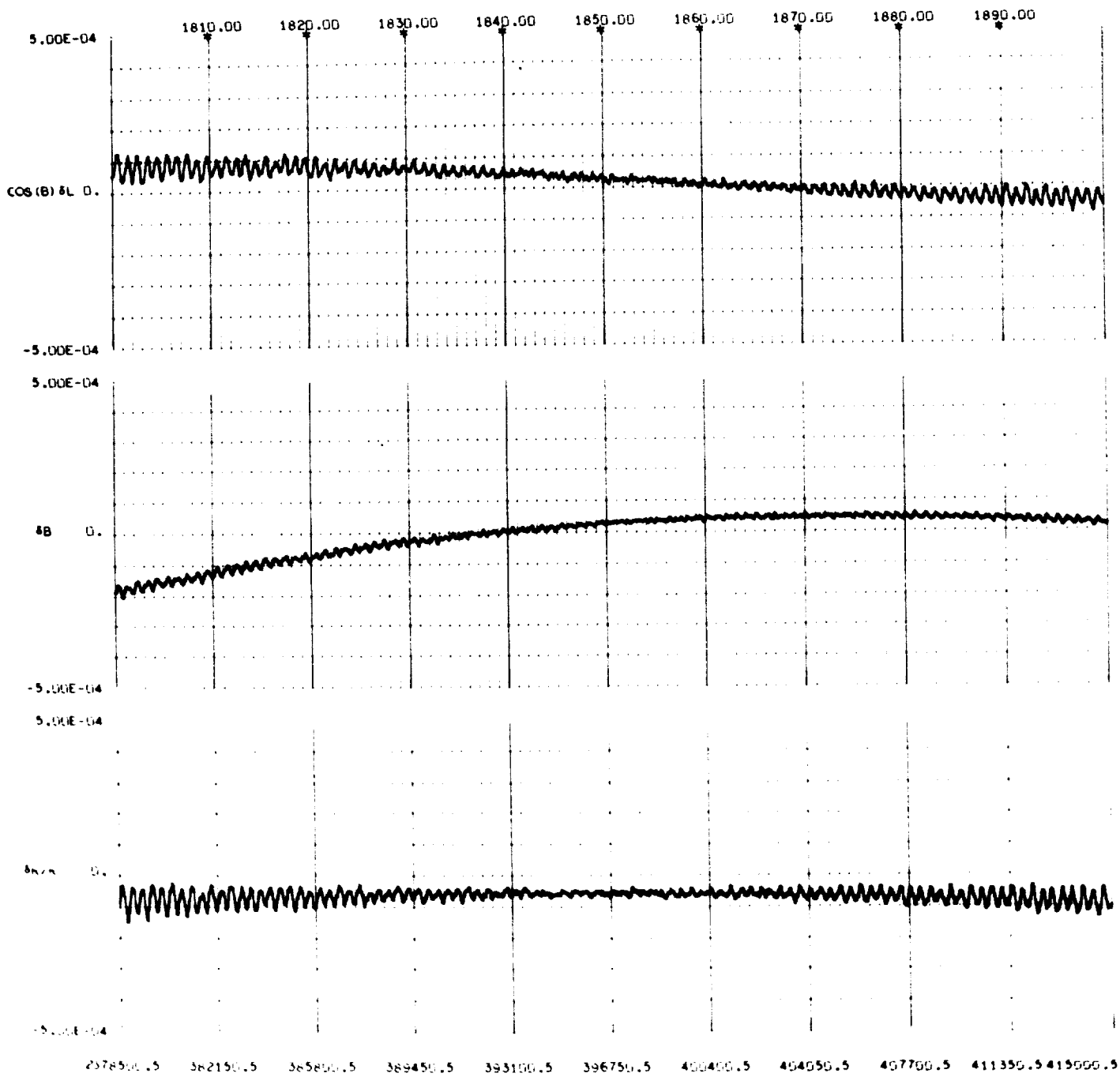


Fig. C-20. Pluto angular residuals (modified source minus DE28) for 1800–2000

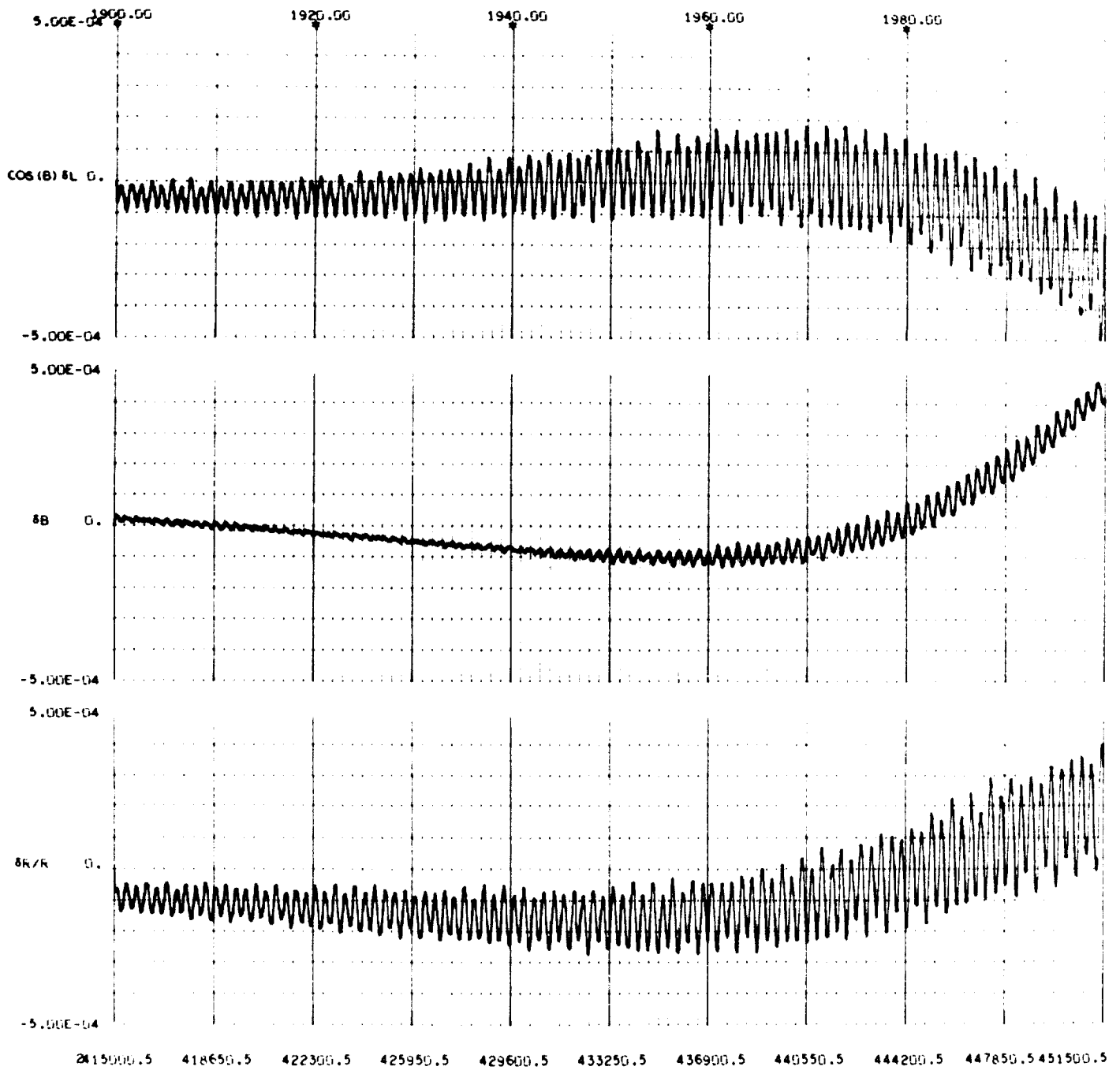


Fig. C-20 (contd)

