NASA TECH BRIEF

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Concept for Simplified Serial Digital Decoder

The problem:

To develop a method of decoding the first order Reed–Muller codes. The decoding apparatus should function as a maximum-likelihood exhaustive-search decoder, and should be a modular implementation to accommodate codes of any length.

The solution:

A modular decoder that lends itself best to special purpose digital equipment using sequential access memories.

How it's done:

The decoder considers a code or matrix. Multilevel symbol confidence levels are considered in a serial decoding scheme. The structure of the dictionary is as follows:

$$A = \begin{bmatrix} +1 & +1 \\ +1 & -1 \end{bmatrix}; \quad B = \begin{bmatrix} A & A \\ A & \bar{A} \end{bmatrix}; \quad N = \begin{bmatrix} (N-1) & (N-1) \\ (N-1) & \overline{(N-1)} \end{bmatrix}$$

The decoding problem is best described by an example.

a,	b,	c,	d		
+1	+1	+1	+1	=	a + b + c + d
+1	−1	+1	−1	=	a − b + c − d
+1	+1	−1	−1	=	a + b − c − d
+1	−1	−1	+1	=	a − b − c + d

In the exhaustive search through the original dictionary a received word (a, b, c, d) is multiplied symbol by symbol with each entry in this dictionary. The result is shown to the right of each entry in the dictionary. A search is then performed between the resultant correlation values. The largest value indicates the word most likely to have been transmitted. For k variables (information bits) the original dictionary contains 2^k code words each including 2^k symbols. A maximum likelihood exhaustive search through this dictionary requires 2^{2k} additions and sorting of 2^k correlation values. However, it can be shown that the original dictionary, or matrix, can be factored into the product of k different matrices. Each of these matrices has only 2^{k+1} nonzero entries and is distinct from each of the other k–1 matrices. Thus, the decoding scheme requires only $k \cdot 2^k$ additions. The same number of resultant values (2^k) have to be sorted for the final result.

The decoding scheme, applied to the previous example, is shown below:

a,	b,	c,	d		
+1	+1	0	0	=	a + b
+1	−1	0	0	=	a − b
0	0	+1	+1	=	c + d
0	0	+1	−1	=	c − d

a + b	a − b	c + d	c − d		
+1	0	+1	0	=	(a + b) + (c + d)
0	+1	0	+1	=	(a − b) + (c − d)
+1	0	−1	0	=	(a + b) − (c + d)
0	+1	0	−1	=	(a − b) − (c − d)

(continued overleaf)

This technique is continued. For k = 3 factor matrices are shown below:

$$\begin{matrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{matrix} \qquad \mathbf{1}$$

$$\begin{matrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{matrix} \qquad \mathbf{2}$$

$$\begin{matrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{matrix} \qquad \mathbf{3}$$

Each factor matrix can be implemented by a digital module, the block diagram of which is shown below.

The value of i ranges from 1 through k. The signals W_{i-1} and $\overline{W}_{i-1}$ are the outputs of the *ith* stage of a digital counter, the first stage, W_0, of which receives a new count pulse at the start of each new symbol of the received code word.

Finally, if the modules are cascaded in increasing order of i, the output of the *ith* module decodes a code arising from i variables. Thus, if a decoder for k variables exists and it becomes necessary to decode a code arising from k + 1 variables, only one new stage, corresponding to i = k + 1, and one stage to the W counter need be added.

Notes:

1. This development was designed, constructed, and approved for the Mariner project.
2. Inquiries concerning this development may be directed to:

   Technology Utilization Officer
   NASA Pasadena Office
   4800 Oak Grove Drive
   Pasadena, California 91103
   Reference: B68-10045

Patent status:

This invention is owned by NASA, and a patent application has been filed. Royalty-free, nonexclusive licenses for its commercial use will be granted by NASA. Inquiries concerning license rights should be made to NASA, Code GP, Washington, D.C. 20546.

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