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Solution of Differential Equations by Application of Transformation Groups

A report has been prepared dealing with the application of transformation groups to the solution of systems of ordinary differential equations and, in particular, partial differential equations. Although these groups are Lie groups in the usual sense, their transformation properties rather than their basic structures are applied.

The principal theorem, referred to as Lie's theorem, gives a method for finding an integrating factor for a system of ordinary differential equations when the appropriate invariance group or groups can be found. Lie's theorem can be extended to partial differential equations by considering a partial differential equation as a continuously infinite system of coupled ordinary differential equations. For a system of ordinary differential equations the integrating factor is a matrix. For a partial differential equation the integrating factor is then a continuously infinite matrix.

The proof of Lie's theorem and its use for partial differential equations depends on constructing an adequate theory of continuously infinite matrices; this is done through the use of distributions or generalized functions.

Chapter II of the report (Chapter I is the introduction) treats systems of ordinary differential equations. Lie's theorem is derived in such a way that it can be readily applied to the discrete approximation of a partial differential equation. Examples given are the discrete approximation to the heat flow and wave equations considered as initial value problems, and it is shown that the limiting form of the solutions obtained are those given by other more familiar techniques. In Chapter III, Lie's theorem is derived for and applied directly to partial differential equations, without the necessity of using a discrete approximation. The heat flow equation is

again used as an example. While the examples given are linear equations, there is nothing in the method that restricts it to linear problems. Lie's theorem can in principle be applied to nonlinear partial differential equations, but in practice it has been difficult to find a nonlinear example.

Chapters II and III are oriented towards the engineer, physicist, or chemist whose prime interest is the solution of practical problems. The proofs or derivations here are not rigorous.

In the Appendix, however, an effort is made to achieve rigor in the proofs given. It is here that the foundation of a theory of continuously infinite matrices based on distribution theory and generalized functions is given.

While the literature has not disclosed applications of Lie's theorem for the solution of partial differential equations (or even for systems of ordinary differential equations), it is unlikely that the material presented in the report is completely new.

Note:

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No patent action is contemplated by NASA.

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