# Appendix D

## Secular Deficiencies of Modified Source Ephemerides

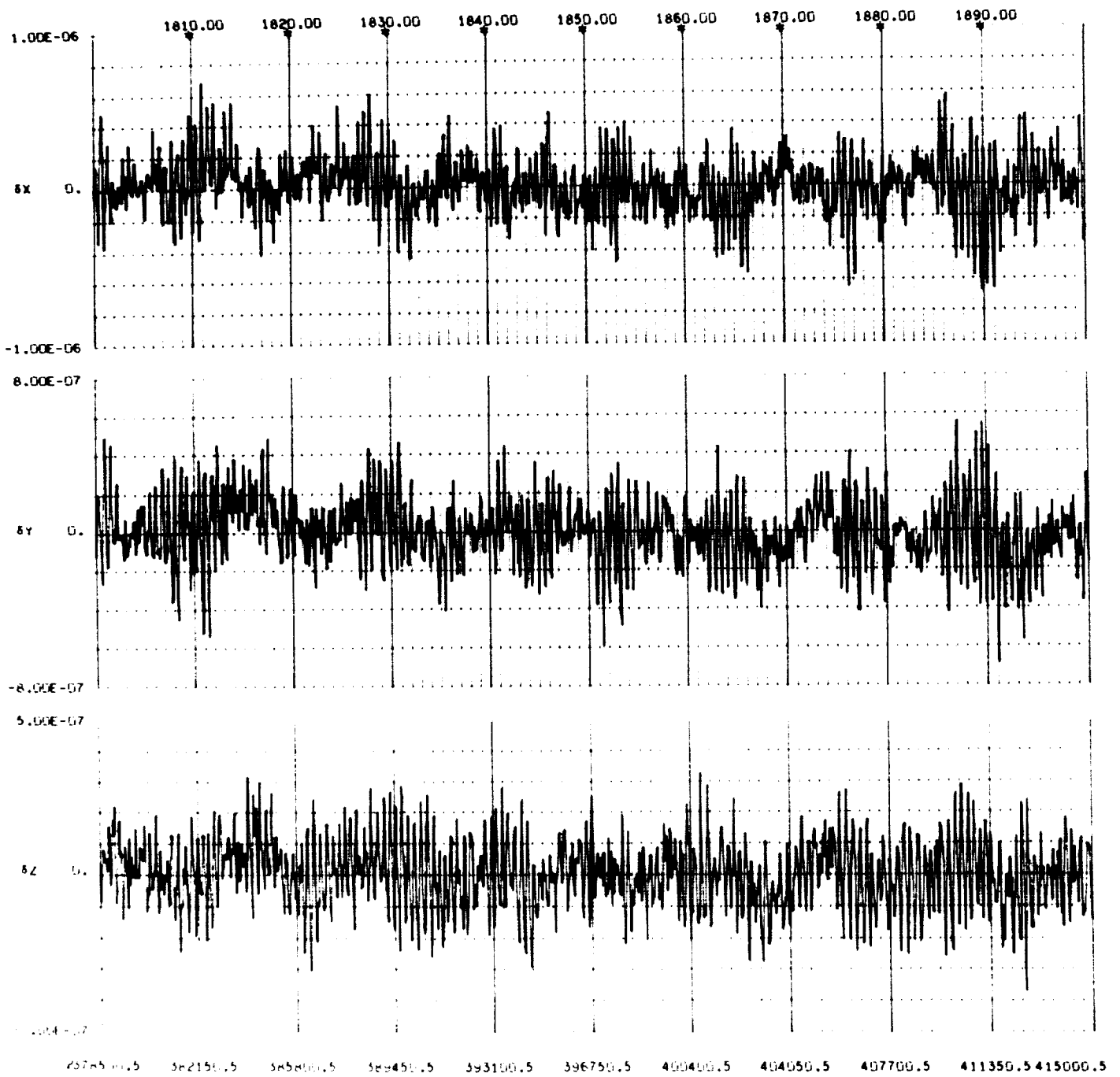


Fig. D-1. Venus rectangular residuals (improved modified source minus DE28) for 1800–2000

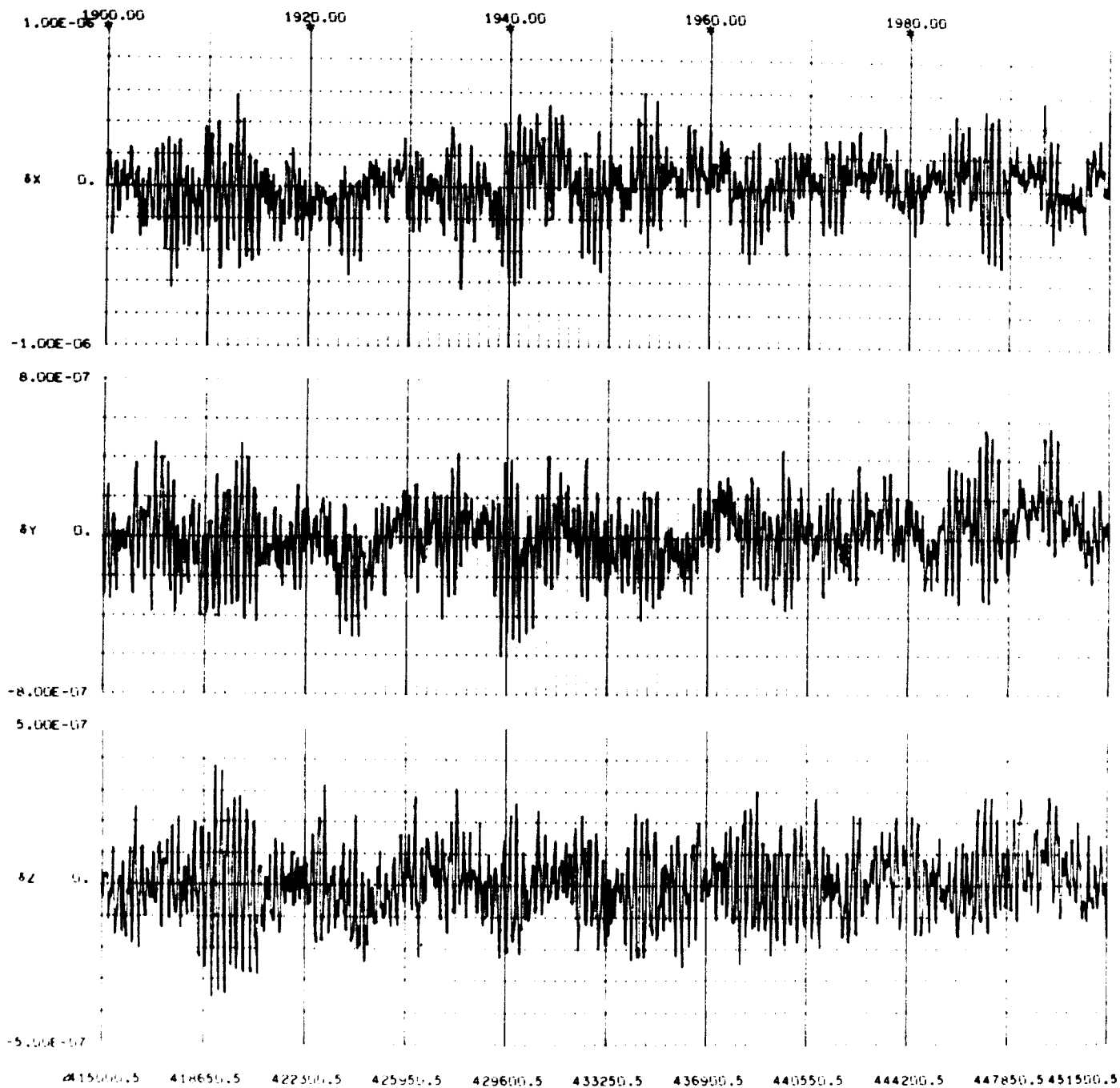


Fig. D-1 (contd)

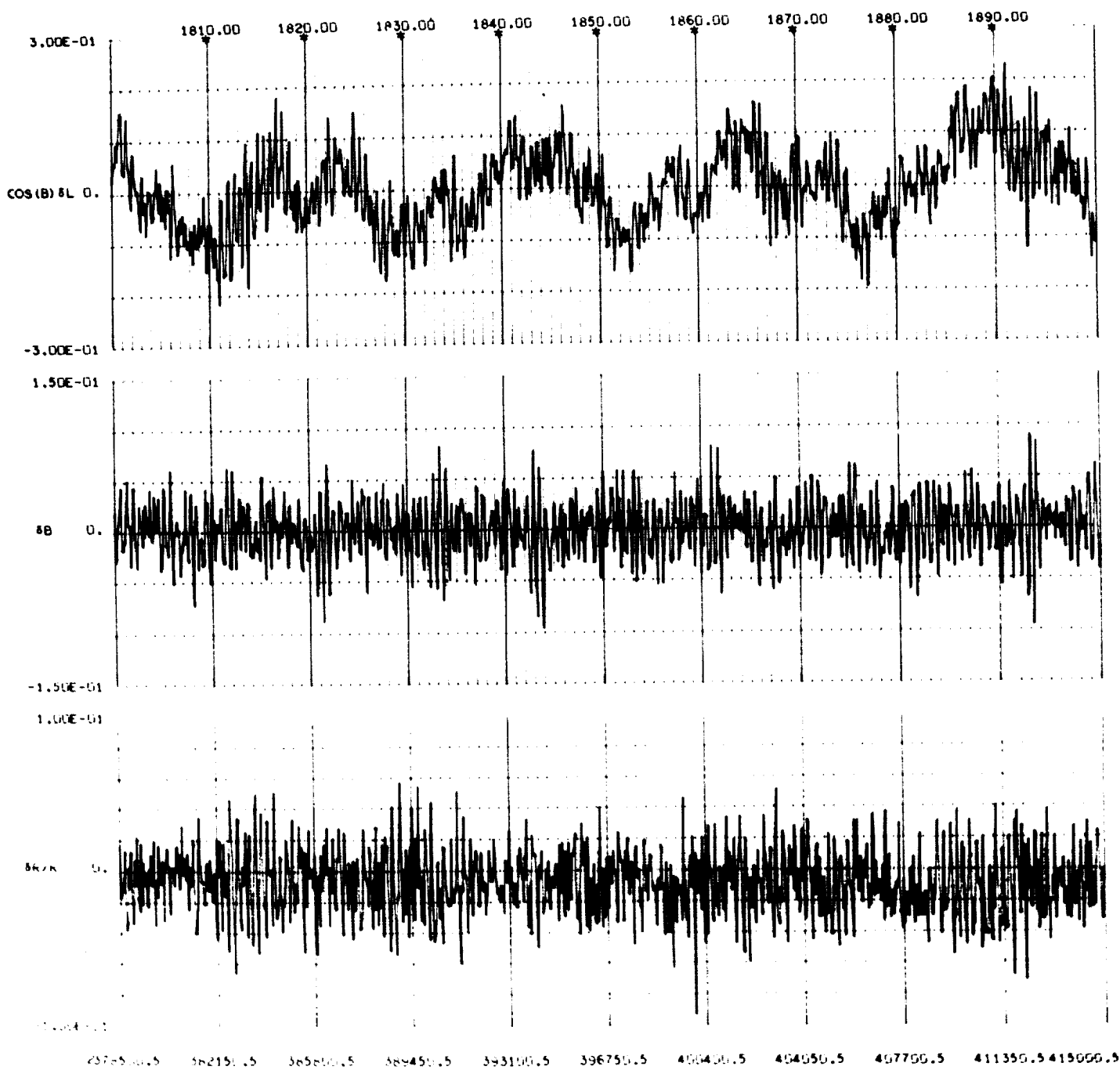


Fig. D-2. Venus angular residuals (improved modified source minus DE28) for 1800-2000

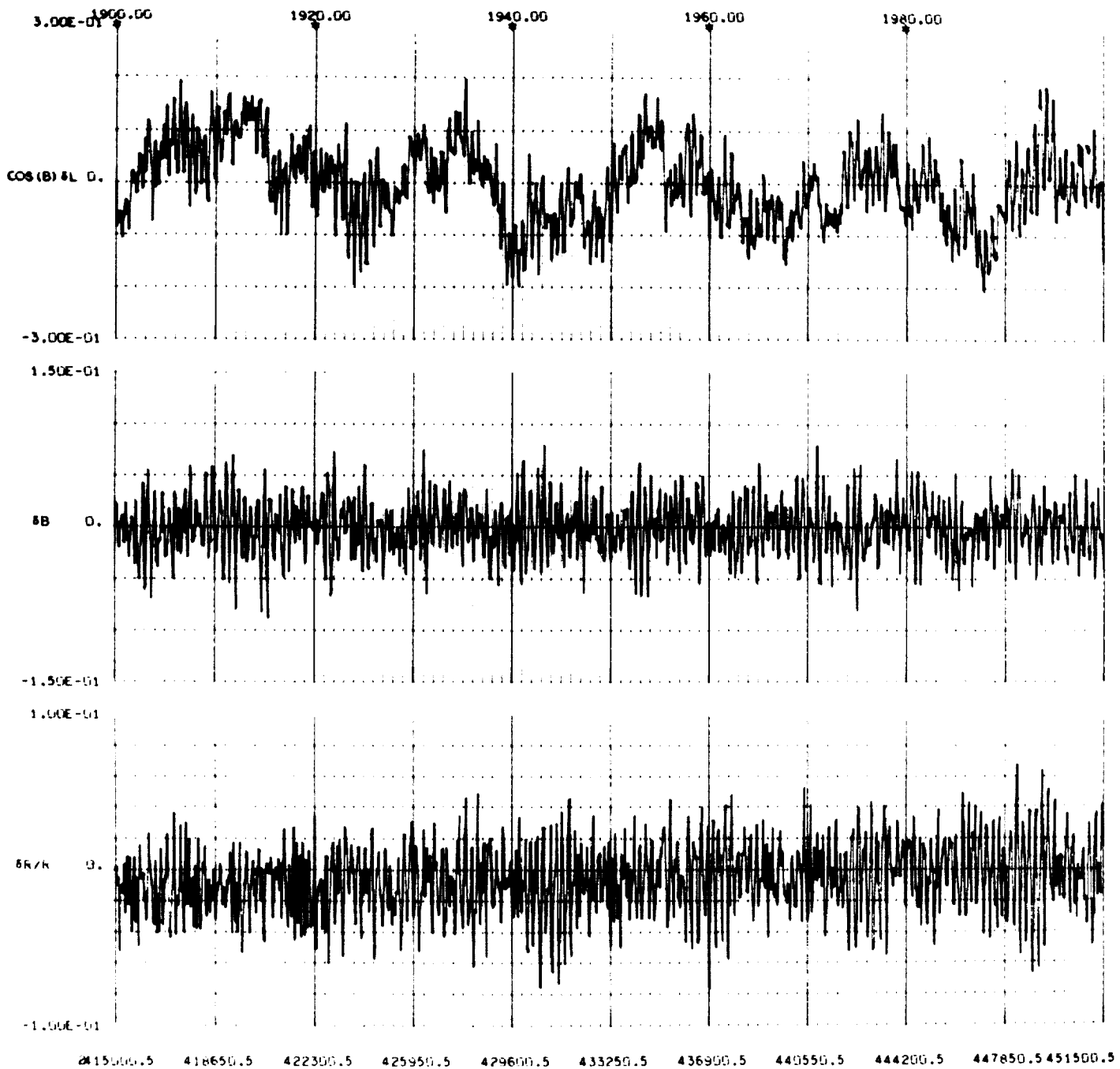


Fig. D-2 (contd)

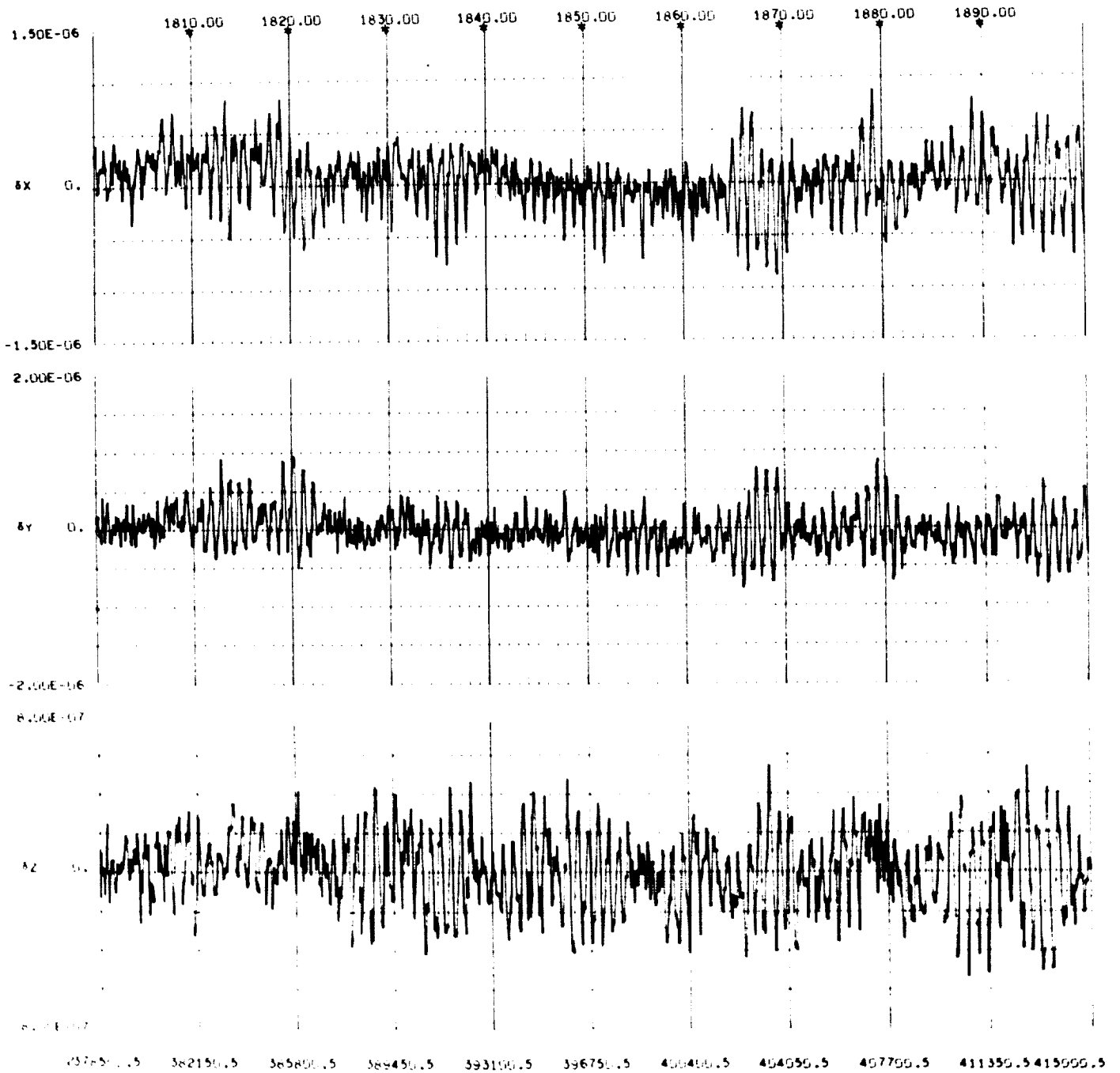


Fig. D-3. Earth-moon barycenter rectangular residuals (improved modified source minus DE28) for 1800–2000



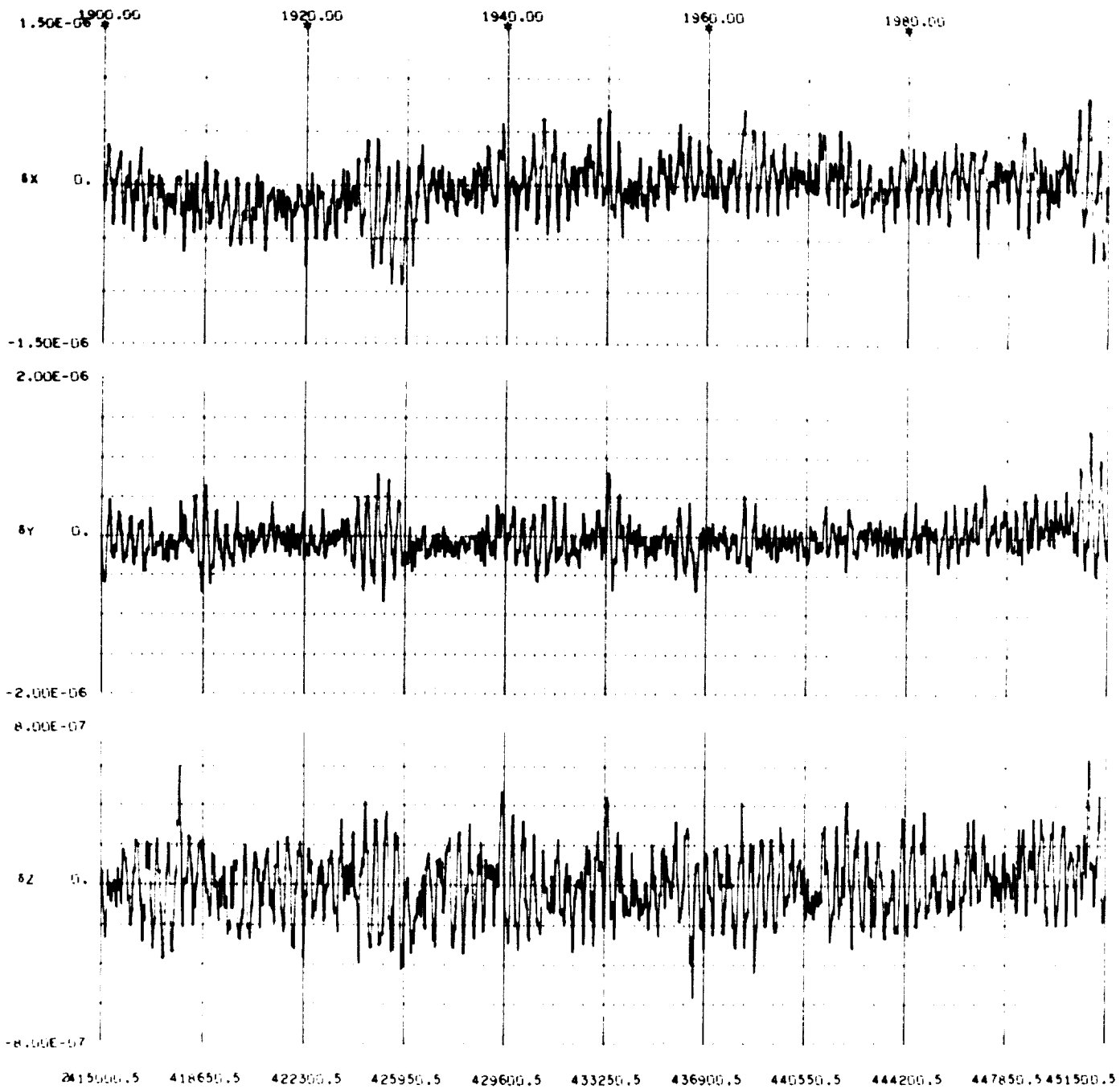
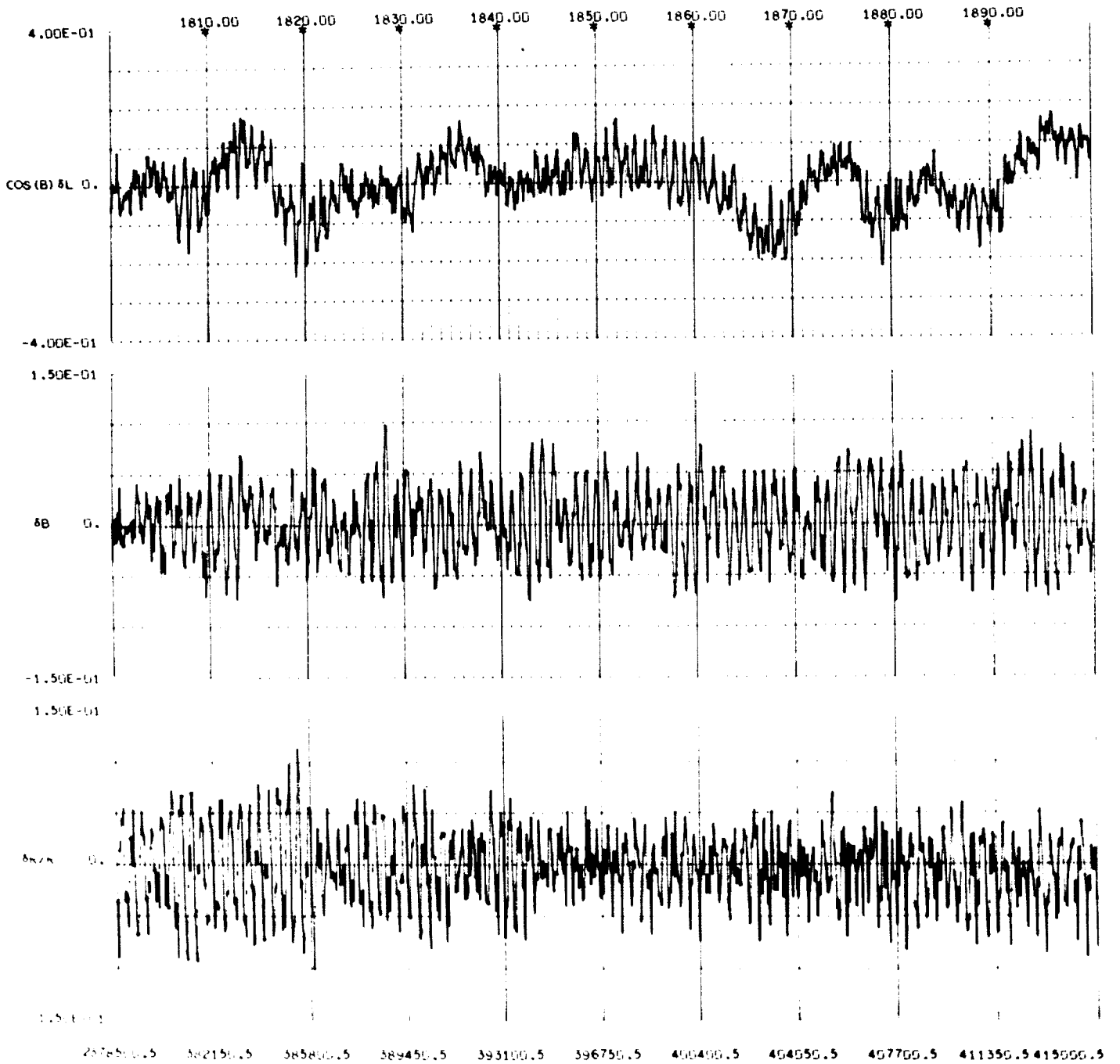


Fig. D-3 (contd)



**Fig. D-4. Earth-moon barycenter angular residuals (improved modified source minus DE28) for 1800–2000**

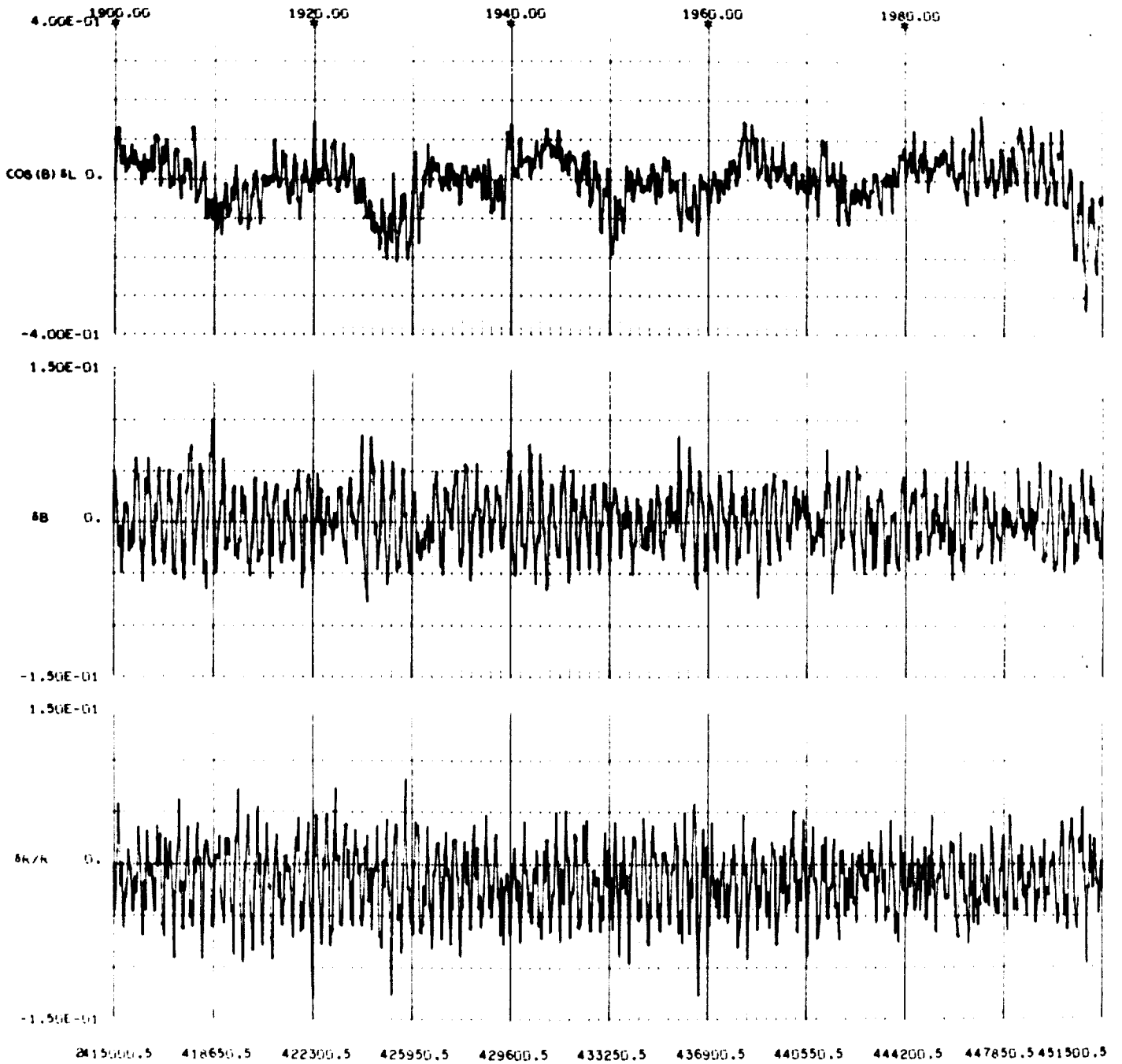
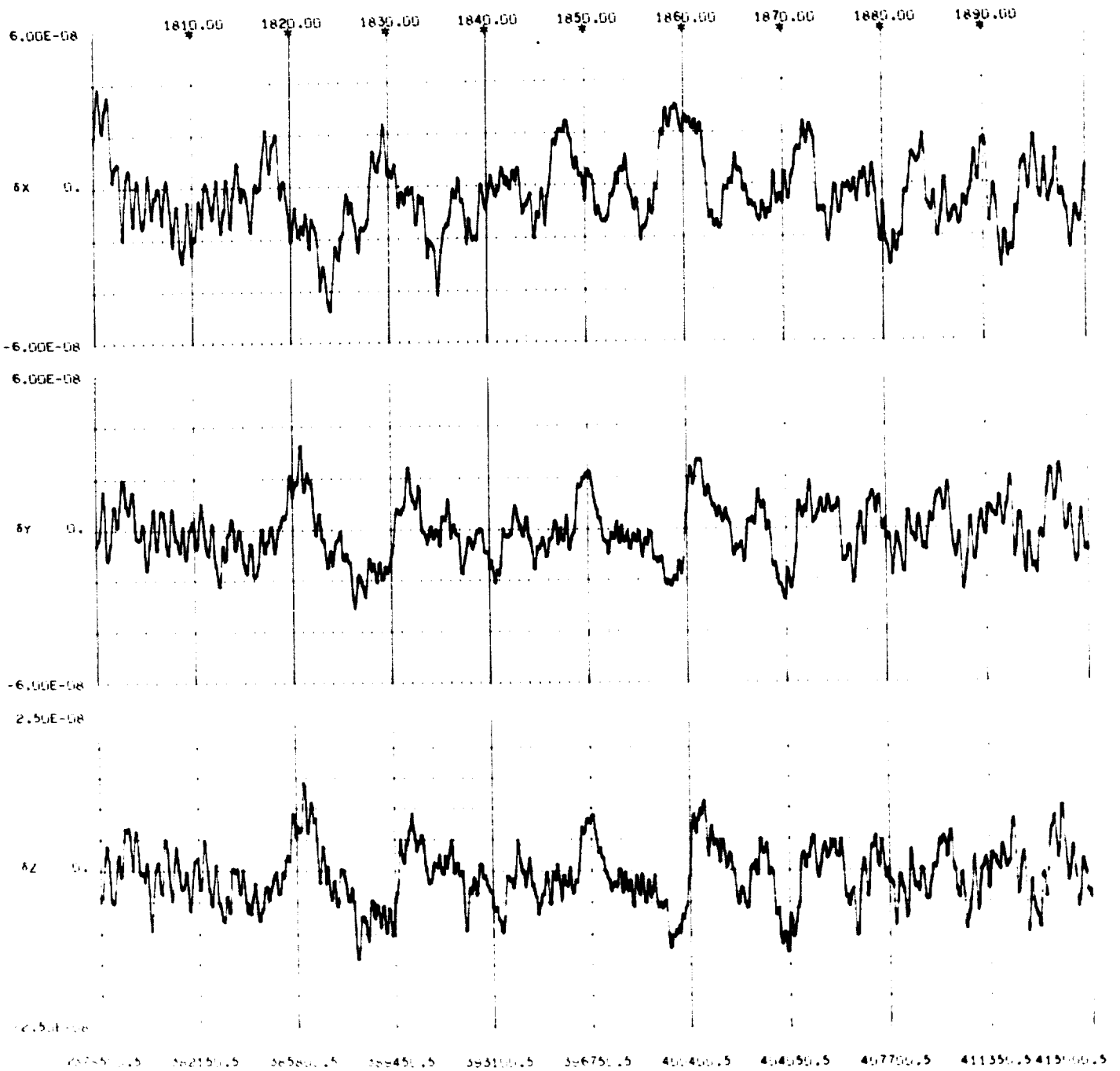


Fig. D-4 (contd)



**Fig. D-5. Jupiter rectangular residuals (improved modified source minus DE28) for 1800–2000**

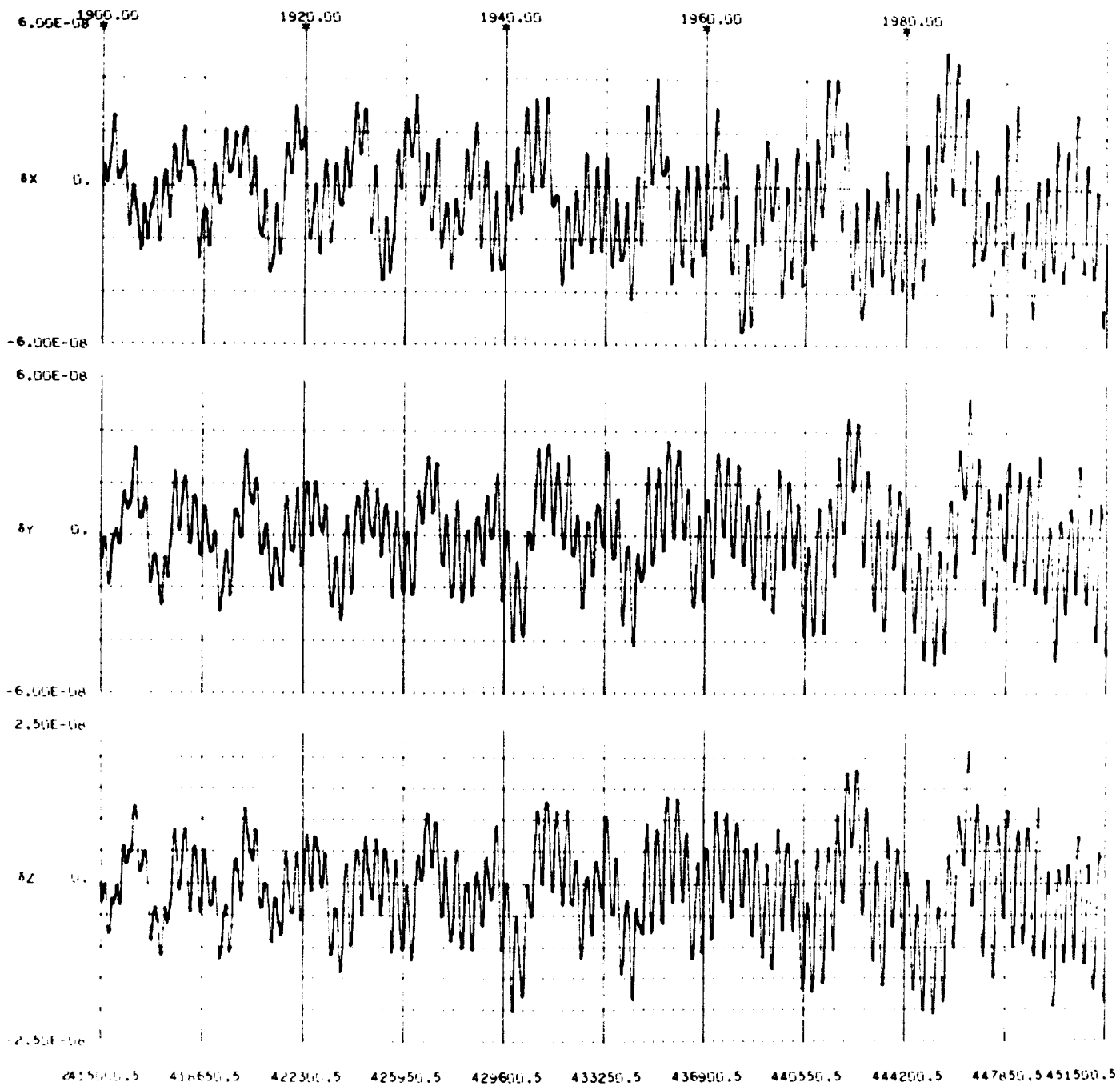


Fig. D-5 (contd)

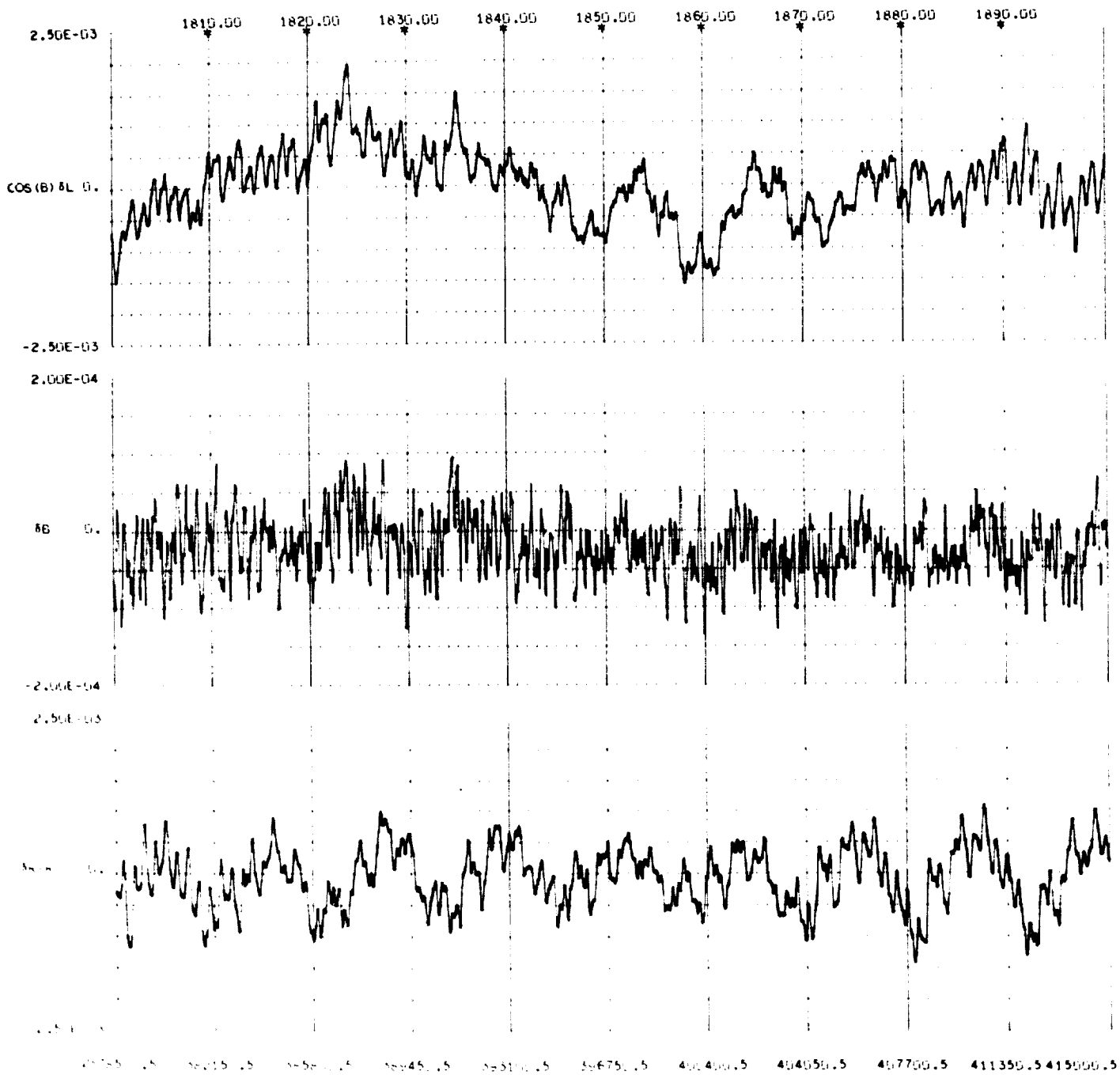


Fig. D-6. Jupiter angular residuals (improved modified source minus DE28) for 1800–2000

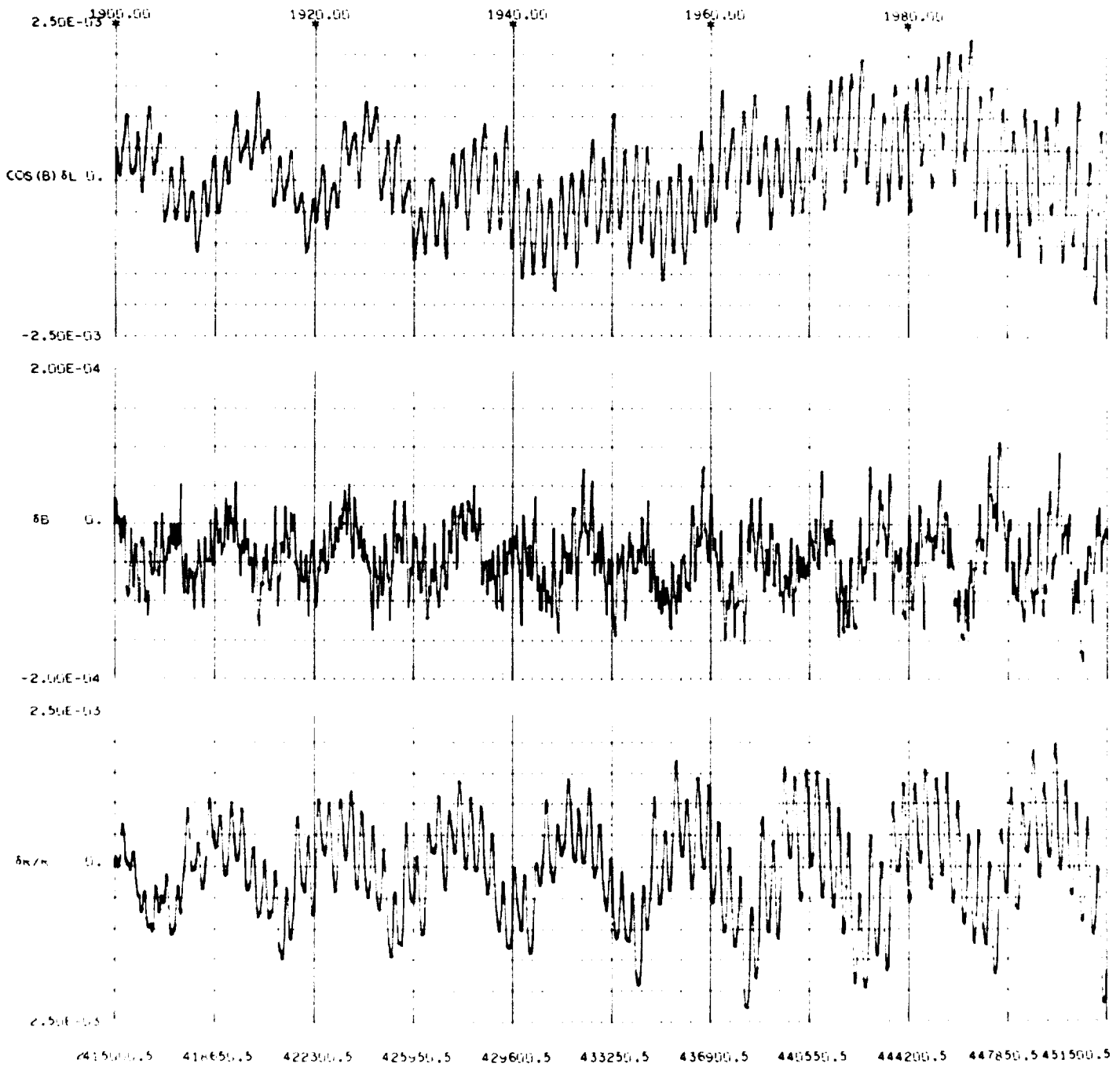


Fig. D-6 (contd)

**Table D-1. Improved modified source statistics for Venus**

Corrections applied to MS ephemeris					
$\Delta e = -(0.0668 \pm 0.0004) T$ $\Delta \tilde{\omega} = -(7.43 \pm 0.06) T$ $\Delta(\sin I \sin \Omega) = +(0.010 \pm 0.001) T$ $\Delta(\sin I \cos \Omega) = -(0.0002 \pm 0.0009) T$					
Statistical summary (IMS minus DE28)					
Residuals in position	Mean square residuals	Mean residuals	Standard deviations	Maximum residuals	Minimum residuals
$dx, AU$	$3.94E-14$	$8.82E-09$	$1.98E-07$	$7.05E-07$	$-8.17E-07$
$dy, AU$	$3.42E-14$	$5.71E-09$	$1.85E-07$	$7.18E-07$	$-7.78E-07$
$dz, AU$	$1.38E-14$	$3.54E-09$	$1.18E-07$	$4.18E-07$	$-4.03E-07$
$\cos B \, dl, ''$	$5.71E-03$	$1.41E-04$	$7.56E-02$	$2.66E-01$	$-2.36E-01$
$dB, ''$	$7.77E-04$	$4.44E-04$	$2.79E-02$	$1.01E-01$	$-1.05E-01$
$dr/r, ''$	$6.19E-04$	$-6.96E-03$	$2.39E-02$	$8.73E-02$	$-9.40E-02$

**Table D-2. Improved modified source statistics for earth-moon barycenter**

Corrections applied to MS ephemeris					
$\Delta e = -(0.0164 \pm 0.0004) T$ $\Delta \tilde{\omega} = +(1.19 \pm 0.03) T$ $\Delta(\sin I \sin \Omega) = +(0.0002 \pm 0.0009) T$ $\Delta(\sin I \cos \Omega) = -(0.0057 \pm 0.0009) T$					
Statistical summary (IMS minus DE28)					
Residuals in position	Mean square residuals	Mean residuals	Standard deviations	Maximum residuals	Minimum residuals
$dx, AU$	$7.33E-14$	$1.16E-08$	$2.71E-07$	$1.17E-06$	$-1.04E-06$
$dy, AU$	$6.75E-14$	$-2.70E-08$	$2.58E-07$	$1.50E-06$	$-8.42E-07$
$dz, AU$	$3.17E-14$	$2.42E-10$	$1.78E-07$	$6.39E-07$	$-6.50E-07$
$\cos B \, dl, ''$	$4.96E-03$	$9.76E-05$	$7.05E-02$	$2.31E-01$	$-3.34E-01$
$dB, ''$	$1.07E-03$	$2.26E-03$	$3.27E-02$	$1.13E-01$	$-9.81E-02$
$dr/r, ''$	$1.32E-03$	$-1.04E-02$	$3.48E-02$	$1.41E-01$	$-1.46E-01$



**Table D-3. Improved modified source statistics for Jupiter**

Corrections applied to MS ephemeris					
$\Delta e = -(0''.00183 \pm 0''.00001) T$ $\Delta \vec{\omega} = +(0''.0099 \pm 0''.0003) T$ $\Delta(\sin I \sin \Omega) = -(5''.0 \pm 31''.0) \times 10^{-6} T$ $\Delta(\sin I \cos \Omega) = +(1''.3 \pm 0''.3) \times 10^{-4} T$					
Statistical summary (IMS minus DE28)					
Residuals in position	Mean square residuals	Mean residuals	Standard deviations	Maximum residuals	Minimum residuals
$dx, \text{ AU}$	2.98E-16	-3.88E-09	1.68E-08	5.28E-08	-5.48E-08
$dy, \text{ AU}$	2.25E-16	-9.20E-10	1.50E-08	5.27E-08	-4.91E-08
$dz, \text{ AU}$	3.98E-17	-1.11E-09	6.21E-09	2.09E-08	-2.06E-08
$\cos B \, dL, ''$	4.59E-07	3.45E-05	6.77E-04	2.28E-03	-2.00E-03
$dB, ''$	3.43E-09	-3.29E-05	4.85E-05	1.06E-04	-1.85E-04
$dr/r, ''$	4.31E-07	-6.66E-05	6.53E-04	1.99E-03	-2.28E-03

